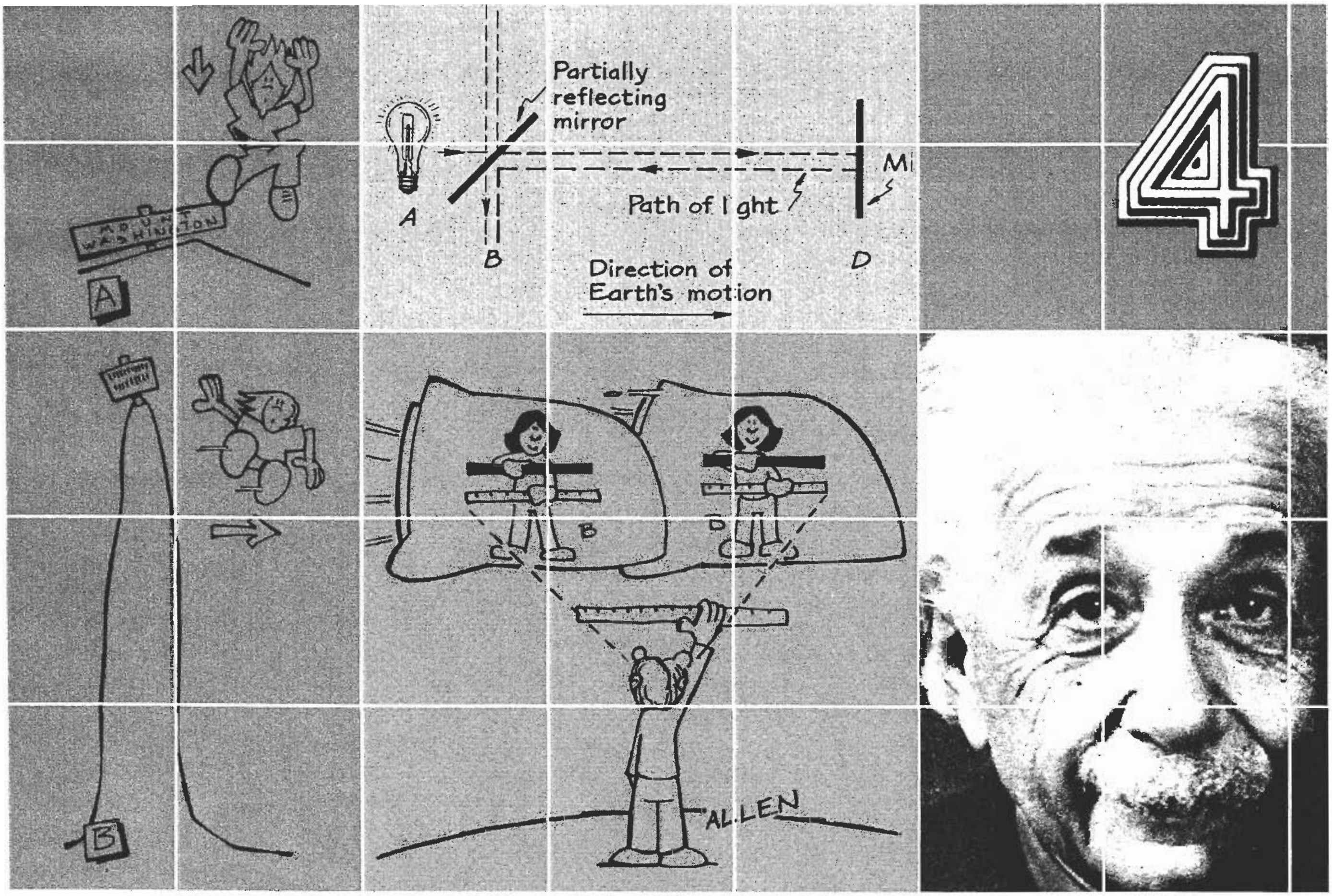


Special Theory of Relativity

What difference does it make anyway, whether we move faster or whether the object becomes shorter? I have to go ten blocks to get to the post office, and if I step harder on the pedals the blocks become shorter and I get there quicker.

George Gamow, *Mr. Tompkins in Paperback*

Faster speeds? Shorter blocks? Logically it makes no difference. A greater speed or a shorter distance will get you to the post office more quickly. However, never having seen a city block become shorter, we rely upon greater speeds. The excerpt from Gamow's book seems absurd—not because it violates our logic, but because it violates our common sense.

Our common sense includes an absolute concept of space and time. While position and time clearly are relative concepts, other concepts that are derived from measurements of position and time seem absolute. Take the concept of duration, for example. If you leave New York at 8:00 A.M. EST and arrive in Kansas City at 11:00 A.M. EST (Eastern Standard Time), the duration of your trip is 3 hours. Measured in a different reference frame, such as Central Standard Time, you would leave New York City at 7:00 A.M. CST and arrive in Kansas City at 10:00 A.M. CST. The duration of your trip is still 3 hours. While the departure and arrival times will be different in different reference frames, the *duration* of the event seems to be constant. The same can be said about the length of an object. The position of each end of a city block is described differently in coordinate systems that use different origins, but the *length* of the city block—found by subtracting the coordinates of each end—seems to be constant. In the last chapter we saw that measurements of the velocity of the stagecoach or the velocity of the robbers would be different in different reference frames. But the *relative velocity* between the stagecoach and robbers—found by subtracting their velocities relative to the earth—seems to be constant. This experience of constant length, constant duration, and constant relative velocities allows us to imagine an absolute reference frame against which we really can know the position or velocity of an object or the time of an event.

At the turn of this century, many physicists imagined, either consciously or unconsciously, an absolute reference frame for light. They expected the speed of light measured here on earth to reflect the motion of the earth through this absolute reference frame. An experiment performed by Albert Michelson and Edward Morley in 1887, called the *Michelson-Morley experiment,* shattered those beliefs when it showed that the speed of light is the same in all reference frames. In 1905 Albert Einstein introduced the *special theory of relativity,* which assumes that the speed of light is constant in all reference frames. In so doing, Einstein did away with all notions of an absolute reference frame—forcing us to rethink our concepts of position, time, and motion. Length, duration, and relative speeds vary with the motion of the reference frame, though imperceptibly at ordinary speeds. At speeds near the speed of light, city blocks do get shorter.

WHY A NEW THEORY?

Most people expect physical theories to develop in a stepwise fashion, almost in the manner that a computer executes a program. In fact, such development seldom occurs; the special theory of relativity is one such example.

Measurements by Michelson and Morley raised questions about the nature of light. The speed of light seemed to be constant in all reference frames. Apparently unaware of those measurements, Albert Einstein proposed a remarkable theory based upon the assumption that the speed of light was constant in all reference frames. In a sense, he had answered a question without knowing that it had been asked. We begin with this question.

Speed of Light and Relative Speed

By 1887 the speed of light had been measured rather accurately. Light travels at about 300,000,000 meters/second (3×10^8 m/s) in a vacuum. In materials like glass or water, light slows down a bit but always in predictable ways. The surprise comes when we measure the speed of light emitted from a source which is moving relative to us.

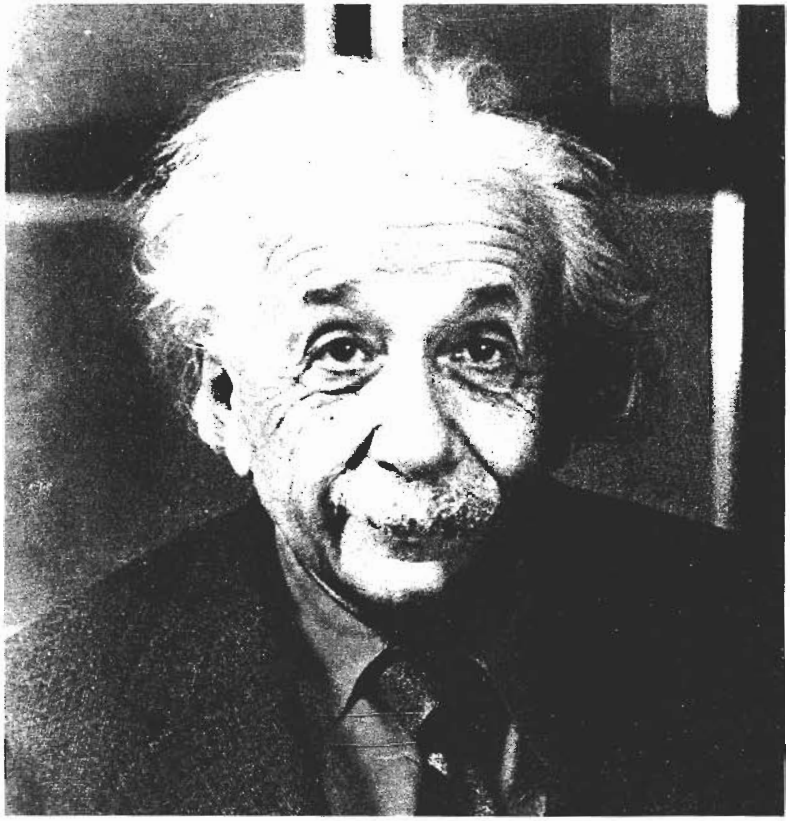


Figure 4-1
Albert Einstein.

Suppose you are driving past a sign advertising hamburgers. Light travels from the sign to you as you move along in much the same way that the robbers traveled toward the moving stagecoach (Chapter 3). Suppose your speed relative to the earth is 20 m/s. The speed of light relative to the earth is about 300,000,000 m/s. Using the concept of relative speeds and velocities, we would *expect* the speed of light relative to you to be 300,000,000 m/s + 20 m/s, or 300,000,020 m/s, as you approach the sign. If you pass the sign and move away from it, then we would *expect* the speed of light relative to you to be 300,000,000 m/s − 20 m/s, or 299,999,980 m/s. If you decrease your speed to 10 m/s, the *expected* speed of light relative to you would be 300,000,010 m/s as you move toward the sign and 299,999,990 m/s as you move away from it. We *expect* the speed of light relative to ourselves to depend upon our motion. As we shall see, these *expectations* are contradicted by experiments.

Michelson-Morley Experiment

When we talk about the speed of light relative to ourselves, we mean the speed of light that we would measure as we move along. Admittedly, a speed of 10 m/s or 20 m/s seems insignificant compared to the speed of light, but experiments can be performed in which the observer is moving fast enough to produce measurable differences. Albert Michelson and Edward Morley com-

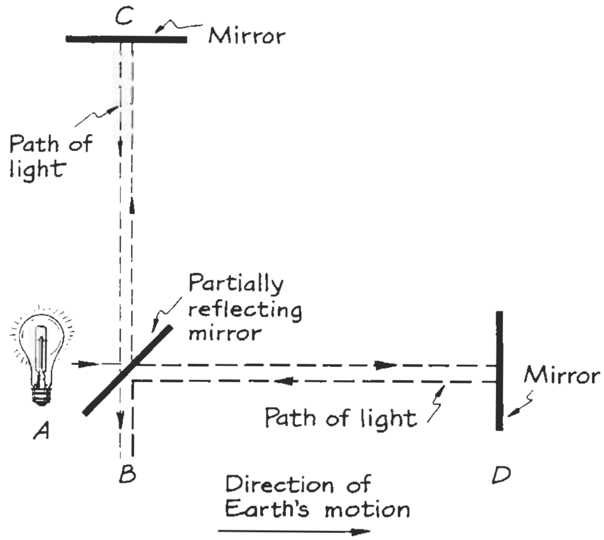


Figure 4-2
Michelson and Morley expected light to travel at a different speed to *D* and back than to *C* and back. Instead they found that light travels at the same speed along both paths.

pleted such an experiment in 1887 using the earth's motion as it revolves about the sun (30,000 m/s) to provide the moving reference frame.

A sketch of the **Michelson-Morley experiment** is shown in Figure 4-2. An observer at B is watching two light beams that travel identical lengths. The equipment is arranged so that the path *ABD* is in the direction of the earth's motion as it orbits the sun, while *BC* is perpendicular to the earth's motion. Since in one case the light is moving perpendicular to the motion of the observer, the analysis of relative speeds is not as simple as in our earlier examples. But we would still expect the speed of light to be different along the two paths. When actually measured, however, the speed of light was exactly the *same* along the two paths.

Because they contradicted our conventional view of relative speed, these results were staggering. We expect speeds to vary according to the motion of the reference frame from which they are observed, but the Michelson-Morley experiment tells us that the speed of light is always 300,000,000 m/s, regardless of the observer's motion. In terms of our example, light travels toward us at the same speed whether we are standing still or moving toward or away from the hamburger sign. The speed of light is independent of the observer's motion.

Special Theory of Relativity

By assuming that the speed of light is constant in all reference frames, the special theory of relativity offers us a solution to the apparent contradictions posed by the Michelson-Morley experiment. It begins with just two postulates:

1. The principles of physics are the same in all reference frames moving at a constant velocity relative to one another.
2. The speed of light in a vacuum is the same value regardless of the motion of the observer relative to the source of light.

The special theory of relativity provides new meaning to the concepts of relative speed, time, and length. At low speeds these concepts are essentially identical to the common-sense concepts developed in Chapters 1, 2, and 3. At speeds near the speed of light, however, the special theory alters these concepts radically. Since we have never traveled at such high speeds relative to nearby objects, we have no everyday experience against which to check the predictions. So don't be dismayed if these concepts seem strange.

When Einstein proposed the special theory of relativity in 1905, the only significant experience with high speeds had been the measurements of the speed of light. Lacking direct experience, Einstein and others developed their ideas using thought experiments. A **thought experiment** is an experiment conducted in the mind, so to speak: We set forth postulates and then consider what would happen under a variety of circumstances. In special relativity, a thought experiment deals with a situation involving motion at speeds very near the speed of light. While such an experiment is imaginary in the sense that we cannot actually perform it, it does provide us with a logical conclusion and suggests real experiments to verify its predictions. In the decades since the special theory of relativity was proposed, observations of high-speed particles and distant galaxies have provided real experimental verification for many of the thought experiments you will encounter throughout the remainder of the chapter.

RELATIVE VELOCITY AT HIGH SPEEDS

Measurements of the speed of light in a variety of experiments confirmed Einstein's postulate that the speed of light is constant in all reference frames moving at constant velocity relative to one another. Such measurements contradicted the concept of relative speed and velocity we encountered in low speed examples in Chapter 3. To explain these results, the special theory of relativity replaces our low-speed definition of relative velocity with one that can be applied to both low-speed and high-speed situations.

In Chapter 3 we developed a general rule for calculating the relative velocity between two objects given their velocities relative to the earth. When the robbers and stagecoach were moving toward one another, we added the magnitudes of their velocities relative to the earth. Suppose we replace the stagecoach and robbers with two spaceships, A and B, moving toward one another at speeds near the speed of light. Our low-speed definition of relative velocity predicts that the velocity of A relative to B is simply the sum of the magnitudes of their velocities relative to the earth. The special theory of relativity modifies this definition by adding a term that limits this sum at higher speeds. To get some feeling for how relative velocities change at high speeds, let's look at some specific examples.

Table 4-1 contrasts the low-speed and high-speed predictions of the relative velocity between our two spaceships, A and B. The low-speed column lists a relative velocity that is the sum of the magnitudes of the velocities of the two spaceships. The high-speed column shows that these relative velocities have to be modified at higher speeds. At ordinary walking speeds, 1 m/s, both the low-speed and high-speed definitions of relative velocity predict the same result—the velocity of one spaceship relative to the other is 2 m/s. The two definitions continue to predict roughly the same relative velocities up to 10,000 m/s (10^4 m/s). At higher speeds the relative velocity predicted by the high-speed definition begins to be smaller than that predicted by the low-speed definition. The difference between the relative velocities predicted by the two definitions seems small up to about one-third the speed of light. At still higher speeds, however, the difference becomes substantial. If both spaceships move toward one another at two-thirds the speed of light (2×10^8 m/s), their relative velocity is 2.77×10^8 m/s rather than 4×10^8 m/s.

If we replace spaceship B by a beam of light, then we see how the high-speed definition of relative velocity correctly predicts Einstein's second postulate. If observers in spaceship A could move at the speed of light, 3×10^8 m/s, they would measure the speed of the light beam moving toward them to be 3×10^8 m/s—the same speed measured by a stationary observer. The speed of light is 3×10^8 m/s regardless of the motion of the observers in spaceship A. The high-speed definition of relative velocity is consistent with our low-speed observations, consistent with the results of the Michelson-Morley experiment and consistent with Einstein's second postulate.

Table 4-1 Relative Velocity

v_A (m/s)	v_B (m/s)	Low-Speed Definition (m/s)	High-Speed Definition (m/s)
1	1	2	2
10	10	20	20
100	100	200	200
1×10^3	1×10^3	2×10^3	2×10^3
1×10^4	1×10^4	2×10^4	2×10^4
1×10^5	1×10^5	2×10^5	1.99999×10^5
1×10^6	1×10^6	2×10^6	1.99998×10^6
1×10^7	1×10^7	2×10^7	1.99778×10^7
1×10^8	1×10^8	2×10^8	1.8×10^8
2×10^8	2×10^8	4×10^8	2.77×10^8
3×10^8	3×10^8	6×10^8	3×10^8

SELF-CHECK 4A

An observer in spaceship A, moving at a speed of 1×10^8 m/s, measures the speed of a beam of light coming toward him. The speed of light relative to the earth is 3×10^8 m/s. What do the low-speed and high-speed definitions of relative velocity predict for the speed of light the observer measures? Is the high-speed result consistent with Einstein's second postulate?

SIMULTANEITY AT HIGH SPEEDS

Our concept of time (derived from experiences at low speeds) is absolute. We imagine that time proceeds at the same rate for all observers. Two events that occur simultaneously in one reference frame will be simultaneous in other reference frames as well. At speeds near the speed of light, however, this concept of simultaneity changes. We examine the reason for these changes with the help of a thought experiment involving two space travelers, Barbara and Allen.

As shown in Figure 4-3(a), Allen stands on the surface of the earth, while Barbara sits in a spaceship. A lamp has been mounted in the center of the spaceship. Each time the lamp flashes, light moves toward both ends of the spaceship. Since the distance to each end of the spaceship is equal, we expect light to reach the ends simultaneously. While the spaceship is stationary, Barbara and Allen will agree that light reaches the end of the spaceship simultaneously. When the spaceship moves past Allen at speeds near the speed of light, they no longer agree. Let us look at what each sees.

Figure 4-3(b) shows what Barbara sees in her reference frame. The lamp is motionless relative to Barbara and the spaceship—they are all moving together. Since the lamp always remains in the middle of the spaceship, light travels the same distance in reaching the front and the back of the spaceship. As stated in the second postulate, light travels at the same speed in all reference frames. Consequently, light reaches the front and back of the spaceship simultaneously. Barbara's report is the same as when the spaceship was stationary.

As shown in Figure 4-3(c), the spaceship moves past Allen as light travels from the lamp to each end of the spaceship. After the lamp flashes, the back of the spaceship moves toward the light and the front moves away from the light. The distance traveled by the light moving to the back is half the length of the spaceship minus the distance traveled by the spaceship while the light is in transit. Allen sees the light hit the back of the spaceship before he sees it hit the front. Why? The light traveling toward the front must travel half the length of the spaceship plus the extra distance traveled by the spaceship while the light is in transit. Light traveling to the back of the spaceship travels a shorter distance than light traveling toward the front. If the spaceship could

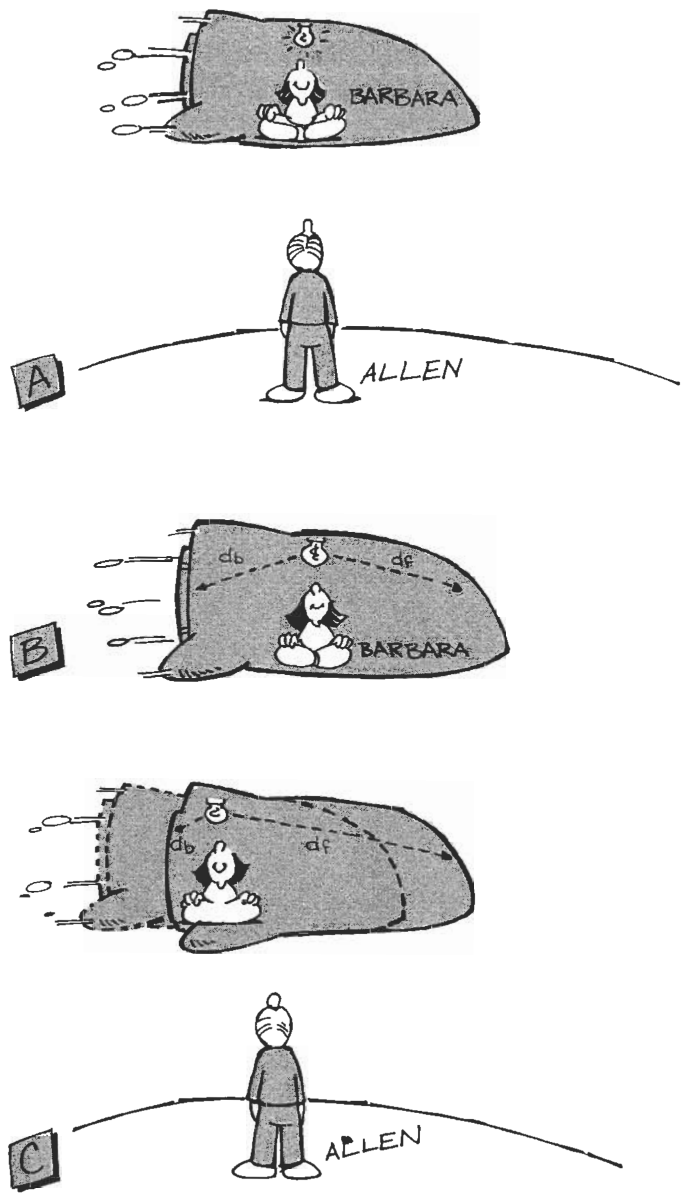


Figure 4-3
In Barbara's reference frame, light reaches the front and back of the spaceship simultaneously. In Allen's reference frame, light reaches the back before it reaches the front. Events that are simultaneous in one reference frame will not be in another that is moving at a constant velocity relative to the first.

travel at the speed of light (which it cannot, as we will see later), the light would never catch up to the front of the spaceship. As it is, Allen no longer reports that light strikes the front and back of the spaceship simultaneously.

Events that are simultaneous in Barbara's reference frame are no longer simultaneous in Allen's reference frame. Their disagreement cannot be resolved—both could produce measurements to substantiate their statements. They can, however, understand the reason for their disagreement. If the speed of light is the same in both reference frames, events that are simultaneous in one reference frame cannot be simultaneous in a second reference frame moving at a constant relative velocity. At low speeds, the difference is so slight that observers in both reference frames continue to report that the events are simultaneous. At speeds near the speed of light, however, the differences are considerable. Observers in the two reference frames no longer agree.

SELF-CHECK 4B

Barbara stands in the center of a supertrain. A lamp mounted in the center sends light toward both ends. As the supertrain moves past Allen at speeds near the speed of light, do Allen and Barbara agree as to whether light reaches the ends of the train simultaneously? Describe what each observes and why.

TIME INTERVALS AT HIGH SPEEDS

Most of our measurements of time involve duration rather than simultaneity. We measure the time needed to travel to Kansas City, the time for the earth to rotate on its axis and so forth. If our concept of simultaneous time depends on the reference frame, we would expect the duration of an event to be different in different reference frames as well.

An Experiment with Time

One measurement of duration, performed first in the 1940s and repeated in the 1960s, involved small, fast-moving particles called *muons*, created in the earth's upper atmosphere by collisions between atoms and high-speed particles from outer space. Once created, the muons do not exist very long; they spontaneously change into other particles, called *electrons* and *neutrinos*. The time of a muon's existence is measured by the *half-life*—the time it takes for one-half the muons in a group to change into electrons and neutrinos. On earth we measure the half-life of stationary muons to be about 1.53 microseconds (μs) (0.00000153 s, or 1.53×10^{-6} s). Thus, if we start with 1200 muons, 1.53 μs later we will have only 600 left; another 1.53 μs later we will have only 300 left; and so forth. The muons' behavior provides us with a convenient clock. Knowing the initial number of muons, physicists can measure duration in terms of the number of muons left at some later time.

As muons are created in the upper atmosphere, they move toward the surface of the earth at speeds of up to more than 99% the speed of light. By measuring the number of muons at various heights as they move toward earth, we can measure the half-life of the muons in this moving reference frame and compare it to the half-life of 1.53 μs measured in our stationary reference frame on earth.

In an experiment performed in the 1960s, physicists measured the number of muons found at the top of Mount Washington, New Hampshire, (altitude = 1907 m) and at sea level. The muons were traveling at 99.2% of the

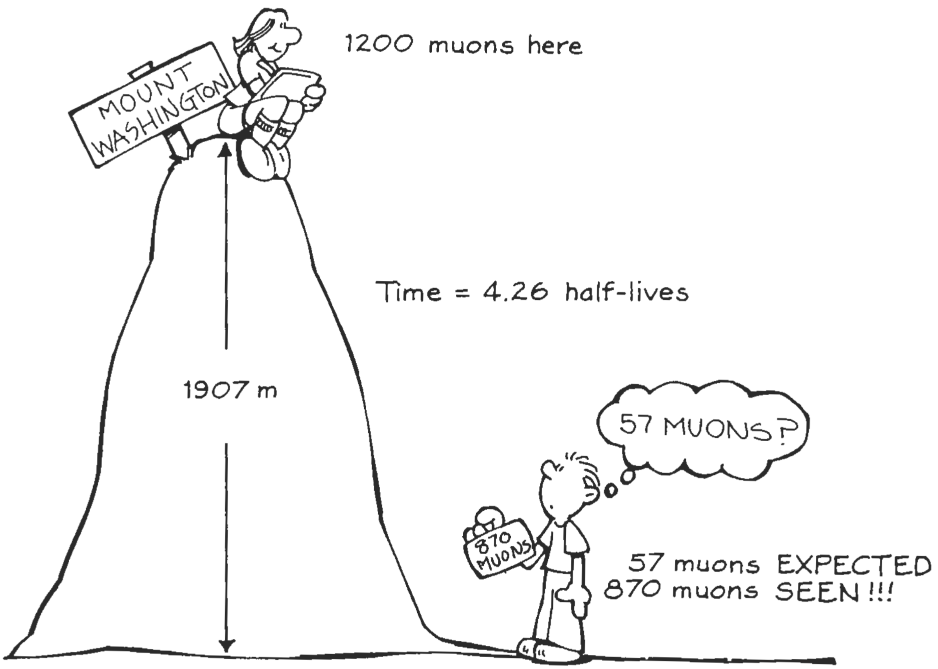


Figure 4-4
Muons traveled at over 90% the speed of light as they fell toward the earth's surface. If time is constant in the muons' and earth's reference frames, we should see 57 muons at the bottom of Mount Washington. Instead we see 870 muons. Time moves more slowly in the muons' reference frame.

speed of light. Since we know the speed of the muons ($0.992 \times 3 \times 10^8$ m/s $= 2.976 \times 10^8$ m/s) and the distance traveled (1907 m), we can calculate the time required for the muons to make the trip by dividing the distance by the speed:

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{1907\ \text{m}}{2.976 \times 10^8\ \text{m/s}} = 6.41 \times 10^{-6}\ \text{s}$$

The time of travel, 6.41 μs, is more than four times the half-life of the muon particles, so we expect to see far fewer muons at sea level than at the top of the mountain. As shown in Figure 4-4, if 1200 muons are detected at the top of Mount Washington, only 57 should be detected at sea level if their half-life is the same as in the laboratory. Actual measurements, however, detected 870 muons at sea level. Clearly, the half-life of the muons in their moving reference frame is longer than the half-life measured in the laboratory. We investigate the reasons for this with another thought experiment involving Barbara and Allen.

Barbara proposes an experiment in which she and Allen measure the duration of an event—the time it takes light to travel from a lamp to a mirror and back to the lamp (Figure 4-5). Allen will measure the duration of this event in two reference frames: one in which Barbara's spaceship is stationary relative to him and a second in which she is moving past him at 80% the speed of light. This is analogous to measuring the muon's half-life when the muons are stationary and when they are moving toward the earth.

Figure 4-6 helps us compare what Allen will observe in the two reference frames. When the spaceship is stationary relative to Allen, the light travels to the mirror, is reflected, and travels back to the lamp along the vertical path shown in Figure 4-6(a). When the spaceship moves past Allen,

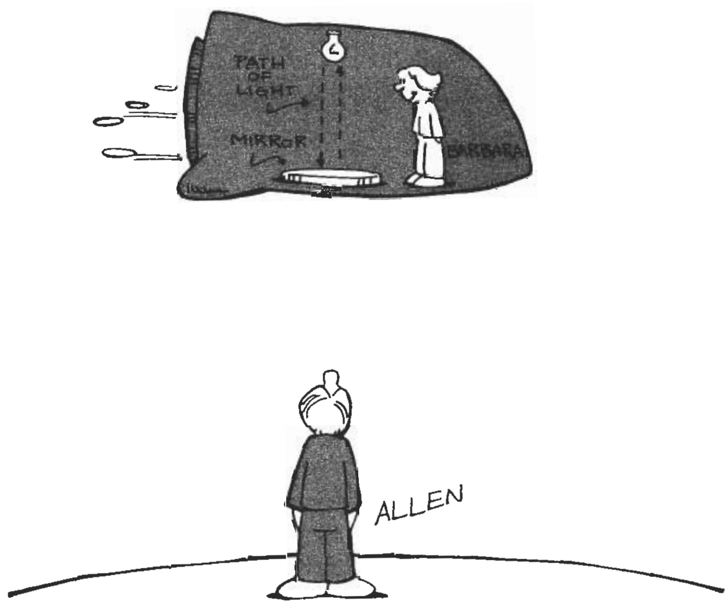


Figure 4-5

Allen will measure the time it takes for light to travel from the lamp to the mirror and back in two reference frames: one in which the spaceship is stationary relative to him and a second in which it moves past him.

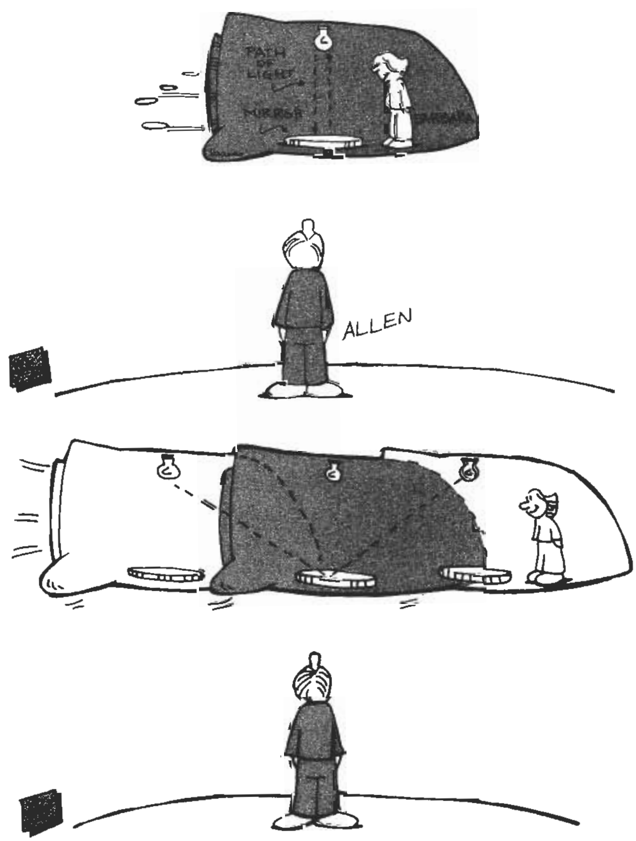


Figure 4-6

When the spaceship moves past Allen, light travels from the lamp to the mirror and back along the diagonal in (b) rather than along the vertical path in (a). Since the distance is greater, the time interval will be greater.

light travels along the diagonal path shown in Figure 4-6(b). We have to add the horizontal motion of the spaceship to the vertical path of the light as it moves to the mirror and back. The distance along the diagonal path is greater than the distance along the vertical path. Since the speed of light is the same in both reference frames, Allen will report that it took the light longer to travel to the mirror and back in the moving reference frame than in the stationary reference frame.

Time Dilation

The same event occurs in both reference frames. Light is emitted by a lamp, reflected by a mirror, and returned to the lamp. However, the time between the beginning of the event (emission of light) and the end of the event (return of the light) is different. The duration of events is longer in the moving reference frame than in the stationary reference frame. This slowing of events is called **time dilation.** When observers in one reference frame measure time in another, they find that time varies with the relative speed of the two reference frames. Moving clocks run slower than stationary ones.

Table 4-2 allows you to compare the duration of events an observer measured in the observer's own stationary reference frame with the duration

Table 4-2 Time Dilation

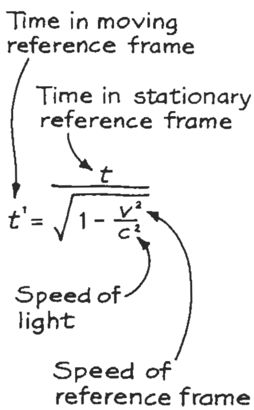
Speed of Moving Frame: Fraction of Light Speed	Speed of Moving Frame: (m/s)	Time Duration in Stationary Observer's Frame (s)	Time Duration in Moving Frame as Measured by Stationary Observer (s)
0.0001	3×10^4	1.00	1.000000005
0.001	3×10^5	1.00	1.0000005
0.01	3×10^6	1.00	1.00005
0.1	3×10^7	1.00	1.005
0.2	6×10^7	1.00	1.02
0.4	1.2×10^8	1.00	1.09
0.6	1.8×10^8	1.00	1.25
0.8	2.4×10^8	1.00	1.67
0.9	2.7×10^8	1.00	2.29
0.99	2.97×10^8	1.00	7.09
0.999	2.997×10^8	1.00	22.37
0.9999	2.9997×10^8	1.00	70.71

observed in reference frames moving past at different speeds. The speed of the moving reference frame has been described in two ways: as a fraction of the speed of light and in meters per second. A reference frame that moves at 30,000 m/s is moving at 0.0001, or one ten-thousandth, the speed of light. At relatively low speeds, like 30,000 m/s, the difference between the two measurements of time is insignificant. We never notice time dilation in everyday life—not even when traveling by jet at hundreds of kilometers per hour. As objects begin to move at speeds near the speed of light, however, time dilation becomes impossible to ignore. A lamp that flashes once per second when motionless relative to you will flash once in 7.09 s when moving past you at 99% the speed of light. It is somewhat like watching a slow-motion movie. Compared to events in our stationary reference frame, events in the moving reference frame take longer. We conclude that time moves more slowly in the moving reference frame.

We can use these results to understand the surprising results of the muon experiment. When the muon is motionless relative to the experiment-

A STEP FURTHER—MATH

TIME DRAGS ON



Using some geometry and algebra, we can derive an exact expression, called the time-dilation equation, that relates the time intervals measured in different reference frames.

$$\text{Time between two events in moving reference frame} = \frac{\text{time between events in stationary reference frame}}{\sqrt{1 - \frac{(\text{speed of frame})^2}{(\text{speed of light})^2}}}$$

Applied to our thought experiment, this equation tells us that the time Allen measures as Barbara moves past, called t', is equal to the time he would measure if Barbara were stationary relative to him divided by $\sqrt{1 - v^2/c^2}$.

We use this expression to compare the measurements Allen makes in the two reference frames. Suppose it takes light 3 s to travel to the mirror and back when the spaceship is stationary relative to Allen. When the spaceship moves past at 80% the speed of light ($v = 0.8c$), Allen measures a time of:

$$t' = \frac{t}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{3\text{ s}}{\sqrt{1 - \left[\frac{(0.8)(3 \times 10^8\text{ m/s})}{(3 \times 10^8\text{ m/s})}\right]^2}} = 5\text{ s}$$

An event that takes 3 s in a stationary reference frame takes 5 s if you watch it in a reference frame moving past you at 80% the speed of light. What if Barbara goes even faster? At 90% the speed of light, the 3 s stretch into 7 s. At 99.9% the speed of light, they become more than a minute. What about 99.99% the speed of light? Try it and see!

ers, its half-life is 1.53 μs. As the muons travel at 99% the speed of light relative to the observers, their half-life (as measured by the observers) is 7.09 times longer, 10.85 μs. Since the trip from the top of Mount Washington to sea level only lasted 6.41 μs, the muons had not been through even a single half-life. With the half-life of 10.85 μs predicted by time dilation, 870 of the 1200 muons would be left after 6.41 μs. This matched what had been measured at sea level!

SELF-CHECK 4C

An average cockroach has a lifetime of 3 months on earth. Suppose you observe the cockroach from the earth as it travels past you in a spaceship. What lifetime would you measure at each of the following relative speeds: 0.1, 0.4, 0.8, 0.9 times the speed of light?

Symmetry of Time Dilation

In thinking about time dilation, we must distinguish between what Barbara measures and what Allen measures. Our thought experiment described what Allen would see. If, for example, Allen sees an event take place in 3 s when Barbara is stationary relative to him, he will see the same event take longer—for instance, 5 s—when Barbara moves past him at 80% the speed of light. Consider the measurements that Barbara might make. In her own reference frame, the light will take 3 s to go down and back regardless of whether she is stationary or moving relative to Allen. Her measurements will agree with Allen's when she is at rest relative to him, but they disagree when she is moving relative to him.

If we ask Barbara to measure an event on earth, she will notice the same change in time duration as Allen. An event on earth that takes 3 s when she is stationary will take 5 s as she moves past at 80% the speed of light. When measuring events on earth, Barbara reports that earth time slows down as she moves relative to the earth. When measuring events on Barbara's spaceship, Allen reports that time slows down as she moves relative to the earth. Each views the other's time to be proceeding more slowly.

SELF-CHECK 4D

Allen sows a tomato seed. Three months later he picks the first ripe tomato. Barbara, traveling at 80% the speed of light relative to Allen, sows an identical tomato seed. In Allen's reference frame, how much time elapses before Barbara picks her first ripe tomato? In Barbara's reference frame, how much time elapses before Allen picks his first tomato?

Twin Paradox

While a bit bizarre, the fact that Allen sees the light take longer to bounce off mirrors does not seem impossible to believe. For most of us, the next step—from light clocks to biological clocks—is a much more difficult one to accept. If Allen sees time proceed more slowly in Barbara's reference frame as she speeds by, then he sees all events proceed more slowly—including the time between heartbeats. Allen sees Barbara age more slowly.

One of the more startling outcomes of time dilation is a thought experiment that came to be called the **twin paradox.** Jackie takes off in a rocket, travels to a nearby star at a speed near the speed of light, turns around, and comes back to earth. Her twin, Steve, stays home. During the trip out, each views the other's time to be proceeding more slowly than their own. Steve expects Jackie to be younger than he upon her return. Jackie expects Steve to be younger. When they meet again on earth, they cannot both be younger!

The solution to the paradox lies in the fact that the two reference frames did not move at a constant velocity relative to one another. Seen from Steve's earthbound reference frame, Jackie's spaceship accelerated to its cruising speed, continued at this speed until it reached the star, accelerated as it turned around and headed back to earth, cruised at a constant speed back to earth, and finally accelerated (negatively) as it landed. During the periods in which Jackie was moving at a constant velocity relative to Steve, each was seeing time proceed more slowly in the other's reference frame. During acceleration, however, Jackie's time was distorted, while Steve's was not. Jackie will return the younger twin. The symmetry of time dilation is broken once Jackie accelerates.

LENGTH CONTRACTION

Any measurement of length ultimately involves our concept of position. Typically, we place a meterstick along the side of the object and mark the position of each corner of the object relative to the meterstick. The length is then the difference between these two positions. While we think of position as a relative concept, most of us imagine length to be absolute. We expect the length of the object to be the same, regardless of the motion of the reference frame. But, as with our concept of time, the concept of length is altered in reference frames moving at high speeds.

Length in the Muons' Reference Frame

Consider a thought experiment with the muons that we introduced in the last section. At the top of Mount Washington, we counted 1200 muons; at the bottom, 870. Since the muons were moving at an enormous speed relative to us, we explained this surprising result in terms of time dilation. Relative to us in a stationary reference frame, time slows down in the moving reference frame.

Imagine that we can move into the muons' reference frame and travel at 99.2% the speed of light. Now we are motionless relative to the muons. We

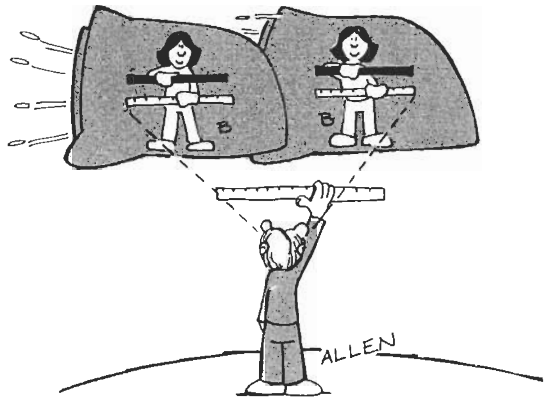


Figure 4-7
Allen sees two parts of the pole simultaneously, but the pole is measured to be shorter than in Barbara's reference frame.

describe the muons and ourselves as stationary and Mount Washington as moving upward at 99.2% the speed of light. When we see the top of Mount Washington pass us, we count 1200 muons, and when we see the bottom (sea level) pass us, we count 870. A change in reference frame cannot cause muons to appear or disappear; the number we count must be the same in all reference frames. But here a contradiction arises. Since we are stationary relative to the muons, we once again measure their half-life to be 1.53 μs. To determine the duration of the descent, we divide the height of Mount Washington by our speed. Thus, we *expect* the descent time to be (1907 m)/(2.98×10^8 m/s), or 6.41 μs—more than four muon half-lives. If this time calculation is correct, only 57 muons should be left.

In both reference frames—the one in which the muons are stationary on earth and the one in which we are moving with the muons at 99.2% the speed of light—the half-life must be 1.53 μs. The number of muons counted at the top and at the bottom of Mount Washington must also be the same in all reference frames. Consequently the only incorrect reasoning is the calculation of the travel time. To arrive at this value we assumed that the height of Mount Washington (1907 m) is the same in our new (moving) reference frame as it was in our old (stationary) reference frame. Therein lies the problem. Our measurement of length depends on the motion of the reference frame.

Length Contraction

Normally, we expect objects to remain stationary as we measure their length. We lay the pole down next to the meterstick, mark the left end, look over, and mark the right end. Implicit in our actions is the belief that the left end does not move while we measure the right end. As long as we measure objects that are stationary relative to ourselves, our assumption is valid. But when the pole moves past us, we have to invent another strategy for measuring its length. We must locate the two ends of the pole simultaneously, that is, at the same time. Ultimately, our measurement of length depends on our concept of simultaneity.

To understand the limitations that simultaneity places on our measurements of length, contrast Barbara's and Allen's measurements of the length of a pole. Figure 4-7 shows Barbara holding a pole as her spaceship moves past Allen. As she holds the pole up next to a meterstick, she sees the two ends of the pole simultaneously and measures its length. On earth, Allen looks up and sees the pole simultaneously as well. However, since the spaceship moves past him as he looks, Allen does not see the pole exactly as Barbara does. Light from the left end reaches him before light from the right end, for example. As shown in the figure, this effect causes Allen's measurement of the pole's length to be smaller than Barbara's measurement. Rapidly moving objects are shorter to the stationary observer. The change in length that occurs when objects move at speeds near the speed of light is called **length contraction.**

Table 4-3 allows you to compare the length of the object an observer measures in his or her own stationary reference frame with the length he or she observes in reference frames moving past at different speeds. The speed of the moving reference frame has been described in meters per second and as a fraction of the speed of light. At relatively low speeds, like 30,000 m/s, the difference in length is not noticeable. Clearly, we never notice length contraction in everyday life. At speeds near the speed of light, however, length contraction becomes significant. An object that is 10 m long in a reference frame stationary relative to the observer will be 6 m long when moving past the observer at 80% the speed of light.

Table 4-3 Length Contraction

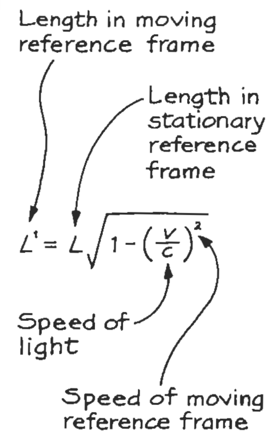
Speed of Moving Frame		Length in Stationary Observer's Frame (m)	Length in Moving Frame as Measured by Stationary Observer (m)
Fraction of Light Speed	(m/s)		
0.0001	3×10^4	1.00	0.999999995
0.001	3×10^5	1.00	0.9999995
0.01	3×10^6	1.00	0.99995
0.1	3×10^7	1.00	0.995
0.2	6×10^7	1.00	0.98
0.4	1.2×10^8	1.00	0.92
0.6	1.8×10^8	1.00	0.80
0.8	2.4×10^8	1.00	0.60
0.9	2.7×10^8	1.00	0.44
0.99	2.97×10^8	1.00	0.14
0.999	2.997×10^8	1.00	0.04
0.9999	2.9997×10^8	1.00	0.01

A STEP FURTHER—MATH

FISK'S DISK

As we did for time dilation, we can develop an equation that relates the lengths measured in the different reference frames.

$$\text{Length measured for moving object} = \text{length of stationary object}\sqrt{1 - \left(\frac{\text{speed of object}}{\text{speed of light}}\right)^2}$$



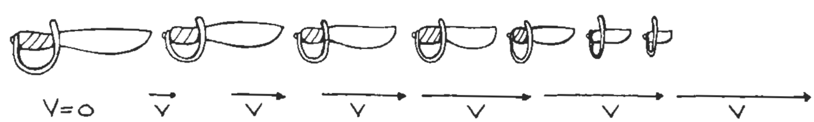
Called the Lorentz-FitzGerald contraction, after two physicists who first proposed it, this expression tells us that the length Allen measures as Barbara moves past, called L', is equal to the length he would measure if Barbara were stationary relative to him multiplied by $\sqrt{1 - v^2/c^2}$.

We use this expression to compare the measurements Allen makes for the two reference frames. Suppose Barbara has a 2-m rod in her spaceship. When Barbara is stationary relative to Allen, he reports that the rod is 2 m long. When the spaceship moves past him at 80% the speed of light ($v = 0.8c$), Allen measures a length of

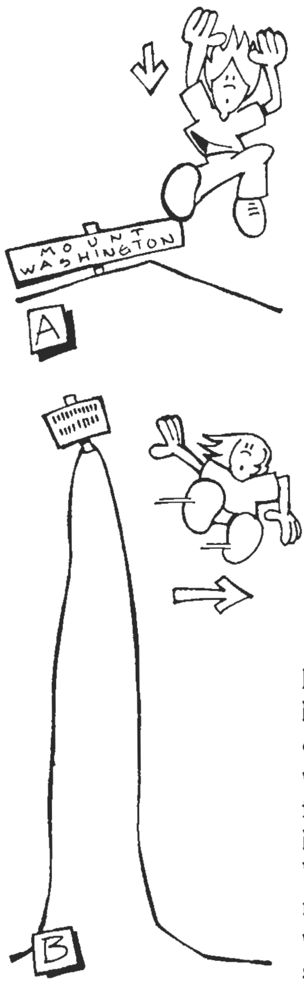
$$L' = L\sqrt{1 - v^2/c^2} = (2\text{ m})\sqrt{1 - \left(\frac{(0.8)(3 \times 10^8\text{ m/s})}{(3 \times 10^8\text{ m/s})}\right)^2}$$

$$= 1.2\text{ m}$$

A rod that is 2 m long in a stationary reference frame is 1.2 m long if you measure it in a reference frame moving past you at 80% the speed of light. What if Barbara goes even faster? At 90% the speed of light, the 2-m rod contracts to 0.88 m. At 99.9% the speed of light, it measures 0.08 m. What do we have left at 99.99% the speed of light?



There was a young fencer
named Fisk
Whose thrust was
exceedingly brisk
So fast was his action
The Lorentz-FitzGerald contraction
Reduced his rapier
to a disk



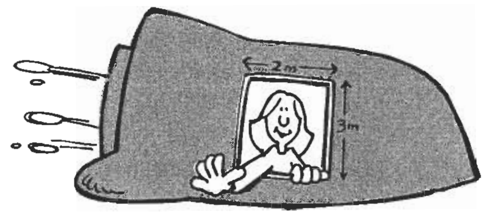


Figure 4-9

Figure 4-8
Only the dimensions along the direction of motion are affected. **(a)** In a reference frame moving down, the height of Mount Washington changes but the width remains the same. **(b)** In a reference frame moving along the earth's surface, the width changes but not the height.

Like time dilation, length contraction is symmetric. If Allen reports that a 10-m pole is only 6 m long as Barbara moves past him at 80% the speed of light, Barbara reports that a 10-m pole in Allen's reference frame is only 6 m long as she moves past him at 80% the speed of light. If we return to our experiment with the muons, this tells us that Mount Washington is shorter as we move with the muons at 99.2% the speed of light. If we actually performed the calculation, Mount Washington would be only 241 m high. Now, let us determine the time it takes the muons to travel from the top of Mount Washington to sea level. The duration of the descent is the distance the muons travel divided by their speed—$(241 \text{ m})/(3 \times 10^8 \text{ m/s}) = 0.81\ \mu\text{s}$. It would take 0.81 μs to make the complete trip from top to bottom, less than a single half-life of muons. This is consistent with the 870 muons we measure at the bottom of Mount Washington.

Length contraction occurs only in the direction in which the object (or the reference frame) is moving. As shown in Figure 4-8(a), Mount Washington appears shorter but the same width as we move downward toward the surface of the earth. If we were to move along the surface of the earth at the speed of a muon (Figure 4-8(b)) Mount Washington would appear thinner, but its height would remain the same. Only the dimension in the direction of motion varies with the relative speed of the object.

SELF-CHECK 4E

A window in Barbara's spaceship is 2 m long and 3 m high (see Figure 4-9). What length and height does Allen measure for the window if Barbara passes by him at the following speeds: 0.1, 0.4, 0.8, and 0.9 times the speed of light?

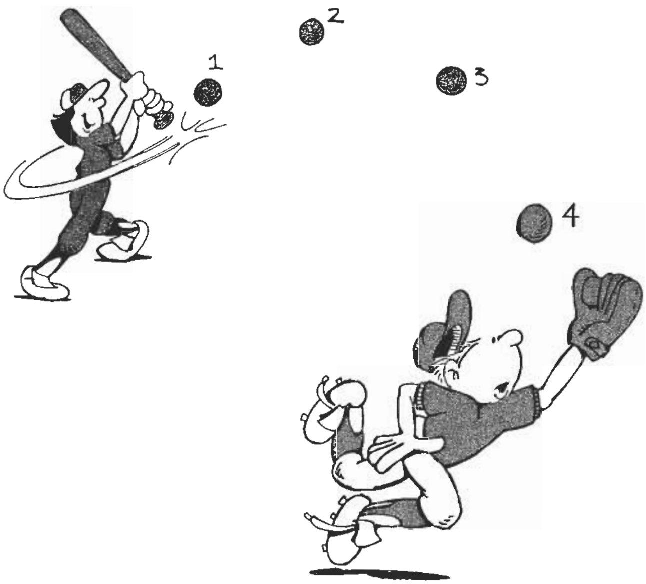


Figure 4-10
If a baseball could travel faster than light, the fielder would have to catch the ball before he knew that it had been hit.

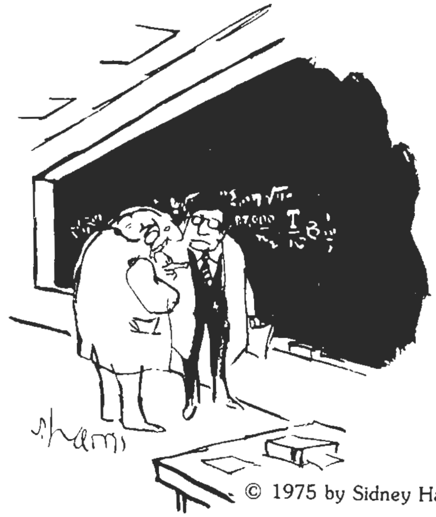
HOW FAST IS FASTEST?

The speed of light is the greatest speed we have encountered. The special theory of relativity suggests that the speed of light is the upper limit to all motion. Nothing can move faster. To understand the reasons for believing that such a limit exists, consider another thought experiment.

Suppose you are playing center field in a baseball game. The batter swings and sends a fly ball out to you, as illustrated in Figure 4-10. In order for you to see the ball, sunlight must be reflected from the baseball into your eyes. Since light travels very, very fast relative to the baseball, the reflected light reaches your eyes almost instantaneously and you can track the ball along its path. Imagine what you would see if the baseball traveled faster than the speed of light. Light travels at a constant speed, so the light that travels the shortest distance will reach you first. Because the ball moves faster than light, the ball reaches you before the light reflected from the ball at points 1, 2, 3, or 4. In fact, you must catch the ball before you see it. After catching it you would see the light reflected from points 4, 3, 2, and 1—in that order. You would catch the ball, then see it travel back to the batter. Finally, the batter would hit it.

Absurd? Yes! This result violates the basic logic of causality. A ball traveling faster than light puts the cart before the horse. The effect (catching the ball) occurs before the cause (hitting the ball). Either we must reject cause and effect or conclude that all objects travel more slowly than light. So far, experiments agree with these expectations. No one has discovered an object that exceeds the speed of light—3×10^8 m/s, or 186,000 miles per second.

Faster speeds? Shorter blocks? Let's return to the bicyclist whose delightful argument opened the chapter. In writing this story, George Gamow imagined what the world would look like if light moved at ordinary speeds—such as 5 m/s. Relative to this speed of light, the bicyclist certainly could



"That 187,000 miles per second makes me a bit skeptical about the whole thing."

pedal hard enough to see the blocks get shorter or even time slow down. But do the blocks *really* get shorter and does time *really* slow down? Our concepts of the length of a city block and the duration of a second—of space and time—depend on motion. Gamow's bicyclist catches one glimpse of space-time. Standing on the street corner, we catch another. Neither space nor time is absolute.

CHAPTER SUMMARY

Our experiences with objects at low speeds suggest that while position, time, and motion are relative concepts, relative speed, duration, and length *seem* to be constant in all reference frames moving at constant velocity relative to each other. In an experiment performed in 1887, Albert Michelson and Edward Morley showed the speed of light to be constant in all reference frames. This contradicted our low-speed concept of relative velocity. In 1905 Albert Einstein proposed the *special theory of relativity*. By assuming that the principle of relativity (Chapter 3) is valid and that the speed of light is constant in all reference frames, Einstein revised our concepts of relative velocity, simultaneity, time duration, and length.

The changes made by the special theory of relativity are summarized below:

Concept	*Low-Speed Definition*	*High-Speed Definition*
Relative velocity	Relative velocity is the same in all reference frames.	Relative velocity changes with the motion of the reference frame. The relative velocity can never exceed the speed of light.

Simultaneity	Events that are simultaneous in one reference frame are simultaneous in another.	Simultaneity depends on the motion of the reference frame. At high speeds, events seen as simultaneous in one reference frame will not be simultaneous in a reference frame moving at a high speed relative to it.
Time	Duration of an event is the same in all reference frames.	*Time dilation:* An observer sees time moving more slowly when the event occurs in a reference frame moving past him or her.
Length	Length of an object is the same in all reference frames.	*Length contraction:* An observer measures the length of an object to be shorter in its direction of motion.

At low speeds these new concepts are identical to those introduced in Chapters 1, 2, and 3. At speeds near the speed of light, the new concepts predict radically different results. Since we have little experience at such speeds, the concept that time slows down or that objects contract seems strange. Measurements at high speeds, however, demonstrate the validity of the theory.

The special theory of relativity proposes that the speed of light is the limit beyond which objects cannot move faster. If we imagine that objects move faster than light, then we see an event after its cause, and our logic of causality is destroyed. Consequently, we accept the speed of light as the upper limit for motion in the universe.

ANSWERS TO SELF-CHECKS

4A. Low-speed definition: speed of light is 4×10^8 m/s. The high-speed definition predicts that the observer measures the speed of light to be 3×10^8 m/s, the same as the speed of light relative to the earth. It does agree with Einstein's second postulate.

4B. Barbara and Allen do not agree. Barbara reports that light reaches the two ends of the train simultaneously. Allen reports that it does not. Because of the motion of the train past him, Allen reports that light reaches the back of the train before it reaches the front.

4C. 0.1 speed of light, 3.015 months; 0.4 speed of light, 3.27 months; 0.8 speed of light, 5.00 months; 0.9 speed of light, 6.88 months.

4D. In Allen's reference frame, 5 months elapse before Barbara picks her first ripe tomato. In Barbara's reference frame, 5 months elapse before Allen picks his first ripe tomato.

4E. The spaceship moves in the direction of the length of the window. Consequently, the window remains 3 m high. The length of the window depends on the speed of the spaceship: 0.1 speed of light, 1.99 m; 0.4 speed of light, 1.84 m; 0.8 speed of light, 1.20 m; 0.9 speed of light, 0.88 m.

PROBLEMS AND QUESTIONS

A. Review of Chapter Material

A1. What was the importance of the Michelson-Morley experiment to the special theory of relativity?

A2. State the two postulates of the special theory of relativity.

A3. What is a thought experiment?

A4. Describe what happens to the relative velocity between two objects as their velocities relative to the earth approach the velocity of light.

A5. Describe a thought experiment which shows that two events which are simultaneous in reference frame A are not simultaneous in a second reference frame moving at a constant velocity relative to A.

A6. State the equation for time dilation. Describe how measurements of the duration of an event vary with the velocity of the reference frame relative to the observer.

A7. Describe both a thought experiment and an actual experiment which show that the duration of an event changes in reference frames moving at high speeds.

A8. What do we mean when we say that time dilation is symmetric?

A9. State the equation for length contraction. Describe how measurements of length vary with the velocity of the object relative to the observer.

A10. Why do we think that light cannot travel faster than the speed of light?

B. Using the Chapter Material

B1. Edward is flying past you at a velocity of 200,000 km/s, east. He reports that the lights on the front and back of his spaceship are flashing simultaneously. Do you see them flashing simultaneously? When Edward is directly north of you, which one will you see flash first—front or back?

B2. Fran looks at the lights on Edward's spaceship (Problem B1). She reports that the ship's lights are flashing simultaneously. What is her velocity relative to Edward? What is her velocity relative to you?

B3. Barbara and Allen are moving toward each other at speeds (measured relative to the earth) that are two-thirds the speed of light. What is their relative velocity?

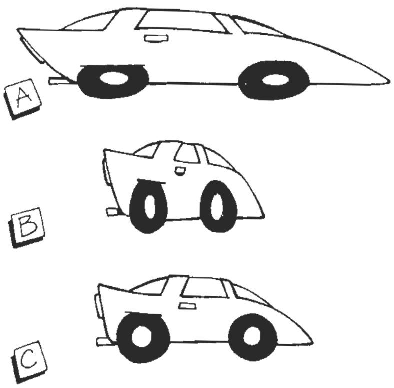
B4. During the flights to the moon the Apollo spacecraft averaged 55,000 m/s. If the astronauts had measured the speed of light reflected from the moon, what value would they have obtained?

B5. A muon moving at 80% the speed of light is located at the top of Mount Everest (altitude = 8848 m). In the muon's reference frame, how long is the distance to sea level?

B6. Suppose that the muon experiment had been conducted with muons moving at speeds of 0.6 the speed of light. What would be the half-life measured for these muons by an earthbound observer?

B7. Astronomers observe distant galaxies which are moving away from the earth at speeds of 270,000 km/h. At this speed an atom's vibration, which takes 1 μs on the earth, takes 2.29 μs when observed in the moving galaxy. Suppose an astronomer in the distant galaxy is looking back at the earth. What time measurement would she make for this same vibration on earth? What measurement would she obtain for an atom sitting motionless next to her?

B8. Three identical high-speed cars move past us. We see them as shown in Figure 4-B8. Which car is moving most rapidly? Which is moving most slowly?



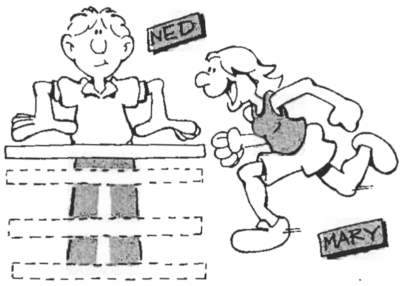
B9. A circle with a diameter of 1 m is moving to your right. Sketch the shape of the circle for each of the following speeds (given as a fraction of the speed of light): 0.4, 0.8, 0.9, and 0.999.

B10. An inventor claims that he can build an airplane which will travel from New York to Los Angeles at three times the speed of light. He asks you to invest money in his company to build his airplane. Explain why this might be a bad investment.

C. Extensions to New Situations

C1. Ned is holding a very long board as shown in Figure 4-C1. As Mary speeds by at 0.9 the speed of light, Ned drops the board so that in his reference frame both ends reach the ground at the same time.

a. What is the length of the board in Mary's reference frame?



b. In Mary's reference frame, do both ends reach the ground simultaneously? Why or why not?

c. If the answer to (b) is no, which end reaches the ground first?

d. Sketch a strobelike drawing of how the board must look in Mary's reference frame as it falls.

C2. Time dilation causes a difference in aging between high-speed space travelers and earthbound people. Consider a spaceship traveling at 0.99 the speed of light relative to the earth.

a. The space traveler's heart beats once per second as measured by the traveler. What time elapses on earth between the heart beats?

b. In 1 h, earth time, whose heart beats the most—the earthbound person or the space traveler?

c. The number of times the heart has beat is a good measure of aging. With each heartbeat we get a little older. In the earthbound reference frame, which person is aging more quickly—the earthbound observer or the space traveler?

d. Answer questions (a), (b), and (c) from the reference frame of the space traveler.

C3. The twin paradox is related to the result of Problem 4-C2. Jackie remains on earth while her twin brother Steve takes a high-speed space trip. He travels away from earth at 90% the speed of light, turns around, and returns to earth at 90% the speed of light. While traveling at a constant speed, Jackie and Steve each observe the other to be aging more slowly. Using the special theory of relativity, each would conclude that the other would be younger when they meet again. Thus, we have an impossible situation. Experiments with very sensitive clocks show that Steve will be younger. This result does not disagree with the special theory of relativity because one of its postulates is violated during the trip. Which one? How is it violated?

C4. Here is another paradox. A very high-speed airplane with a brave (or foolish) pilot is to fly through a tunnel in a mountain. When measured in a reference frame stationary relative to the tunnel, the airplane is longer than the tunnel. From the airplane's reference frame, the tunnel becomes even shorter at high speeds. In the earth's reference frame, an observer closes doors on both ends of the tunnel while the airplane is inside. This event could never happen in the airplane's reference frame. To see how this paradox is resolved, answer the following questions.
   a. The two doors are closed simultaneously in the earth's reference frame. Do the doors close simultaneously in the airplane's reference frame?
   b. Which door closes first in the airplane's reference frame?
   c. Use the results of (a) and (b) to describe the order of events as the airplane flies through the tunnel.
   d. How does the dependence of simultaneity on reference frame resolve the paradox?

C5. The volume of a box is its length times its width times its height. In a reference frame stationary relative to a box, its dimensions are: length = 2 m; width = 4 m; height = 6 m.
   a. What is the volume of the box in this reference frame?
   b. What volume do you measure when the box moves at 0.8 the speed of light relative to you in the direction of its length?
   c. What volume do you measure when the box moves at 0.8 the speed of light relative to you in the direction of its height?

C6. The nearest star, Alpha Centauri, is 4.3 light years (4.1×10^{16} m) from the earth.
   a. How long does light require to travel from Alpha Centauri to earth?
   b. Suppose space travelers move from earth to Alpha Centauri at the top speed of the space shuttle, 350 km/h. How long in earth time would the trip to Alpha Centauri take?
   c. How long, earth time, would the trip require if the travelers moved at 0.8 the speed of light?
   d. How far would Alpha Centauri be in their reference frame?
   e. How long, in the space traveler's time, would the trip take?
   f. Do trips to nearby stars seem feasible at speeds close to the speed of light?

C7. To see if trips to other galaxies might be possible, answer the questions in Problem 4-C6 for the Andromeda Galaxy, which is 2,200,000 light years (2.1×10^{22} m) away.

C8. Suppose two muons are coming toward each other. One is moving relative to the earth at 0.8 the speed of light, east, and the other is moving relative to the earth at 0.8 the speed of light, west.
   a. What is the relative velocity between the two muons?
   b. What will be their relative velocity if they move away from each other rather than toward each other?

D. Activities

D1. The quotation which opened this chapter is taken from "Mr. Tompkins in Wonderland" by George Gamow. In this story, Mr. Tompkins dreams that the speed of light is 10 mi/h. Describe some of the effects Mr. Tompkins will see as he pedals his bicycle around Wonderland. Then, read the story and compare your description with Dr. Gamow's. (The story has been published as part of *Mr. Tompkins in Paperback* (Cambridge University Press, 1969.))

D2. Gene Rodenberry, the creator of the science-fiction television and film series *Star Trek* was accused of not understanding physics because starships in the series moved at speeds greater than the speed of light. Rodenberry replied that he had no choice. If they did not move faster than light, the starships could never travel between galaxies. Discuss his answer using the results of Problem 4-C7.