

## Scaffolding Schema Induction During Problem Solving Through Analogical Encoding

David Jonassen, MU Ed Psych  
Carlos Wexler, MU Physics  
Sanjay Rebello, KSU Physics  
Gregory Triplett, MU EE  
Zdeslev Hrepic, FHSU Physics



Research supported by  
#DUE-0618459



## Default Method for Teaching Problem Solving in the Sciences: Worked Examples

PROBLEM TEXT: For a ballot box contain 3 red balls and 2 white balls, two balls are randomly drawn. The chose balls are not put back in the ballot box. What is the probability that a read ball is drawn first and a white ball second?

SOLUTION

STBP 1:

Total number of balls:	5
Number of red balls:	3
Probability of red ball on first draw:	3/5

STBP 2:

Total number of balls after first draw:	4
Number of white balls:	2
Probability of white ball on second draw:	2/4

STBP 3:

Probability that a red ball is drawn first and a white ball second:	$3/5 \cdot 2/4 = 3/10$
---	------------------------

ANSWER: The probability that a red ball is drawn first and a white ball second is 3/10.

## Rationale for Worked Examples

- Learning and problem solving facilitated by directing attention appropriately and reducing cognitive load (i.e., not require students to mentally integrate multiple sources of information).
- Worked examples improve performance on similar problems because of schema acquisition. Later, transfer improves because of rule automation.

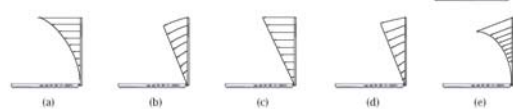
## Effects of Worked Examples

- Overgeneralization from single analogue
- No conceptual schema acquisition, only process schema
  - Problem solving learned as a procedure to be memorized, practiced, and habituated that emphasizes answer getting, not meaning making (Wilson, Fernandez, & Hadaway, 2001).
  - tendency to generalize problem solutions based on surface level similarities among problems
  - Minimal transfer of problem solutions based on single example (Loewenstein, Thompson, & Gentner, 1999)

- and inability to answer conceptual questions, such as...

### Question 2

With reference to the figure on the right, consider the motion of the laptop's lid when closing. Select which of the figures labeled (a) through (e) correctly represents the velocity of points along the lid at the instant the lid is in the vertical position shown.



## A Solution: Analogical Encoding

- Process of mapping structural properties between multiple analogues
- Instructional use of multiple analogies (Gick & Holyoak, 1983)
- Comprehension, schema induction, and long term transfer across contexts greatly enhanced by comparing two analogues for structural alignment (Gentner et al, 1997, 1999, 2003, 2005)

## Theoretical Rationale for Analogical Encoding

- structure mapping theory (Gentner, 1983, 1989)
- structural alignment - analogues compared for their relational (structural) similarity
- structurally aligned analogues must have matching causal relationships but not necessarily common objects
- analogues have same systems of relations

## Structural Mapping Process

- analogues compared for their relational (structural) similarity - relationships are mapped from source to target
- matching relations must have matching arguments and one-to-one correspondence

<b>Problem 1:</b>	<b>Problem 2:</b>
A 2kg block on surface compresses a horizontal spring by 0.2m. When the block is released, the spring makes it slide across the surface, so that when it leaves contact with the spring it is moving at a speed of 4m/s. Neglecting friction, what is the stiffness constant of the spring?	A spring of stiffness constant 530N/m hangs vertically so that the lower end of the spring is 0.15m above the ground. A 2.5-kg mass is now attached to the spring. Neglecting air resistance, how far above the ground is the lower end of the spring?

- Then, solve transfer problem

## But...

- Research on analogical encoding used relatively simple, domain-neutral contexts (Duncker's radiation problem, 1945).
- Not attempted with domain-specific problems
- Little research on quality of schemas constructed

## Duncker's Base Problem

- A small country was controlled by a dictator. The dictator ruled the country from a strong fortress. The fortress was situated in the middle of the country, surrounded by farms and village. Many roads radiated outward from the fortress like spokes on a wheel. To celebrate the anniversary of his rise to power, the dictator ordered his general to conduct a full-scale military parade. On the morning of the anniversary, the general's troops were gathered at the head of one of the roads leading to the fortress, ready to march. However, a lieutenant brought the general a disturbing report. The dictator was demanding that this parade had to be more impressive than any previous parade. He wanted his army to be seen and heard at one time in every region of the country. Further, the dictator was threatening that if the parade was not sufficiently impressive he was going to strip the general of his medals and reduce him to the rank of private. But it seemed impossible to have a parade that could be seen and heard in every part of the country

## Duncker's Target Problem

- Suppose you were a doctor faced with a patient who has a malignant tumor in his stomach. It is impossible to operate on the patient, but unless the tumor is removed, the patient will die. A kind of ray, at a sufficiently high intensity, will destroy the tumor. Unfortunately, at this intensity, the healthy tissue that the ray travels through on its way to the tumor will also be destroyed. At lower intensities, the rays are harmless to healthy tissue, but will not destroy the tumor. How can the ray be used to destroy the tumor without harming healthy tissue?
- Solution: Aim multiple, low intensity, rays at the tumor, from various angles, so that they all meet and combine in intensity at the tumor.

## But, will it work with...

<b>Problem 1:</b>	<b>Problem 2:</b>
A 2kg block on surface compresses a horizontal spring by 0.2m. When the block is released, the spring makes it slide across the surface, so that when it leaves contact with the spring it is moving at a speed of 4m/s. Neglecting friction, what is the stiffness constant of the spring?	A spring of stiffness constant 530N/m hangs vertically so that the lower end of the spring is 0.15m above the ground. A 2.5-kg mass is now attached to the spring. Neglecting air resistance, how far above the ground is the lower end of the spring?

- Comparing cases is not automatic. Merely reading multiple cases is not enough to produce comparison effects that need to be trained (Loewenstein, Thompson, Gentner, 1999)
- Making relational structure explicit during encoding promotes appropriate transfer.

## Scaffolding Analogical Encoding

- Because analogical encoding is not an automatic process, we examined how to scaffold the process through
  - Questions
  - Structure Mapping

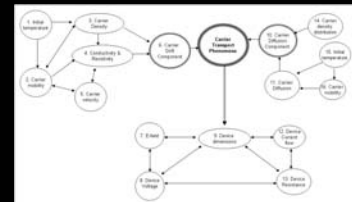
## Research Venues

- Experiment 1
  - Electrical Engineering 3610, N=50, Semiconductors and Devices, University of Missouri
- Experiment 2
  - Physics 1210, N=207 at start), Introductory Physics, University of Missouri
- Experiment 3
  - Physics 1510, Kansas State University

## Question Treatment - Exp 1

<p><b>Problem 5.36</b> In GaAs, the donor impurity concentration varies as <math>N_d \exp(-x/L)</math> for <math>0 &lt; x &lt; L</math>, where <math>L=0.1 \mu\text{m}</math> and <math>N_d=5 \times 10^{16} \text{ cm}^{-3}</math>. Assume <math>\mu_n=6000 \text{ cm}^2/\text{Vs}</math> and <math>T=300\text{K}</math>. (a) Derive the expression for the electron diffusion current density versus distance over the given range of <math>x</math>. (b) Determine the induced electric field that generates a drift current that compensates the diffusion current density.</p>	<p><b>Problem 5.43</b> Consider a GaAs sample at <math>T=300\text{K}</math>. A Hall effect device has been fabricated with the following geometry: <math>d=0.01 \text{ cm}</math>, <math>W=0.05 \text{ cm}</math>, and <math>L=0.5 \text{ cm}</math>. The electrical parameters are: <math>I_x=2.5 \text{ mA}</math>, <math>V_x=2.2 \text{ V}</math>, and <math>B_z=2.5 \times 10^{-2} \text{ tesla}</math>. The Hall voltage is <math>V_H=4.5 \text{ mV}</math>. Find: (a) the conductivity type, (b) the majority carrier concentration (c) mobility, and (d) resistivity.</p>
<p>Q1-1 What part(s) of the structure map is/are best representative of "the donor impurity concentration varies as <math>N_d \exp(-x/L)</math> for <math>0 &lt; x &lt; L</math>, where <math>L=0.1 \mu\text{m}</math> and <math>N_d=5 \times 10^{16} \text{ cm}^{-3}</math>"?</p>	<p>Q2-1 What part(s) of the structure map is/are best representative of "A Hall effect device with the following geometry: <math>d=0.01 \text{ cm}</math>, <math>W=0.05 \text{ cm}</math>, and <math>L=0.5 \text{ cm}</math>"?</p>
<p>Q1-2 What part(s) of the structure map is/are best representative of "Assume <math>\mu_n=6000 \text{ cm}^2/\text{Vs}</math> and <math>T=300\text{K}</math>"?</p>	<p>Q2-2 What part(s) of the structure map is/are best representative of "<math>I_x=2.5 \text{ mA}</math>, <math>V_x=2.2 \text{ V}</math>, and <math>B_z=2.5 \times 10^{-2} \text{ tesla}</math>, and Hall voltage is <math>V_H=4.5 \text{ mV}</math>"?</p>
<p>Q1-3 What part(s) of the structure map is/are best representative of "induced electric field that generates a drift current that compensates the diffusion current density"?</p>	<p>Q2-3 What part(s) of the structure map is/are best representative of "the conductivity type and majority carrier concentration"?</p>

## Map Treatment - EE



<p><b>Problem 5.36</b> A donor impurity has a cross-sectional area of <math>0.001 \text{ cm}^2</math> and a length of <math>10^{-3} \text{ cm}</math> is deposited on the surface of a GaAs sample. In <math>T=300\text{K}</math>, we want a current of <math>10 \text{ mA}</math> in the device. Calculate the average drift velocity.</p>	<p><b>Problem 5.43</b> A Hall effect device is in the shape of a rectangular bar with a cross-sectional area of <math>100 \mu\text{m}^2</math>, a length of <math>0.5 \text{ cm}</math>, and a width of <math>0.01 \text{ cm}</math>. Assume <math>\mu_n=6000 \text{ cm}^2/\text{Vs}</math> and <math>T=300\text{K}</math>. The temperature is <math>300\text{K}</math>. Determine the carrier concentration and the average drift velocity.</p>
<p>What part(s) of the structure map is/are best representative of "the donor impurity concentration varies as <math>N_d \exp(-x/L)</math> for <math>0 &lt; x &lt; L</math>, where <math>L=0.1 \mu\text{m}</math> and <math>N_d=5 \times 10^{16} \text{ cm}^{-3}</math>"?</p>	<p>What part(s) of the structure map is/are best representative of "A Hall effect device with the following geometry: <math>d=0.01 \text{ cm}</math>, <math>W=0.05 \text{ cm}</math>, and <math>L=0.5 \text{ cm}</math>"?</p>
<p>Q1-1</p>	<p>Q2-1</p>
<p>Q1-2</p>	<p>Q2-2</p>
<p>Q1-3</p>	<p>Q2-3</p>

## Assessments

- Transfer problem to solve

**Problem 1:** Three volts is applied across a 1-cm long semiconductor bar. The average drift velocity is  $10,000 \text{ cm/s}$ . (a) Find the electron mobility. (b) if the electron mobility in part (a) were  $800 \text{ cm}^2/\text{Vs}$ , what is the average drift velocity?

- Posttest Exam
- Far Transfer (question on final exam)

## Results - Exp 1

- Online practice

	N	Mean %	SD
Set 1 - Mapping	26	75.64	22.72
Set 2 - Mapping	26	62.09	21.35
Set 3 - Mapping	26	76.07	19.29
Set 1 - Question	24	58.52	17.07
Set 2 - Question	24	52.08	14.36
Set 3 - Question	24	63.99	16.38

$t(48) = 3.12, p = .003$

- Tests

	Analogical Practice %	Homework Problem	Test 3 Scores	Final Exam Question
Question Treatment	58.25 (11.31)	9.48 (1.04)	85.51 (9.33)	9.50 (1.02)
Structure Mapping Treatment	71.50 (17.75)	9.46 (1.33)	87.50 (13.20)	9.08 (2.28)


## Discussion - Exp 1

- Students found the structure mapping treatment easier than questions that focused on conceptual relations
- No effect of treatment on test performance.
- Did not adequately assess conceptual understanding
- Extent of treatment insufficient to overcome established study methods or to focus students' attention on structural nature of problem

## Treatment - Exp 2

**PROBLEM 1**

Two pendulums bob (see figure) are made of soft clay so that they stick together after impact. The mass of bob A is half that of bob B. Bobs A and B are initially at rest, with bob A starting at a height  $h$  relative to bob B. What is the merged bob (A+B) speed immediately after the collision?




**5 conceptual exercises on WebCT**

**Dynamics, work-energy, linear momentum, fluids thermodynamics**

**PROBLEM 2**

A 10 kg bullet traveling at a speed  $u_0 = 70 \text{ m/s}$  is fired west. The bullet penetrates the block of wood where system immediately after the collision?



**ANSWER: A, B**

Q1-Q4-PROBLEM 1: After the collision happens, in which direction does each B, which of the following physical quantities change? (Select all that apply.)

- a) Gravitational Potential Energy
- b) Kinetic Energy
- c) Mechanical Energy
- d) Linear momentum
- e) None of the above

**ANSWER: A, B, d**

Q1-Q4-PROBLEM 2: Immediately after the collision the bullet + block are moving. Which of the following physical quantities change during this process? (Select all that apply.)

- a) Gravitational Potential Energy
- b) Kinetic Energy
- c) Mechanical Energy
- d) Linear momentum
- e) None of the above

## Results - Exp 2

Model	SumSquares	df	MeanSq.	F	Sig.	R <sup>2</sup>
Exam 1 Regression	1589.47	1	1589.47	26.64	.000	.115
Exam 2 Regression	139.97	1	139.97	9.60	.002	.048
Exam 3 Regression	1249.99	1	1249.99	15.04	.000	.075
Exam 4 Regression	1610.76	1	1610.76	41.59	.000	.192
Exam 5 Regression	615.15	1	615.15	21.58	.000	.109

## Results - Exp 2

Model	Sum Squares	df	MeanSq	F	Sig.	R <sup>2</sup>
Exam 1 Problem Solving - Regression	1012.63	1	1012.63	29.09	.000	.124
Exam 1 Conceptual - Regression	64.75	1	64.75	4.03	.046	.019
Exam 2 Problem Solving - Regression	746.77	1	746.77	18.76	.000	.087
Exam 2 Conceptual - Regression	64.50	1	64.50	2.51	.115	.008
Exam 3 Problem Solving - Regression	351.76	1	351.76	21.51	.000	.109
Exam 3 Conceptual - Regression	36.56	1	36.56	4.21	.041	.023

### Broken down by question type

# Pretest - Posttest- Force Concept Inventory

		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	FCI1	9.10	153	3.820	.309
	FCI2	17.22	153	4.841	.391

Week 1 and Week 8

## Discussion - Exp 2

- Lots of limitations
  - No control group
  - Heavy course requirements
  - Predictable study scripts
  - Student aversion to innovation, and other vagaries of classroom research)
- Analogical encoding improved problem solving
- conceptually oriented analysis of problems better supported traditional problem solving than conceptual understanding (unlike Hung & Jonassen, 2006)

# Experiment 3 - Treatment 1

**Potential Energy Problems**

<p><b>Problem 1:</b></p> <p>A 2kg block on a surface compresses a horizontal spring by 0.2m. When the block is released, the spring makes it slide across the surface, so that after it leaves contact with the spring it is moving at a speed of 3m/s.</p> <p>Beginning from rest, what is the stiffness constant of the spring?</p>	<p><b>Problem 2:</b></p> <p>A spring of stiffness constant 0.049N/m hangs vertically so that the lower end of the spring is 1.10m above the ground. A 2.5-kg mass is now attached to the spring.</p> <p>Neglecting air resistance, how far above the ground is the lower end of the spring?</p>
---	---

**Q2-3:** Which of the following physical quantities change in **Problem 11**?

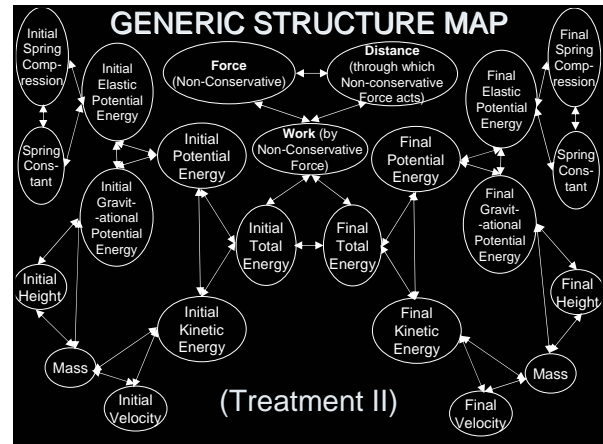
Select all that apply.

- ☐ a) Kinetic Energy of the block.
- ☐ b) Elastic Potential Energy of the block.
- ☐ c) Gravitational Potential Energy of the block.
- ☐ d) None of the above.

Optional comment regarding your answer:

Work-energy problems

[Submit answer\(s\)](#)



# Training Problem Pairs

(Problem #s are from Giancoli, 6<sup>th</sup> Ed.)

- TYPE 1 : Work – Energy Principle
  - Problem # 6-19
  - Problem # 6-23 (Change % change to actual number)
- TYPE 2 : Potential Energy
  - Problem # 6-29
  - Problem # 6-32
- TYPE 3 : Conservation of Mechanical Energy
  - Problem # 6-37 (To be revised/replaced)
  - Problem # 6-38

## Experiment 3 - Results

	N	Mean	%	SD
Set 1 - Mapping	30	54.80	25.29	
Set 2 - Mapping	30	43.79	23.22	
Set 3 - Mapping	28	54.95	25.0	
Set 1 - Question	28	55.82	7.33	
Set 2 - Question	18	56.94	10.71	
Set 3 - Question	18	55.75	16.76	


	Analogical Practice	%	Homework	Problem	Final Exam	Question
Question Treatment	56.49	(11.31)	10.021	10.86	(1.51)	2.39 (1.72)
Structure Mapping Treatment	51.18	(22.59)	10.35	(2.25)	2.74	(1.65)

## What Next? Future Studies

- Compare with unscaffolded (control) analogical encoding
- Effects of feedback on analogical encoding
- Effects of analogical encoding on question generation and effects on transfer
- Assess schema quality using:
  - Text editing
  - Problem classification
  - Recall problem details
  - Problem similarity

# Questions, arguments

<http://www.coe.missouri.edu/~jonassen/>



University of Missouri-Columbia