MATHEMATICS AS STRUCTURAL LANGUAGE OF PHYSICAL THOUGHT

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INTRODUCTION

Scientific knowledge diverges considerably from that which is used in everyday life. Some aspects make this very clear, as the particularity of science making use of abstract concepts that establish an indirect relationship with familiar objects and situations. Some authors use the expression "scientific culture" (Joshua and Dupin, 1993, Lemke, 1998a, Astolfi, 1994, Bachelard, 1938) in opposition to "common sense culture" as a way to stress the difference between both these types of knowledge. In comparing these two cultures, the language used is a considerable source of differentiation. Given that science generally uses mathematics to express its ideas, contrary to what commonly takes place. Among the experimental sciences, physics is the most formalized one. Since the XVII century, physics has treaded an ample and safe course towards crescent mathematical practices. (Paty, 1999). Currently, Mathematics is definitely lodged in the core of physics, hence becoming evident when encountering the products of its scientific activities. In the literature, as books and articles, one can perceive that mathematics enters the physical discourse by means of functions, equations, graphics, vectors, tensors, inequations, geometries, among others. Owing to the important role mathematics has played in the organization of physical theories, some authors view its adoption as a criterion of rationality, and not merely as indicative of conventionality or empiricism (Simon, 2005). Paty[2] approached a similar theme when he proposed that intensifying mathematical knowledge, particularly that which is employed in the organization of physical problems, broadens human rationality. This is apparent in the debates concerning mathematical systems to represent theories. Far from comprising simple choices of convenience, the definition of formal systems has been the object of questions that involve physical signification of theories (Silva and Martins, 2002).

Researchers from all areas of Physics have no doubt that without Mathematical cognizance, by no means simple knowledge, it is unfeasible to produce good physics seeing the progress being made towards breaking research[3].

In physics teaching, mathematics is often considered responsible for scholastic failure. It is customary to hear from teachers that their students do not understand physics on account of their fragility in mathematical understanding. Many consider that a solid mathematical base in the years that precede physics teaching guarantees successful learning.

The training for physics research presumes that mathematics is definitively lodged in the body of sciences and the university curriculums reflect such perception, with substantial em-

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[1] Financial support from Brazilian Science and Technology Council.

[2] Paty, 2000

[3] Some attempts came far over the last century. See, for exemple, **The Mathematical Training of Social Scientists, Report of the Boulder Symposium,** *Econometrica*, Vol. 18, No. 2 (Apr., 1950), pp. 193- 205, (http://links.jstor.org/sici?sici=0012-9682(195004)18:2%3C193:TMTOSS%3E2.0.CO;2-A)

phasis on mathematical preparation. The study programmes in physics include many mathematics subjects (linear algebra, geometry, advanced calculus, etc.) and, often, it is difficult to differentiate them from courses in mathematics[4]. The overabundance of mathematics is often held accountable for demotivating some of the students entering such courses. The mathematical barrier is too high for many students intending to study physics, thus contributing to high dropout rates and course transference.

The relationship inferred between physics and mathematics from university curriculums reflects a type of “professional pre-requisite” hierarchy: to study physics one must know mathematics, that being the case, let us teach it first! However, posing the question like this covers up the problem of knowing how mathematics should be taught when the intention is to use it as an instrument of thought in physics.

In general education, the situation does not differ much. As a rule, teachers and students alike agree that mathematics is one of the greatest problems in physics teaching/learning at school. For example, kinematics, which is widely taught in secondary education, is strongly based on the knowledge of *functions*. It is not uncommon for teachers to strive in the physical interpretation of problems, even presenting the function that represents the problems solution to then say: “from now on it is only mathematics and the solution to this was already presented to you in a previous subject”. This implies that once the problem has been understood, from a physical point of view, from then on such competencies are no longer that teacher’s responsibility. The transformation of the problem is a mathematical algorithm and solving this would depend on skills learned in other subjects. Frequently, physics teachers end up ascribing to mathematics the accountability for learning difficulties and not to what they teach. Errors by students in solving high school-level equations, calculus of angular coefficients of curves in graphs, solving systems of equations, etc. are common, hence reinforcing the concept that we are dealing with a lack of mathematical knowledge.

Redish (2005) discusses this standpoint by saying that “…the language of mathematics we use in physics is not the same as the one taught by mathematicians. There are many notable differences” (page 1). Admitting that a many of the problems in physics teaching are found in commanding Mathematics reflects a naive epistemological positioning and ends up considering the latter an instrument of the former! It is vital to specify the role played by mathematics in building up physical cognizance, because as long as constitutive knowledge of natural sciences exists, just like a boulder, there is an eternal dilemma[5] of placing between *concrete* and *abstract*, between *reason* and *experience*. Many consider it merely as an instrument of empirical method, while for others; it is the very essence of actuality, with physics as the method by which to attain it. The eventual solutions will find support in deeper analyses of the relationship that physics maintains with mathematics. There is a need to probe deeper with regards to the formation of physics cognizance, in order to better evaluate the function of mathematics in teaching. The manner by which this has been approached within the context of physics education transforms mathematics teaching into a *pedagogical-obstacle*[6]. To collaborate in order to overcome it is this work’s proposal.

[4] It is usual to have a basic two year course for all science courses, as Mathematics, Physics, Chemistry or Engineering.
[5] Paty states that is a "drama between the Real and the symbolic abstract". Paty, 1988, pp 234
[6] See Astolfi 1994.

HISTORICAL APPROACH TO THE RELATIONSHIP BETWEEN PHYSICS AND MATHEMATICS

We learn to see physical laws expressed in mathematical language. Although it seems natural that it is so, a more critical view can reveal that it is a relationship built over time. For Bunge (1973, 1985), the capacity of mathematical systems to correctly represent natural phenomenons is the outcome of historical success, that is, a result of a process legitimized by time. For Lautman, "there is a physical reality and the miracle in explaining this is the need for more developed mathematical theories to explain it"[7]. Going back to history offers the opportunity to evaluate other manners of expressing "laws" of the natural world. In ancient times, in the Middle Ages and Renaissance, natural phenomenons as the falling of bodies and the movement of the stars were interpreted by means of conceptual systems very different from those sustained by modern science. The Egyptians and the Babylonians elaborated cosmological systems, including forecasting celestial events and calendars that did not comprise mathematics as their main basis. (Kuhn, 1957) Refraining from deviating too much from the modern scientific tradition, we can take as an example the "Aristotelian Physics", whose interpretation of the physical world was based on the idea of a *natural place* and in the law that asserts that the bodies sought out *their natural place* in the universe: bodies with an *earth* type essence would be closer to the center of the Universe, contrary to the *fire* types, which would be in the peripherical part of the terrestrial sphere (Ibid). Also, the arguments apart from these thoughts, by Buridan, Oresme and other medieval scholars were not mathematical either. Originally, such thoughts were described in Medieval Latin, making use of refined arguments, but presented in non-formal language, as in most of the pre-scientific treatises of that time.

It was in the XVII century, with the coming of modern science that natural phenomenons began to be systematically expressed by means of mathematical relationships. Such practice became the Pythagorean traditional heritage. In it, nature was conceived by means of analogies between phenomenons and the idealized relationships that were elicited. Geometry was nature's language *par excellence*, with the world as its arena for inspiration and application of the relationships produced there. Mathematics was the lining of ideal forms that were believed to be in the very essence of nature[8]. Galileo introduced a small modification in the Pythagorean tradition. For him, Mathematics was an understanding that enabled direct interpretation of nature (Paty, 1988)[9]. Galileo presented his idea on the language of nature in the *Il Saggiatore* text.[10] With regards to the Galilean conception Paty states that:

To justify the mathematical character of magnitudes and laws in physics, Galileo invoked the idea that the «Book of Nature» is written in the language of figures and numbers. "Its type letters", he wrote, speaking of the Universe "are triangles, circles, and other geometrical figures, without which it would be impossible to a human being to understand a single world of it". And he added that all properties of external bodies in nature can be attributed, in ultimate analysis, to the notions of "magnitudes, figures, numbers, and slow or fast, and those

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[7] Albert Lutman, in Paty (1989), pp 235.
[8] Paty, 1988
[9] For Galileo this language was basically geometry. The history of science shows that algebra took the place of geometry, in particular with the advent of the Newtonian mechanics base on the idea of instantaneous action at distance.
[10] Galileo, in *Il Saggiatore* (Galileo [1623]).

have effects on our sensorial perceptions, and are, so to speak, the true essence of the things" [11].

Within the Galilean exemplar, geometry maintains its status as nature's preferential language, but now as a resort of thought for its theoretical structure. This process is outlined as a "mathematical translation", where the scientist is the translator owing to his capacity to traverse between both "languages": that of nature and that of mathematics. Another scientist with an important role in this subject is Newton. For him, the laws of geometrical mechanics and gravitation accomplish his intentions, which are directly asserted from his book's title, *The mathematical principles of natural philosophy*[12]. The mathematical principles, referenced in the title of his most widely acknowledged work, were related to a synthetic geometry, tributary, to some extent, of his conceptions concerning mathematization of mechanics and physical laws. Mathematics in mechanics was a consequence of the world's neo-platonic conception, requiring to be described in terms of absolute, actual and mathematical concepts, as *space, time* and *force* among others. (Paty, 1999)

The succeeding developments of mechanics, as in the works of Leonhard Euler, Alexis Clairaut and Jean le Rond d'Alembert in the XVIII century, although tributary of the original Newtonian approach, legitimize mathematization in other terms. The neo-platonic conception and fundamental mathematical greatness were substituted by a more neutral metaphysics that expunged the idea of *force* from the theoretical body of physics, among others. Subsequently, a tradition of "physics-mathematics" is installed, in which mathematical translation is properly constituted in physics *mediation*. Within this context, mathematization is conceived as inherent to the concepts and support for their construction. Ampère (XIX century) was a backer of this new type of conception, since his procedure intended to "choose the most radical mode of a conceptual approach of mathematization (of experimental knowledge)… of maximally curtailing the distance between the mathematical discourse and concrete data that he was destined to inform and elucidate"[13]. This tradition was implanted in physics research by use, demonstrating the symbolic intensity on its own, and currently reaching its greatest refinement through modern physics theories, where it is impossible to think empirically without the aid of highly sophisticated mathematical symbolism.

Redish states that in physics, "the purpose [of using mathematics] is a representative meaning of systems rather than expressing abstract relationships" (Redish, 2005, page 1). A number of recent historical studies offer interesting examples of how this takes place. For example Silva and Pietrocola (submitted) studied how mathematics extracted from well defined domains in the XIX century served as basis for structuring electromagnetism. They exposed the way William Thomson, James C. Maxwell and others developed models and analogies to explain both electric and magnetic phenomena based on the existence of ether. Works by Thomson and Maxwell provide an interesting opportunity to evaluate the use of analogies in the construction of physical understanding. In this case, construction was done from *material* and *formal* analogies. In the first case, the analogy was based on the idea that electromagnetic ether was similar, as for example, an elastic solid. In the second case, the analogy was based on the fact that the mathematical formulation of known laws on thermal phenomena was the same that governed electromagnetic phenomenons. (Silva e Pietrocola, submitted).

[11] Paty, 1999, p. 9
[12] Newton, 1687. See also Whiteside [1970]
[13] Merleau-Ponty, in Paty 1988, pp 234, note 3.

In another historical episode, the choice of mathematical formalisms denotes the search for signification to appropriately interpret physical phenomenons. Silva & Martins (2002) have produced a historical study concerning the debate of choosing the best mathematical formalism to organize the electromagnetic theory during the 19th century. The choice of vectorial formalism in opposition to the formalism of “quaternions” is grounded on arguments that include elegance and simplicity, but above all, the possibility for rigorous and faithful formulations to the ideas in the electromagnetic theory. This sort of debate discards the idea that it was only about deciding between equivalent ideas. What was at stake at that time was the choice for a better adapted formalism to learn the essence of the electromagnetic phenomenons under study. The vector theory, for instance, lends its meanings and structure to the electromagnetic theory; i.e., we will apply that the nature of electromagnetic entities is given by the vector idea. The arrow above a letter, for example, $\vec{E}$, indicates that the electrical field is a physical quantity with magnitude and direction[14]. The vector language has its own grammar, syntax and spelling which are the axioms, theorems, lemmas, rules and so on. When a physical concept, such as electrical field, is written as $\vec{E}$, it also assumes all the rules of vector language.

Paty emphasizes the constructive role of mathematics in physics with the following: “ the physical thought … is constituted by constructing its concepts from mathematical greatness: laden with a “physical”, “real” or “empiric” content, they retain what is essential of the formal contents they were granted (or redeemed, in the case when Physics was the object of its formation), meaning its construction in the purest mathematical sense ” (Paty, 1999, Pag.1).

MATHEMATICS AS THE LANGUAGE OF PHYSICS

A productive manner of pondering over the relationships between mathematical language and teaching scientific knowledge was considering the historical evolution of thoughts of the natural world. As previously discussed, centuries, if not millennia, were necessary for scientific thought to find support in mathematized language[15]. From the Greeks to the French illuminists, historical episodes reveal the difficulties scientific thought encountered to structure itself from geometry, algebra and other logical systems, striving to interpret natural phenomenons. Expecting that our students naturally incorporate the mathematical language as an instrument of scientific thought is to accredit the view that science merely describes an inherently organized world. It is to believe that the existence of mathematically structured theories on science prescinded previous stages, when the mathematical symbols did not yet represent concepts. Regarding this aspect, there are two important considerations.

The first one concerns the fact that there is a very distinct rupture between the type of language for regular everyday use and for science (Lemke 2002, Astolfi, 1994). Science is neither constructed nor communicated by language; oral speech or written language. If we consider all of the processes of scientific production, from preliminary stages of knowledge, when consensus, certainties, convictions and norms had not yet been obtained, the scientists’ language was hybrid: incorporating aspects of common language and formalized language. Sutton distinguishes two types of language in science: *interpretative* language – during initial

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14 There are some tricks on the use of an arrow to represent vector quantities. There are two kinds of vector quantities (polar and axial), but just one symbol. For more details, see (Silva and Martins 2002).
15 See Paty, 1989.

stages of research; and *formal* language in the final stages, when the constructive process is complete (Sutton, 1996). The final phases are emphasized in science teaching, distorting the various uses of language by scientists, hence inducing students to consider that a mere description of pre-existing facts was carried out. (Pietrocola, 2005) Such affirmation allow understanding why students associate the work of scientists more with *discovery*, than *invention* (Ryan and Aikenhead, 1992), and suppose that scientific truths pre-exist knowledge (Sully, 1989).

Lemke (1998a) adds that in the case of science, particularly of physics, there is a blend of verbal-*typology* and mathematics-*topology* meaning. The verbal-*typological* menaning, characteristic of spoken and written language, includes such discrete categories as hot and cold, linear momentum and angular momentum, above and below, small and big, and fast and slow. However, the topogical meaning, characteristic of formal language (generally scientific), includes continuous categories, such as temperature, distance, time, volume, velocity, etc. It is probable that science, upon dealing with phenomenons as displacement of bodies, flowing of waters and passing of time, was confronted with a limitation of discrete categories of common language and constructed, historically, a language based on continuous categories. (Lemke, 1998a). By constructing linguistic bridges between both categories, scientists were able to accurately deal with phenomena such as as the launching of projectiles and the oscillation of a pendulum. In describing such phenomena, ideas such as, for example, fast and slow were translated by mathematical relationships, where velocities increase and decrease with time. Thus, adapting the language of science involves the skill of using both types of verbal components: typology and topologic. Verbal discourse mathematical associations, visual and graphical representations must be integrated in such a manner as to interpret correctly the physical world. (Belucci and Carvalho, 2005).

It is absorbing to note how the complex process of integration between common and scientific languages, which characterize the constructive process of scientific knowledge, is hastily forgotten within science. The synthetic power of mathematical language and of the relationships produced between the topologic categories provides scientists with a comfortable operational representation, that can, temporarily, free them from the idiosyncrasy of the typological categories of common language. It would be as if, through mathematized representation, the physicist could “see” what is taking place, even when dealing with phenomenons far from the senses, as in the molecules of heated gas or the electrons in a current carrying conductor. (Roth, 2003) Language standardizes and disappears from the initial phases of research, where language is less reliable and subject to inaccuracies, approximations and doubts (Halliday and Martin, 1993; Sutton, 1996, Pietrocola, 2003).

Vigostiki, describing the pre-history of language, provides arguments very close to those mentioned earlier (Vigotsky, 1985). He states that a child can develop spoken language on his own. However, written language is artificial; therefore, needs to be taught. He also adds that, traditionally, teaching writing was performed technically. Such perception of teaching language as a technique is valid up to the present, and not only for written language but for mathematical language in science as well. Teachers believe that due to students operationally commanding some mathematical systems, such as functions, geometry, Cartesian coordinates, etc. they are adept at dealing with physical phenomenons (Redish, 2005). This prompts them to consider that technical mastery of mathematics is sufficient for the scientific thought in grasping the physical world. They fail to remember that scientific thought does not mathematically describe the world, but initially interprets it to later describe it (Poincaré, 1897; Bachelard, 1938).

Written language and mathematical language are specific systems of symbols and signs, which beyond its technical dimension, represent ideas that transcend its internal meanings – both languages are second order symbolisms. In written language, signs represent sounds and words that reproduce speech, which are signs of associations and entities of the world. In such a case, speech is the intermediary link between the world and written words. In the mathematical language used in physics, mathematical symbols and signs represent concepts, which represent objects of the scientific world. In both languages, the continuous use is directly connected to the world's entities and the relationships it intends to represent. That is, habitual practice makes such languages become first order concerning the world. Once this takes place, thought no longer takes into account the intermediary stage of mathematical symbols, but adapts mathematical language as a structure directly related to that part of the world's functionality that it intends to represent. Subsequently, mathematical language becomes *structural* to scientific thought, hence enabling the organization of knowledge. [16] Physicists do not solely think of using common language (colloquial), but make use also of matrix language, functions of probabilities, etc., which serve to represent atoms and molecules. Cosmologists organize thought in agreement with the dictates of tensorial language. Chemists, biologists, and even economists, employ specific mathematical languages to deal with their specific areas. For most of today's scientists there are no alternatives: the important problems and solutions must be expressed mathematically, that is, in agreement with determined mathematical languages firmly established by use.

DIDACTIC-PEDAGOGIC IMPLICATIONS

Mathematics is an essential part of the necessary knowledge to learn physics. Two modes can be underscored by which teaching mathematics in physics allows learning the physical contents. The first one is grounded on the technical domain of mathematical systems, such as operations with algorithms, construction of graphics, solution of equations, etc. We consider such characteristics as being connected to the internal context of mathematical knowledge and will designate them as *technical skills*, in the sense of being able to deal with specific rules and properties of mathematical systems. The second one is based on the capacity of employing the mathematical knowledge for structuring physical situations. We consider such characteristic to be connected to the organizational use of mathematics in external domains and will designate them as *structural skills*. There is a myth about the relationship between physics teaching and mathematics teaching that can be overturned when there is clarity concerning the differences between both of these skills – while the first skill can be obtained outside physics education, meaning, in subjects exclusively mathematics; the second one cannot be. The capacity of dealing with mathematics within its own situations does not warrant the capacity of using it in other areas of knowledge, as physics. In other words, to have technical command of mathematics does not guarantee the capacity of employing it to structure thought in other domains.

This implies that there must be a didactic-pedagogical intention in preparing the physics students to make structural use of mathematics. The authors of didactic books, the formulators

[16] For more about the structural function of mathematics in Physics see Pietrocola, 2002 and 2005

of curricular programs and physics teachers in general should have clarity with regards to such a need, so as to not underestimate the resulting difficulties. The fact that technical skills in mathematics do not convert into structural skills generates pedagogical-epistemological obstacles. These types of problems have been dealt with in science didactic literature by means of the concept of *objective-obstacle* (Martinand 1986). The structural skill is an obstacle since not attaining it, thus remaining at the technical level, prevents complete appropriation of physics knowledge. Structuring thought in students based on the languages that mathematics offers becomes an objective to be pursued by physics didactics.

With regards to contemporary research, the area of *mathematical modeling* (Bassanezi, 1994) proffers works that serves the proposal of endowing thought with structural skill. For the subject matter of teaching physics, mathematical modeling needs to explicitly incorporate the empirical domain, that is, involve experimental activities. A suitable modeling activity should inevitably be concerned with passing on "raw" data contained in a phenomenon for its conceptual representation. In this direction, Pinheiro et al. (2001) and Pinheiro (1996) submitted activity proposals to introduce mathematical modeling practices to students of primary education using natural phenomena by means of experimental activities. In this article's addendum we presented one of these teaching activities contextualized for a pre-university level work. This type of proposal faces difficulties if it is to be implemented in the scholastic context, given that often it collides with the rigidity of academic curriculums, the excessive context emphasis to be transmitted in the subjects and the traditional manner of conceiving the teaching of physics at this level as consequences of single-solution closed problems.

Another manner of addressing the structural skill with mathematics is to modify the way problem solving is approached in physics. Redish (2005) discusses the critical stages to be considered in mathematics in solving physical problems. One of these stages is to stimulate the students to work with symbols in limiting situations. According to his argument:

"Taking the limiting case of either of the two masses [in a half-Atwood's machine] going to zero (or infinity) is an example of considering an ensemble of experiments rather than just a single one and is also a nice example of physicists' willingness to treat constants (the masses) as variables." (ibd, pg. 4)

FINAL CONSIDERATIONS

Although other living entities also communicate, creative language that interprets, projects ideas, and provides the means of argument belongs exclusively to human beings. Endowed with such use of language, the human mind surpasses the world that is instantly accessible to perception and transcends limits in space and time. Thus, what separates us from other living beings is not language as a form of communication, but the capacity we have of creating a world of ideas through language. The universe of words of a human being is ten thousand times more expansive than a Rhesus monkey. Such difference reflects our capacity to imagine that which we cannot touch, cannot see or access with our senses and which enables us to construct a rich world of ideas. That is, our thoughts are expressed through words that we construct and then use them to communicate with and through them, without intermediation. Words are codified ideas and comprise the raw material of our thoughts. By integrating words in sentences, we express ideas and thoughts. The human language is evidence of how thought deals with ideas, articulating one into another in the construction of meaning.

There is not always a direct correlation between meaning in the world of ideas with those of the real world. In this case, we are in exclusive command of the research that moves our thought towards what is new. In this consensus, language must be understood as the mode we have to structure thought, while striving to interpret the phenomena of the world that call our attention. Science has sophisticated the use of language, determining special modalities for each research stage. More interpretative language fits well the initial moments, when problems and doubts are inevitably part of the process; more descriptive language is well suited for the safer domains studied, where the answers often surpass the question initially formulated; and finally, mathematized language is portrayed when it partakes of moments when science is able to transcend a second order language and starts to reason from its own concepts included in formal systems. In this final stage, a type of thought is activated, which is characteristic of science. Within it, thought departs from the immediate world and releases itself to safely prospect the limits of the known world, achieving intimacy with matter, the limits of the Universe, the borderlands of human perception. Bachelard had already stated that the solidity of mathematics resides in the fact that it is *a safe thought of its language* [17].

Enabling students to perceive the possibilities that scientific thought acquires by means of mathematical language should be part of the objectives of scientific education. However, it must be clear that restricted emphasis on technical command of such language is not enough. It is necessary to teach students to learn the world through the many languages of science, showing the importance of the role and function performed by each of those languages. Particularly, mathematics is increasingly becoming a language of many branches of science. That being the case, it is important that in physics teaching the role of mathematics be contemplated in structuring thought. Without this, it will be difficult for recent knowledge produced by science to become the object of education and learning in schools.

[17] Cited by Paty, 1988.

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