Modeling students' conceptual understanding of force, velocity, and acceleration

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Abstract. We have developed a multiple choice test designed to probe students' conceptual understanding of the relationships among the directions of force, velocity, and acceleration. The test was administered to more than 800 students enrolled in standard or honors introductory physics courses or a second-year physics majors course. The test was found to be reasonably statistically reliable, and correlations of test score with grade, course level, and the Force Concept Inventory were moderate to strong. Further analysis revealed that in addition to the common incorrect response that velocity must be in the direction of the acceleration or net force, up to 30% of students gave "partially correct" responses, for example that velocity can be either opposite to or in the direction of the acceleration or net force but not zero. The data also suggests that for some students their evolution of understanding may progress through this kind of partially incorrect understanding.

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INTRODUCTION

Student difficulties with conceptual questions about the relationships among force, velocity, and acceleration have been extensively documented. For example, many students believe that if an object is moving, then something must be pushing it in the direction of motion [1, 2, 3, 4]. Conversely, students commonly believe that if there is a net force pushing on an object, then the object must be moving in the direction of the push [3, 5]. Similar mistakes in students' understanding of the relationships between the direction of velocity and acceleration have been documented [6, 7]. However, there has been no single systematic study of student understanding of all paired relations among the concepts of force, velocity, and acceleration.

Here, we report on results of a simple test instrument, called the FVA test, that we developed to assess student understanding of the relations between the direction of force, velocity, and acceleration of an object in one dimension. Each item in the test provides a simple scenario indicating the direction of one of the vectors for an object, say acceleration $\vec{a}$, and asks the student what this implies about the direction of one of the other vectors, say velocity $\vec{v}$. We label such a question as $\vec{a} \rightarrow \vec{v}$. Table 1 provides an example of an $\vec{a} \rightarrow \vec{v}$ question type and the answer choices.

TABLE 1. Example of an $\vec{a} \rightarrow \vec{v}$ question. The prompt is "A car is on a hill and the direction of its acceleration is uphill. Which statement best describes the motion of the car?"

a) it is moving uphill	"Misconception"
b) it is moving downhill	
c) it is not moving	
d) both a and b are possible	Cannot-be-Zero
e) both a and c are possible	Cannot-be-Opposite
f) a, b, and c are possible	Correct

The answer choices were developed from our previous work on these questions and typically allow the students to respond with one of six choices: the two vectors are in the same direction, the vectors are in opposite directions, the second vector is zero, or that there are multiple possibilities (for example the second vector could be either in the same direction or zero) [8].

The FVA test is comprised of 17 items, including two questions for each of the relations $\vec{F} \rightarrow \vec{v}$, $\vec{v} \rightarrow \vec{F}$, $\vec{a} \rightarrow \vec{v}$, and $\vec{v} \rightarrow \vec{a}$ and one question each for $\vec{a} \rightarrow \vec{F}$ and $\vec{F} \rightarrow \vec{a}$. The other seven questions are an assortment of filler questions asking about objects that are speeding up and slowing down. These filler questions allow for some variety in the questions and avoid having all questions in the test with the same choice for the correct answer.

While there are six answer choices, most students choose among only four. These are represented in Tables 1 and 2 by a, d, e, and f answer choices. These choices represent

TABLE 2. Example of an $\vec{F} \rightarrow \vec{v}$ question. The prompt is "At a particular instant of time, there are several forces acting on an object in both the positive and negative direction, but the forces in the negative direction (to the left) are greater. Which statement best describes the motion of the object at this instant?"

a) it is moving to the left	"Misconception"
b) it is moving to the right	
c) it is not moving	
d) both a and b are possible	Cannot-be-Zero
e) both a and c are possible	Cannot-be-Opposite
f) a, b, and c are possible	Correct

TABLE 3. FVA Test Statistics

Course	N	Average Score on FVA	K-R 20 α (Cronbach's)	Correlation: Grade/Score	Correlation: Grade/Misconception
Calculus Mechanics Course	228	$37.6 \pm 17.7\%$	0.721	0.311	-0.286
Honors Calculus Mechanics Course	86	$57.9 \pm 21.4\%$	0.801	0.428	-0.508
Second Year Majors	65	$69.2 \pm 22.8\%$	0.849		

respectively the common "misconception" that the two vectors must be aligned, the "Cannot-be-Zero" response which allows for the vectors to be either aligned or opposite, but a non-zero value of the first vector implies that the second cannot be zero, a "Cannot-be-Opposite" response which allows for the possibility that the second vector can be zero, but not opposite, and the correct answer.

TEST VALIDITY AND RELIABILITY

This is a brief summary of some evidence supporting the validity and reliability of the test (see also Table 3). The construct validity of the test questions, answer choices, and format was supported through several stages of interview and testing-based modifications, as reported previously [8]. Correlations of the FVA test with other measures such as course level, course grade, and the FCI (all measures of student knowledge) help to support the criterion validity and the construct validity of the test. First, the average score of the FVA test increases with the course level, with first year student scoring lowest and second year scoring highest. Second, as reported in Table 3, there are moderate (>0.3) correlations of FVA score with grade in the class. Likewise, the FVA "misconception" responses are negatively correlated with grade (avg. r = 0.397). Third, there is a relatively strong correlation of FVA score with FCI score (r = 0.569), while the correlation of FCI with final grade was 0.387.

EVIDENCE FOR EVOLUTION OF STUDENT UNDERSTANDING

Students' responses for the $\vec{v} \rightarrow \vec{F}$, $\vec{a} \rightarrow \vec{v}$, and $\vec{F} \rightarrow \vec{v}$ questions are shown in in Figure 1. They have been broken down into the four common responses and are separated out by class. While the different questions all have different answering choice patterns, there are obvious similarities between questions of the same type and between different classes. For all six questions, the percentage of correct responses increases as class level increases so that the score for the second year major's is on average 0.9 standard deviations above that of the regular mechanics course. A one way ANOVA shows that the difference is significant at $p < 0.001$ for all six questions. Similarly, for all questions, the percentage of misconception responses decreases as class level increases. Thus, the average student in the higher level course does better on the FVA test, by both decreasing their misconception responses and increasing their correct responses, than the average student in the lower level course.

However, the change in misconception score between two classes is not always equal to the change in correct score. This is because the fraction of students choosing Cannot-be-Zero and Cannot-be-Opposite responses depends on the class level. Considering the difference between the regular mechanics course, and the honors mechanics course, it is apparent that the decrease in misconception is greater than the increase in correct responses because students are choosing one of the partially-correct responses. Also, when the difference between the honors (white columns) and second year (lined columns) course is considered, it is apparent that the decrease in misconception is much less than the increase in correct responses because students are not choosing the partially-correct responses as often.

These patterns of response are consistent with the average student evolving, from an initial high level of misconceptions to the correct answer, by passing through a partially-correct response "state", which shows more knowledge than the common misconception but lacks the completeness of the correct response. (For example, question four, which is the questions in Table 1, has Cannot-be-Zero as a commonly picked response for the honors students. This means that the students knew that the velocity could oppose the acceleration in addition to pointing in the same direction, but they did not know that velocity could be zero for a nonzero acceleration.)

Naturally, there is a danger in interpreting the data this way, since it assumes that the different classes can be used as sequential snapshots of a given population of students changing over time. This assumption is not without problems. The students in the honors mechanics section are a selected group from the regular pool of mechanics students. In addition, most of the students who take either the regular or honors calculus mechanics sections will not go on to participate in the second year majors course. Thus, there are selection effects that may significantly change the population of students in each course. Clearly a longitudinal study is needed to more carefully determine how student understanding on this topic evolves. Nonetheless, this data suggests the interesting possibility that students may evolve though a partially-correct "state" on the path to fully understanding the topic.

DIFFERENCES IN $\vec{V} \rightarrow \vec{F}, \vec{A} \rightarrow \vec{V}$ & $\vec{F} \rightarrow \vec{V}$

When the different question types are considered several interesting patterns emerge. First, for $\vec{a} \rightarrow \vec{v}$ and $\vec{F} \rightarrow \vec{v}$ ques-

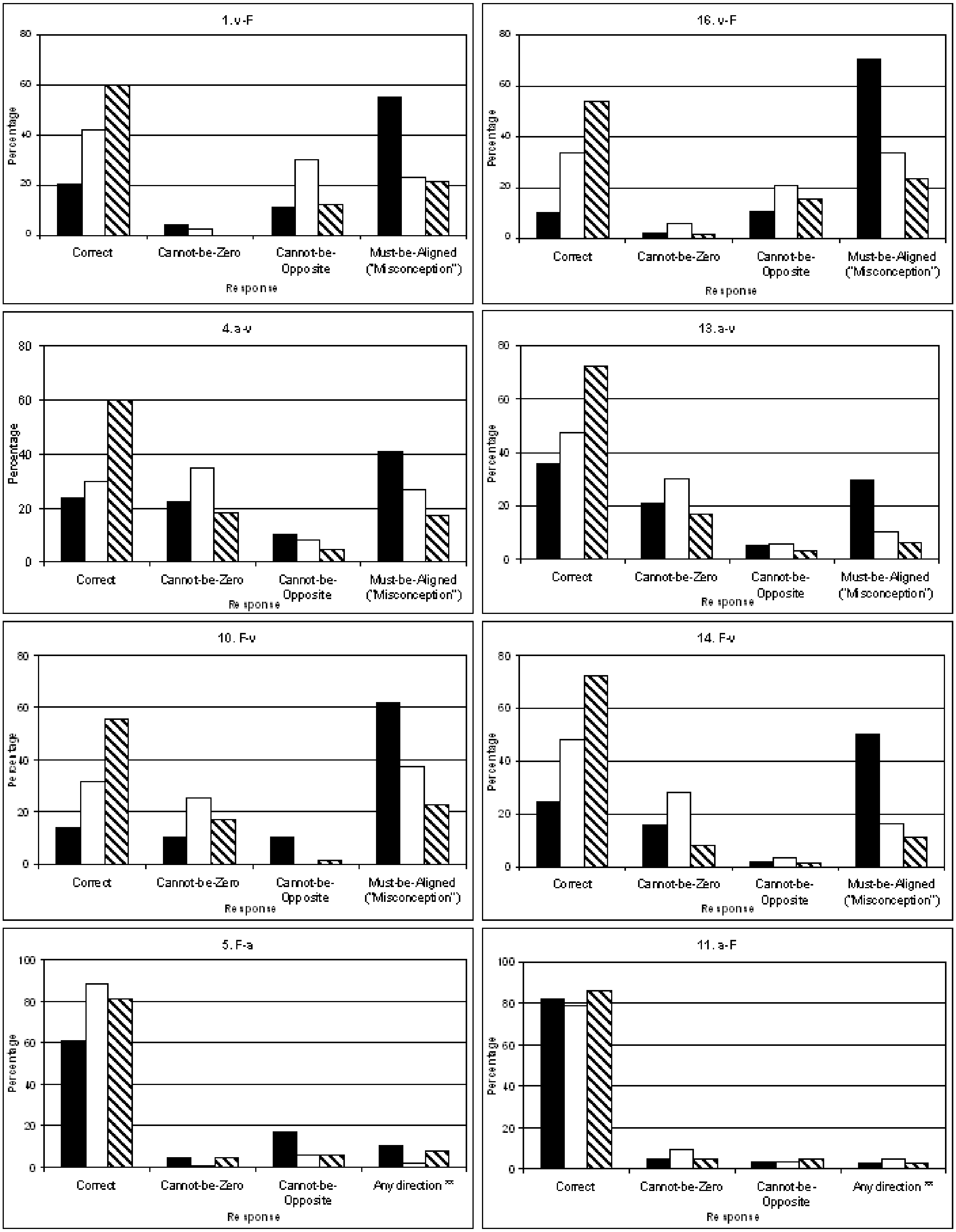

FIGURE 1. FVA Data from 3 Courses. Calculus Mechanics Course: Black Columns (N = 228)
Honors Calculus Mechanics Course: White Columns (N = 86) & Second Year Majors: Striped Columns (N = 65)
**Students respond that $\vec{a}$ and $\vec{F}$ can be aligned, opposite, or one can be zero.

tions students choose the Cannot-be-Zero response more frequently than the Cannot-be-Opposite response while for $\vec{v} \to \vec{F}$ questions the pattern is reversed, namely students choose the Cannot-be-Opposite response more frequently than the Cannot-be-Zero response. Thus when students are told that an object is accelerating or that it experiences a net force in a certain direction, they are more likely to believe that the object can be moving opposite to this direction than they are to believe that it could have zero velocity. Conversely, when they are given the direction of the velocity they are more likely to respond that the force can be zero more often than the force can be opposite the motion. This trend holds for all classes and questions except question 10 for the regular mechanics course. While these differences are relatively small, they were found to be statistically different for the mid-level class, the honors intro mechanics course, with $p \leq 0.003$, but they were not statistically different for the lower or higher level class. This would suggest that there is not a large inherent imbalance, or preference, for the Cannot-be-Zero or Cannot-be-Opposite model that exists in beginning mechanics students or in high level mechanics students. But, for mid-level students there is a large imbalance in which model is preferred. If the use of the three classes as time evolution pictures is valid, then this might suggest that students move through a different middle model for $\vec{a} \to \vec{v}$ and $\vec{F} \to \vec{v}$ questions than for $\vec{v} \to \vec{F}$ questions.

Another interesting pattern to the data is that the differences in answering pattern between question types tends to become less pronounced as class level increases. For example, the regular mechanics course shows about a 27% difference in misconception between $\vec{v} \to \vec{F}$ and $\vec{F} \to \vec{v}$ questions, but the honors mechanics course has only a 10% difference. One might argue that misconception percentages are getting low causing floor effects. However, if the correct responses, which are too low for cieling effects, are considered, there is a 14% difference in correct answering for the regular course but only a 1% difference for the honors course. Similar patterns are seen when comparing $\vec{a} \to \vec{v}$ and $\vec{F} \to \vec{v}$ questions and $\vec{F} \to \vec{a}$ and $\vec{a} \to \vec{F}$ questions. These patterns might suggest differences in how students are thinking about the different quantities. For example, greater reliance on formalisms such as equations should lead to a decrease in differences for $\vec{a} \to \vec{b}$ and $\vec{b} \to \vec{a}$ questions. This might also suggest differences in rates of learning for the different relations. So that, while $\vec{F} \to \vec{v}$ is initially understood better, the $\vec{v} \to \vec{F}$ and $\vec{a} \to \vec{v}$ relations are learned faster, and they catch up to the $\vec{F} \to \vec{v}$ percentage by the time students are at a post honors level. (This again assumes the time evolution model is valid.)

CONCLUSION

We have developed a simple multiple choice test to probe students' understanding of the relationship among the directions of force, velocity, and acceleration. This test is statistically reliable, and it has reasonably strong correlations with other measurements of students' knowledge of mechanics such as score on the FCI and grade in the course. Here we report on particular patterns of "partially correct" student responses in addition to correct and incorrect responses. When given the option in a multiple choice question, 10 to 40% of students will choose one of two partially correct responses. Students respond that one dynamic quantity, such as velocity, need not be aligned with another dynamic quantity, such as net force, because the velocity could also be zero, but they forget that it can be opposite (Cannot-be-Opposite response). Or, students are partially correct by responding that the velocity need not be aligned with the net force because it could be opposite, but they forget that it can be zero (Cannot-be-Zero response). We found that the choice of partially correct responses is largest in the mid-level class, around 30% of the time. In addition, answering pattern, both within a course and between courses, depends on the question types. $\vec{a} \to \vec{v}$ and $\vec{F} \to \vec{v}$ questions both have Cannot-be-Zero as the more populated middle response, but for $\vec{v} \to \vec{F}$ questions the Cannot-be-Opposite middle response is the larger. Also, students in the regular mechanics classes tend to choose the fully incorrect "misconception" response that the vectors must be aligned for the $\vec{v} \to \vec{F}$ questions more frequently than for the $\vec{a} \to \vec{v}$ and $\vec{F} \to \vec{v}$ questions, but these differences are much smaller in the honors and second year courses.

The data from the three course levels suggest that students may evolve in time from the common incorrect "misconception" response to the correct answer by moving through a partially correct response "state". However, this conclusion is confounded by the fact that the three course levels represent different populations. Clearly a longitudinal study is needed to determine the evolution more unambiguously. Nonetheless, the fact that a significant number of students choose the partially correct response has important implications for instruction of mechanics. In particular, omitting the response as a choice in class or on a test may mislead the student, and/or instructor, about his or her level of understanding of the relationships among force, velocity and acceleration.

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