Students' Conceptions About Rolling In Multiple Contexts

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Abstract. Research has shown that students have several misconceptions about rotational motion and rolling [1]. Students often do not understand the relationship between the speeds of various points on a rolling wheel. They also do not understand the relationship between the translational and rotational speeds of a wheel that rolls without slipping. We conducted a study to extend existing research on this topic. Specifically, we explored the reasoning resources that students used with regard to rolling without slipping in three different contexts: a single rolling bicycle wheel, a horizontal plank pushed forward on a rolling drum, and two differently sized wheels in a penny-farthing bicycle. We explored the reasoning resources used by students in two different introductory physics classes at two Midwestern universities. We describe students' reasoning resources about rolling in these different contexts.

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INTRODUCTION

Almost every first semester introductory physics course covers the topic of rolling motion. However, research by Rimoldini and Singh is the only published study that we found on this topic [1]. When asked to compare the velocities of the point at the top and bottom of a rolling wheel, very few students answered the question correctly. Further probing showed that students were uncomfortable with relative motion and unable to shift their reference frame while comparing different velocities. They also seemed to misunderstand what rolling without slipping meant.

A problem that students answered in the Rimoldini and Singh study was about a single rolling wheel. We examine students' conceptions about rolling in three different contexts starting with a context similar to Rimoldini and Singh and expanding to two other contexts. Exploring students' conceptions in different contexts can provide insights into students' reasoning. Our research question is: In what ways do students reason about rolling in these different contexts?

THEORETICAL FRAMEWORK

We adopt the theoretical stance of characterizing student responses in terms of the reasoning resources they use while answering questions. A resource is anything a student uses to answer the question [2]. In using this framework, we do not presuppose students' consistency across different contexts. Rather, students can potentially use different resources to answer questions having the same scientific explanation.

METHODOLOGY

We collected data in two phases. Participants in Phase I were future engineers in a large enrollment calculus-based course at a Midwestern university. Problem 1 (Fig. 1), -- 'Single Wheel', was part of an online homework assignment. Problem 2 (Fig. 2), -- 'Plank on Drum', was on the third of five tests taken by the class. About one-third of 312 students enrolled attempted Problem 1 and about half attempted Problem 2. Since there were multiple versions of the homework and test, not all students attempted both problems. The homework and test were given a week after the topic was covered in class. Problem 3 (Fig. 3) -- 'Penny-Farthing' was posed to 13 students in an interview about two weeks after the test.

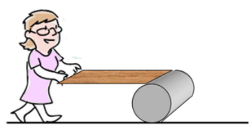
Participants in Phase II were future architects in a large enrollment algebra-based course at a different Midwestern university. We gave a survey with all three problems to about 200 students in the class. Students received extra-credit worth about 1% of the course grade for completing the survey. Unlike Phase I, in Phase II students got all three problems on a single survey so we could investigate the consistency of reasoning resources across contexts. Students in both Phases had completed instruction on rolling and relative velocities before starting this research.

It was *not* our goal to report on the prevalence of various resources. Consistent with our goal, we used phenomenographic analysis [3] which investigates variations in the ways in which students describe their experiences or ideas. We coded students' responses and grouped these into categories based on their meanings. We had no a priori categories. Categories and themes emerged from our analysis. We used this process for each problem.

You and your friend are on a bike ride. On a flat section of the road you chat about the speeds of points on the bicycle wheel with respect to the road. Your friend asks, "At any instant, how does the linear velocity with respect to the ground of the point at the top of the wheel compare with the linear velocity with respect to the ground of the point at the bottom of the wheel?" What is your answer? Explain your reasoning.

FIGURE 1. Problem 1 ('Single Wheel' problem) is similar to the one used by Rimoldini and Singh [1].

You take your kid sister for a walk through the park. For fun, she takes a plank, places it on top of a cylindrical drum and pushes the plank as shown so that the drum rolls forward.



The plank moves along the top edge of the drum without slipping. She asks, "If I want to move the drum two feet forward, how many feet forward would I have to push the plank?" What is your answer? Explain your reasoning.

FIGURE 2. Problem 2 ('Plank on Drum')

You and your friend see a person riding down the street in a penny-farthing bicycle shown. Your friend asks, "At any instant, how does the linear velocity of the

point at the top of the front wheel compare with the linear velocity of the top point of rear wheel?" What is your answer? Explain your reasoning.

FIGURE 3. Problem 3 ('Penny-Farthing')

RESULTS & DISCUSSION

We describe the categories of reasoning resources emerging from the phenomenographic analysis for each problem. Where relevant, we discuss differences between Phase I and Phase II data. We finally compare the resource categories across problems.

Problem 1 ('Single Wheel')

We found two emergent themes in this context, We discuss the reasoning resources that students seemed to use to support these conclusions. Then we compare the resources used across these themes.

Equal Velocities

Students used the following resources to reason that the top and bottom points had the same velocities.

Same radius: The points are at the same distance from the wheel's center so they have the same velocity.

Same unit: The points are part of the same unit, i.e. the wheel, so they move with the same velocity.

Same diameter: The points are are on the same diameter, i.e. directly above one another.

The underlying resource in all these is *sameness*, i.e. two points have something that is the same between them, hence they move at the same speed.

Bottom Point at Rest

Several students who stated that the bottom point of the wheel was at rest used the following resources.

Rolling without slipping: The wheel rolls without slipping so the bottom point is at rest.

Rotating about the point at the bottom: The wheel is in fact rotating about the point at the bottom, so the bottom of the wheel remains stationary.

Radius of point is zero: The point on the wheel right above the ground has radius 0, therefore the linear velocity at this point will equal zero.

Summing velocities: The translational and rotational velocities are equal and opposite at the bottom point, so they cancel out making the bottom point stationary, and the top point is moving at twice the speed of the wheel.

While the last resource is closest to a complete scientifically correct explanation, the other resources also have elements of correct reasoning, such as recognizing how shifting the reference frame can help visualize why the bottom point is at rest.

Comparing Resources across Themes

The 'equal velocities' theme was much more prevalent in the algebra-based (Phase II) data while the 'bottom point at rest' theme was more prevalent in the calculus-based (Phase I) data. In interviews and homework responses some students referred to instructors' statements in class asserting that the bottom point of a rolling wheel was always at rest. Many students seemed to accept this fact. When asked to provide a reason, they used the resources above.

Problem 2 ('Plank On Drum')

We found three emergent themes in this context .

Push The Same Distance

Students who stated that the plank should be pushed the same distance as the drum used the following resources:

Moving together: The plank and the drum move forward *together*, and therefore should cover the same distance. Some variations of this resource referred to the objects moving at the same speed and therefore the distance moved was the same. This resource appears to be somewhat similar to the *same unit* resource that was used by students in Problem 1.

No slip: The plank is not slipping on top of the drum and therefore they had to move the same distance. *Friction* was sometimes cited as a cause for the plank not slipping on the drum.

Circumference: The circumference of the drum seemingly unfolds along the plank, thus they move the same distance.

Push Farther

Some students stated that the plank should be pushed farther than the distance the drum moves, although they were not clear about how far the plank ought to be moved. They used the following resources:

Circumference: The point of contact moves around the circumference of the drum; it travels a greater distance than the drum. This resource was at a surface level similar to the *circumference* resource above in that both referred to the circumference of the drum and going around it. However, students came to very different conclusions based on this observation. Some used the resource to justify that the plank should move the same distance as the drum, while others used it to justify that the plank should move a farther distance than the drum, although they did not specify how much farther.

No slip: The plank is not slipping on top of the drum and is therefore similar to the wheel problem, the plank must move twice the distance than the drum. We found evidence of students explicitly referring to the 'Single Wheel' problem and pointing out similarities while answering the question. In other words, we find evidence of transfer i.e., activating the same resource used on the homework in an exam.

Combination of motion: The drum undergoes rotation and translation, so the plank should be moved to account for both i.e., twice as far as the drum. Variations of this resource referred to linear and angular momentum instead of rotational and translational motion, or rotational and translational energies, rather than motion. All these forms of reasoning shared the notion that the plank's motion must account for two kinds of motion and thus it should move twice as far as the drum.

It Depends On…

Some students did not provide a specific answer to how far the plank ought to be pushed. Rather, they pointed to features of the problem that the answer would depend upon:

Radius: Several students cited that the distance travelled would depend upon the radius of the drum. The larger the drum the farther the plank would have to be pushed.

Circumference: Some students did not refer to the radius of the drum, but rather to the circumference. We contend that both of these students were referring to the notion of the *size* of the drum, but cited different features of the drum as influencing their answer.

Comparing Resources Across Themes

The 'push the same distance' and 'depends upon' themes were much more prevalent in algebra-based (Phase II) data while the 'push farther' theme was more prevalent in the calculus-based (Phase I) data.

Another interesting observation is that students might use the same resource but arrive at completely different results. For instance, we have shown how the *no slip* resource was used to justify both the 'push the same distance' as well as the 'push farther' conclusion. Similarly, the *circumference* resource was used to justify any of the three conclusions as described above.

Problem 3 ('Penny-Farthing')

We found four emergent themes in this context.

Larger Is Faster

Students used the following resources to reason that the top of the larger wheel has a greater linear velocity than the top of the small wheel.

Radius: The larger the radius, the larger the linear velocity because linear velocity is angular velocity times radius, so the top point on the larger wheel moves faster.

Covers more ground: The larger wheel covers more ground in a single rotation, thus the point atop it should have a greater linear velocity.

Smaller Is Faster

Students used the following resources to reason that the top of the smaller wheel has a greater linear velocity than the top of the larger wheel.

Radius: The smaller the radius, the smaller the distance from the axis of rotation, so the faster the wheel rotates. The top point on the smaller wheel moves faster than the top point on the larger wheel.

Travels more to keep up: The smaller wheel rotates faster than the larger wheel to keep up with it. The top point on the larger wheel moves faster than the top point on the smaller wheel.

Different

Students used the *radius* resource to conclude that the linear velocities of the points at the top of the two wheels are different. They made statements noting that the radii are different and must be accounted for, but they did not elaborate on whether it would make the linear velocity atop one of the wheels greater than the linear velocity atop the other wheel.

Same

Students who concluded that the linear velocities at the point atop the two wheels are equal used the following resources:

Move together: The two wheels move together as part of the same system or the same unit and therefore points atop the wheels should have the same linear velocities. This is akin to the *move together* resource used in Problem 2 and the *same unit* resource used in Problem 1.

Same distance, same time: The two wheels cover equal distances in equal times and thus have the same speed. This is akin to the *sameness* resource cited by students in Problem 1.

Comparing Resources Across Themes

There are two interesting features in our resources above. First, we find that the *radius* resource appears across three of the four themes. Students who used this resource did not all come to the same conclusions. Rather, depending upon how they combined it with other resources, they arrived at different conclusions.

Comparing Resources Across Problems

We also find that several resources appeared across different problems. Most prominent among these is the *radius* resource that was commonly used in Problems 2 and 3, but in different ways and often to reach different conclusions in the same problem. We also find that the *same unit* and *move together* resource appear across all three problems. These were typically used to justify conclusions pointing to traveling at the same speed or moving the same distance.

We examined our data in Phase II in which all students were presented with all three problems. However, we found no consistent pattern of use of resources in that students who used a particular resource in a particular way in a problem did not use it the same way in other problems. This result is consistent with the notion that students' reasoning resources change with context. Interestingly, we did find some, albeit limited evidence that students in Phase I attempting Problem 2 on the exam referred back to their reasoning on Problem 1 which they had answered on the homework. This seems to indicate that students are capable of transfer i.e., activating resources consistently across different contests.

CONCLUSION & IMPLICATIONS

Rimoldini and Singh [1] report the following difficulties "(i) They were not comfortable with linear relative motion concepts, and (ii) They did not understand the meaning of rolling without slipping." We do not dispute these assertions, but we submit that these descriptions may not adequately help us understand how students construct their reasoning and what instructional strategies may be most useful in helping them develop an understanding of rolling.

It is also important to note that while we call these reasoning elements 'resources', the resources that Hammer [2] cited were often much more primitive than those discussed here. Nevertheless, a resource-based analysis such as the one presented here can enable to us explore how and context triggers different resources. This is a topic for future study. Furthermore, such investigations would help us find ways to leverage resources that students naturally tend to activate while attempting to answer these questions and design prompts that enable them to use these resources more productively as they construct their reasoning. An example of such an approach is shown by Hammer and Elby. [4] Through appropriately designed questions, they facilitate students to refine their intuition about the forces experienced by a colliding car and truck and begin to reconcile their intuitive ideas with those of Newton's third law. We propose that future studies can explore similar strategies that facilitate students to refine their own intuitive reasoning resources about rolling to improve their conceptual understanding.

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