Thinking about Factors that Affect the Difficulty of a Physics Problem

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Brainstorm

What factors affect the difficulty of a physics problem?

Some things I came up with…

Structure of a Problem/ Type of Problem

- "Textbook" problem
- Ranking task
- Jeopardy question
- Conceptual question
- Multiple Choice
- Multiple representations
- Unstructured problem

Domain of problem

- Inclined planes
- Dynamics
- Circuits
- Quantum Mechanics

Some things I came up with…

Number of steps involved in the solution

Relevant Math

- Algebra
- Calculus
- Vector algebra
- Vector calculus
- Abstract vector spaces
- Complex numbers

What knowledge the solver has

- Intro physics student
- Upper-division physics student
- Graduate student
- Professional Physicist

Accessibility of knowledge

- Relevant knowledge
- Timing of problem

Estimations of Problem Difficulty?

- Star ranking in textbooks
- University of Minnesota – rating the difficulty of context rich problems

Rating Difficulty of Context-Rich Problems

3. Check* for the eleven characteristics that make a problem more difficult:

- unfamiliar context
- hard to learn physics (e.g., circular motion, rolling friction, waves, Gauss's Law)
- more than one approach is needed to solve the problem (e.g., force or kinematics)
- more than two subparts are needed to solve the problem (e.g., two separate force diagrams then onto kinematics)
- the target variable is not specified (i.e., more than one correct way to solve the problem)
- more information is given than needed to solve the problem
- needed information is missing
- assumptions (idealizations) must be made to solve the problem
- the solution involves vector components
- finding the target variable requires trigonometric identities
- the solution requires simultaneous equations or calculus (i.e., non-constant variables)

* As you become more sophisticated, you can give these difficulty characteristics weightings of 0, 1 and 2 instead of simple checks. For example, a problem that requires both the conservation of energy and momentum (weighting of 1) is easier than a problem that requires both circular motion and energy concepts (weighting of 2).

Intrinsic Complexity

Properties of the Problem

- Amount of information given in the problem
- Type of information given in the problem
- Amount of math manipulation involved in the solution
- How well defined is the solution of the problem?
- How many ways can an answer be achieved?

Situated Difficulty

Interaction Between Problem & Solver

- What knowledge does the solver possess that they see as relevant to the problem?
- What knowledge does the solver possess that can be used to solve the problem?
- How familiar is the context? How much interpretation is needed for the solver to parse the problem/create a problem schema?

Thinking about Intrinsic Complexity

'You are asked to divide the story *The Red Balloon* into *episodes*. An episode is an incident in the course of the story which consists of three parts:

1. *Exposition* the setting forth of the initial facts in the episode. It includes the background or surrounding.
2. *Complication* a predicament or dilemma confronting the principal character in an episode and making necessary some decision or action.
3. *Resolution* when the predicament or dilemma is solved or resolved for the principal character, either by his or her actions or by the actions of others, or by fate.'

- Baggett, 1979 – degree of agreement in episodic structure of a story presented in movie and in text.

An ECR Framework for Physics Problems

	Story	Physics Problem
Exposition	Setting forth of initial facts	Set-up of the problem, given quantities
Complication	Predicament or dilemma necessitating a decision or action	The problem statement – what needs to be calculated, the question that needs to be answered.
Resolution	When the predicament or dilemma is solved or resolved	The answer, the value of the quantity that was calculated (numeric or symbolic)

Questions

- Can an ECR framework be used to characterize the intrinsic complexity of a physics problem?

- Can an ECR framework be used to describe a person’s performance in solving a physics problem?

- Compare a scoring of a problem with an ECR rubric with other estimations of problem difficulty.

(Controlling for as much of the situated difficulty as possible)

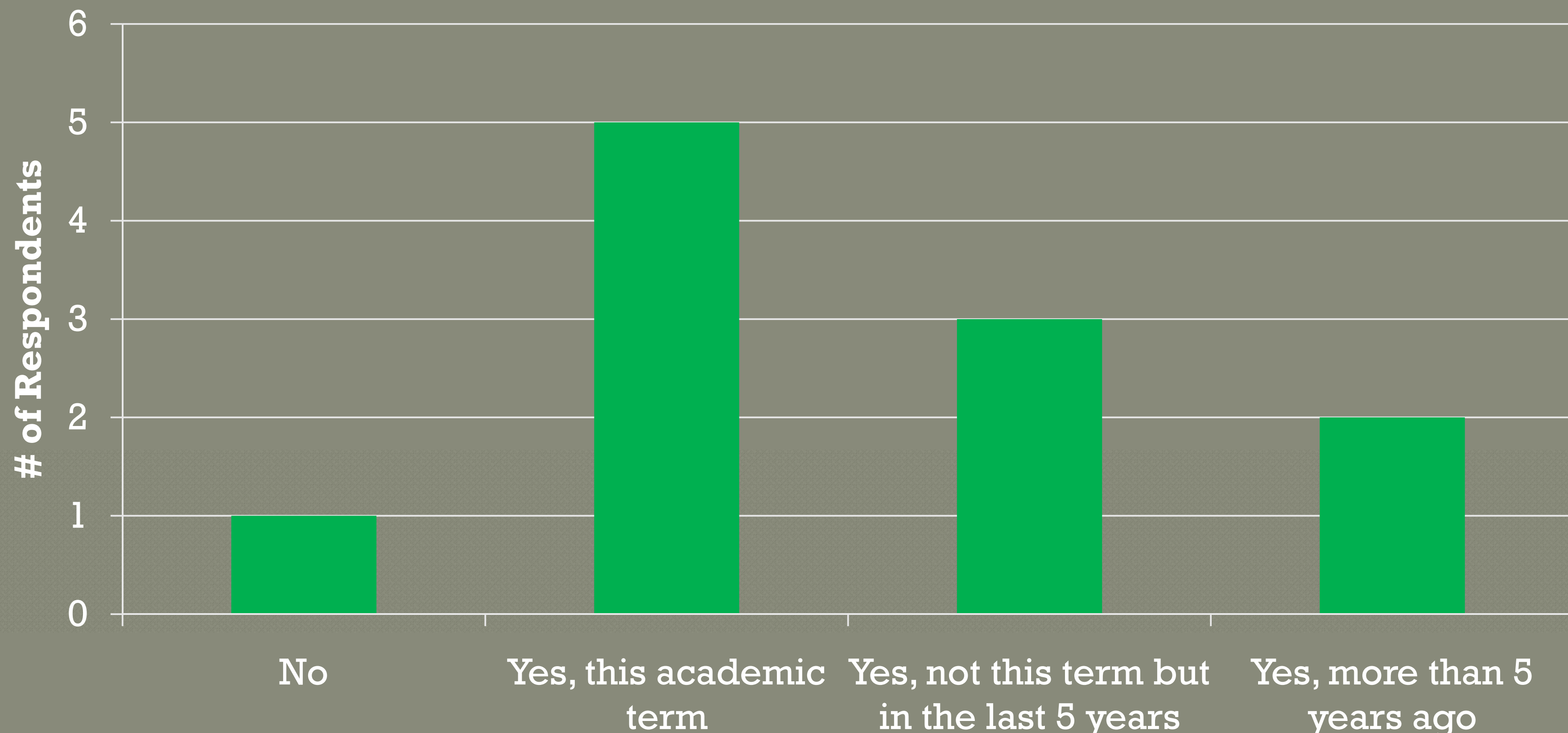
Survey of Problem Difficulty Estimation

- 16 problems
- Halliday & Resnick, 7th Ed.
- Work & Energy
- Single part textbook questions
- 12 problems Likert-Scale

  "How difficult is this problem (for a calc-based intro student)?"
- 4 problems "Give Steps in Solution" & rank difficulty on Likert Scale
- Faculty, EP1 & 2 Studio Instructors, and PER group (n=38 invited, n=11 participated)

Results

Have you ever taught in the context of calculus-based introductory physics?



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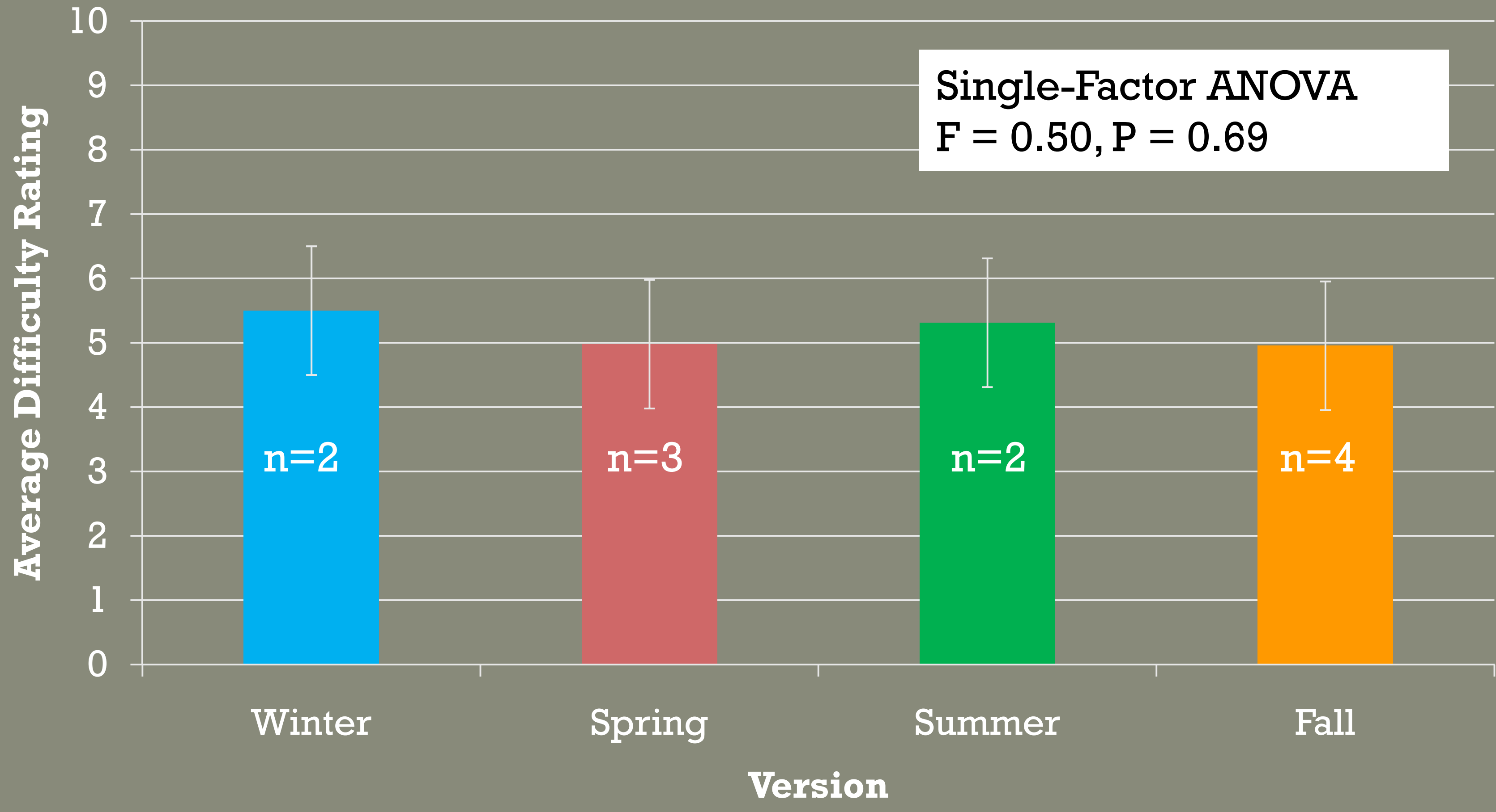
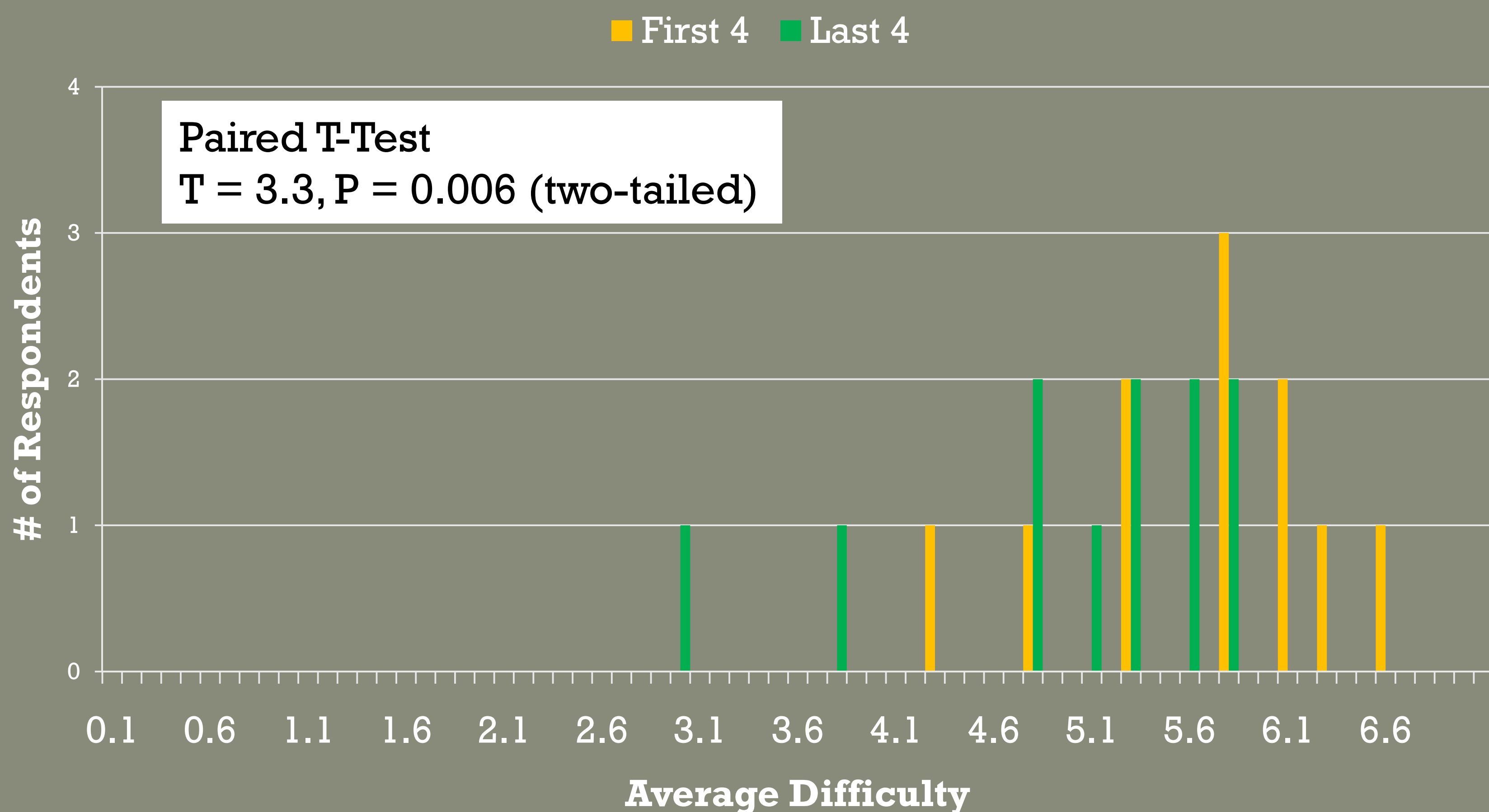


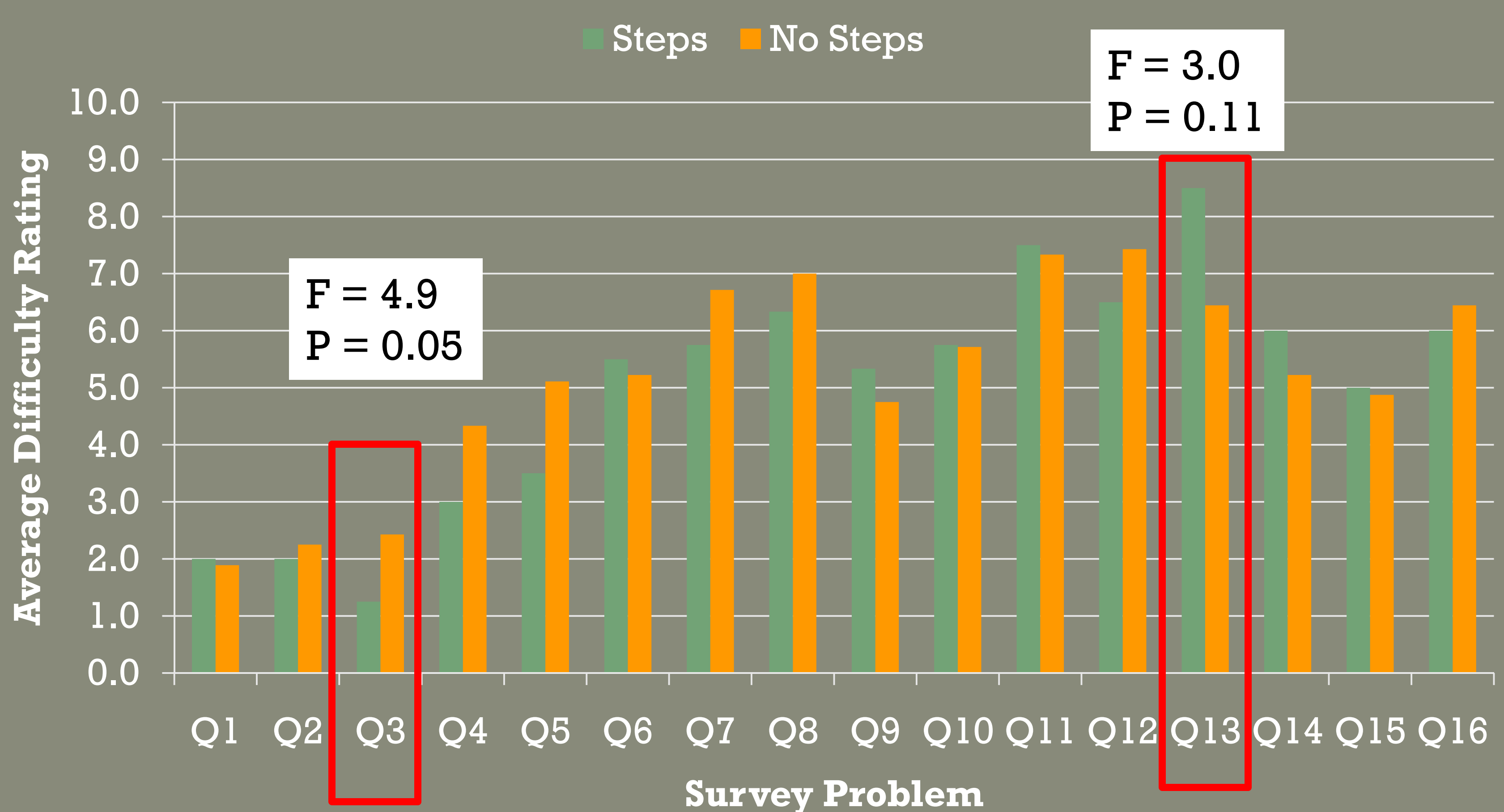
Single-Factor ANOVA
F = 0.50, P = 0.69
Average Difficulty Rating
10
9
8
7
6
5
4
3
2
1
0
n=2
n=3
n=2
n=4
Winter
Spring
Summer
Fall
Version

Average Difficulty Rating Per Problem (St Dev)

First and Last Problems Rated



Solved vs. Ranking Alone



Representation

Word

A 1 kg block is attached to a spring and is oscillating. If the spring constant is k = 200 N/m and the block's speed is 1.2 m/s when the spring compressed 5 cm from its relaxed length,

what is the mechanical energy of the system?

Equation

A single conservative force $F(x)$ acts on a 1.0 kg particle that moves along an x axis. The potential energy $U(x)$ associated with $F(x)$ is given by

$$U(x) = -4x\, e^{-x/4} \text{ J}$$

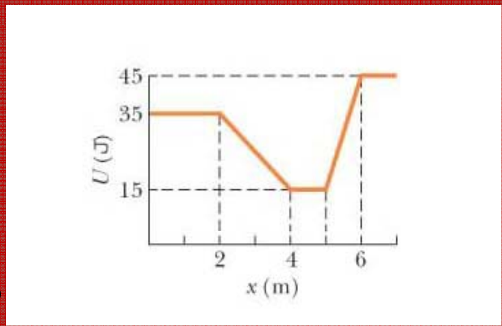
where x is in meters. At x = 5.0 m the particle has a kinetic energy of 2.0 J.

What is the mechanical energy of the system?

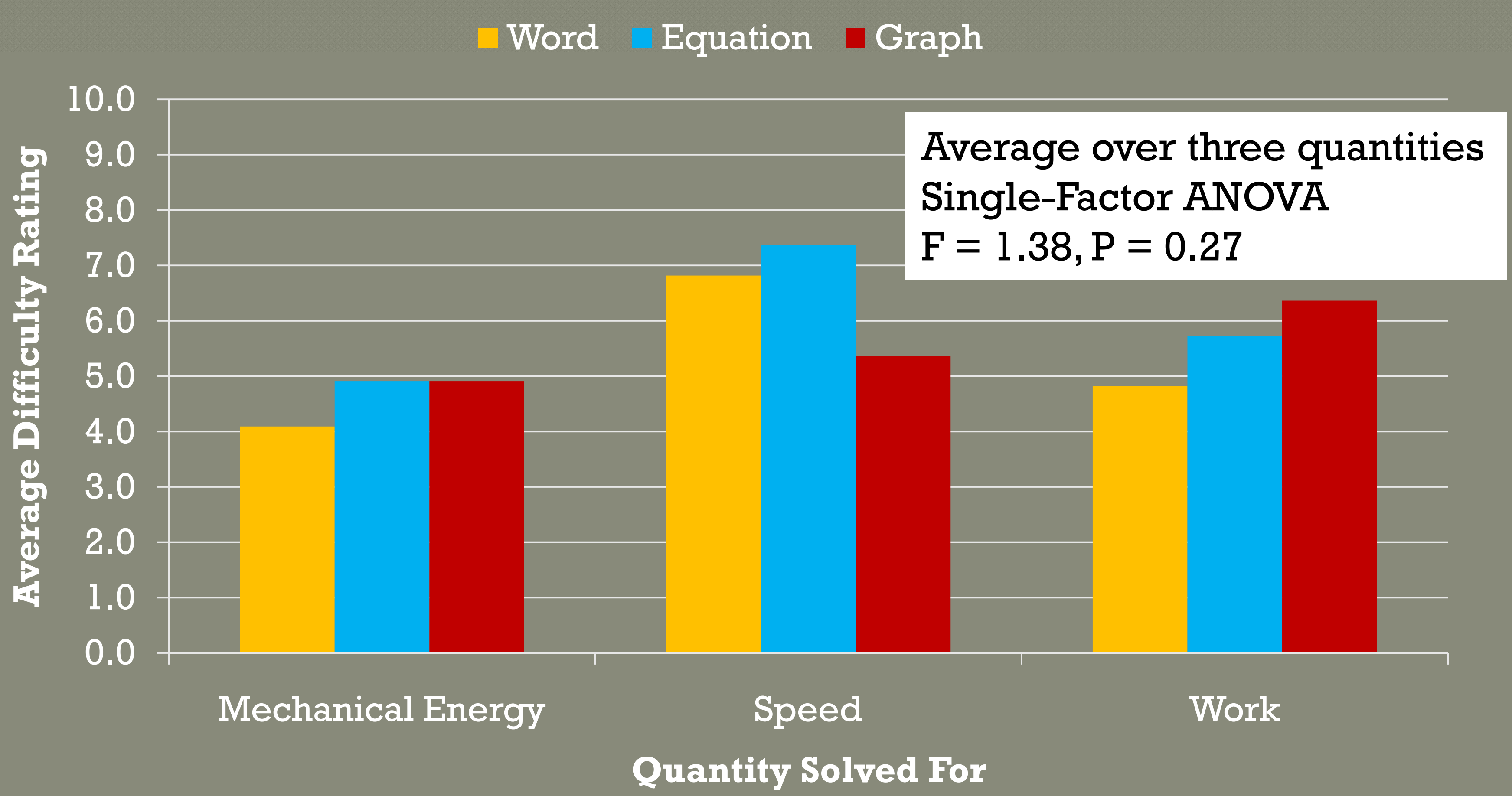
Graph

The figure shows a plot of potential energy U versus position x of a 0.90 kg particle that can travel only along the x axis. (Non-conservative forces are not involved.) The particle is released at x = 4.5 m with an initial speed of 7.0 m/s, headed in the negative x-direction.

What is the mechanical energy of the particle?



Representation



ECR Rubric

Code	Heuristic	Comments
Exposition	For each representation in problem statement	Can occur in solution?
Complication	Any decision that needs to be made	Unpaired? Explicit? Multiple?
Resolution	Any decision that has been made	Unpaired?

Reliability Study:

6 Solved Textbook Examples for Training
10 Solved Textbook Examples for Reliability

Problems from Halliday & Resnick (7th Ed.) and Knight

Sample Problem 7-5

An initially stationary 15.0 kg crate of cheese wheels is pulled, via a cable, a distance $d = 5.70$ m up a frictionless ramp to a height h of 2.50 m, where it stops (Fig. 7-9*a*).

(a) How much work W_g is done on the crate by the gravitational force $\vec{F}_g$ during the lift?

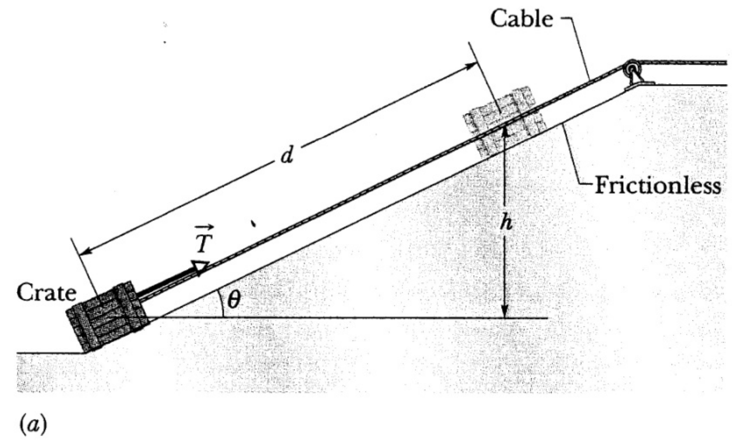
Solution: A **Key Idea** is that we can treat the crate as a particle and thus use Eq. 7-12 ($W_g = mgd\cos\phi$) to find the work W_g done by $\vec{F}_g$. However, we do not know the angle ϕ between the directions of $\vec{F}_g$ and displacement $\vec{d}$. From the crate's free-body diagram in Fig. 7-9*b*, we find that ϕ is $\theta + 90°$, where θ is the (unknown) angle of the ramp. Equation 7-12 then gives us

$$W_g = mgd\cos(\theta + 90°) = -mgd\sin\theta, \qquad (7\text{-}18)$$

where we have used a trigonometic identity to simplify the expression. The result seems to be useless because θ is unknown. But (continuing with physics courage) we see from Fig. 7-9*a* that $d\sin\theta = h$, where h is a known quantity. With this substitution, Eq. 7-18 gives us

$$\begin{aligned} W_g &= -mgh \qquad (7\text{-}19)\\ &= -(15.0\text{ kg})(9.8\text{ m/s}^2)(2.50\text{ m})\\ &= -368\text{ J}. \qquad \text{(Answer)}\end{aligned}$$

Note that Eq. 7-19 tells us that the work W_g done by the gravitational force depends on the vertical displacement but (surprisingly) not on the horizontal displacement. (We return to this point in Chapter 8.)



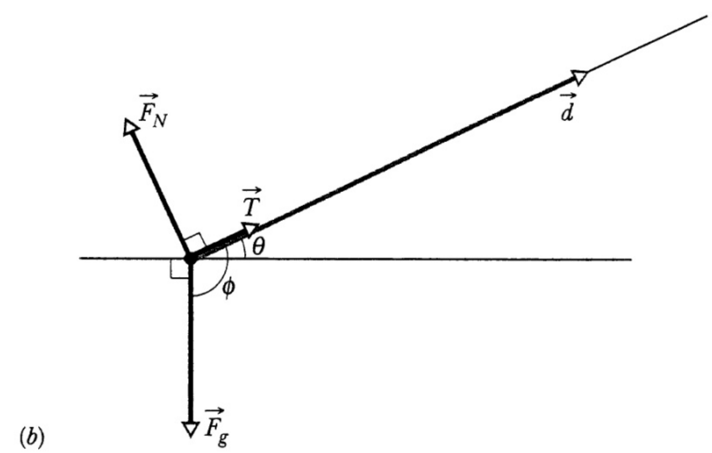


Fig. 7-9 (*a*) A crate is pulled up a frictionless ramp by a force $\vec{T}$ parallel to the ramp. (*b*) A free-body diagram for the crate, showing also the displacement $\vec{d}$.

Reliability

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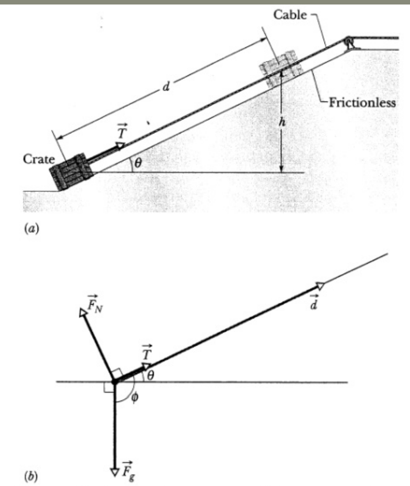

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For all 10 problems:

Reliability = 0.79

Rater 1		Rater 2		
Code	**Description**	**Code**	**Description**	**Agree?**
E	Written problem description	E	written problem description	**A**
E	Picture	E	Picture	**A**
C	Problem statement "How much work…"	C	Calculate work due to gravity	**A**
CR	Key idea : Crate is particle			**D**
CR	W=mgdcos(theta)	CR	Key Idea Use W = mgd cos(theta)	**A**
CR	"However we do not know angle…"	CR	Writing phi in terms of theta	**A**
CR	Fig 7-9 b	CR	Free body diagram	**A**
CR	W=mgdcos(theta+90)	CR	Substituting phi in work	**A**
E	Use of trig identity	CR	Applying Trig Identity	**A**
CR	dsin(theta)=h	CR	h = d sin(theta)	**A**
CR	W==mgh	CR	substitute h	**A**
CR	Substitute values	CR	Plugging in numbers	**A**
R	Compute the values	R	W = -368 J	**A**

What have we done?

- Created an ECR rubric (may need some streamlining up…)
- Created a survey of problem difficulty (SPDE)
- Among Instructors:
  - Different versions same
  - Evidence of a Training Effect
  - No strong "Solving" Effect
  - No strong "Representation" Effect

Questions?

- How do instructors' rankings compare with students?
- Does # of CR pairs correlate with difficulty?
- What effect do different combinations of Expositions have on difficulty?
- Not all CR pairs are equal – how do we capture this?

Average Difficulty Rating Per Problem (Min & Max)