Facilitating Students' Problem Solving across Multiple Representations in Introductory Mechanics

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Abstract. Solving problems presented in multiple representations is an important skill for future physicists and engineers. However, such a task is not easy for most students taking introductory physics courses. We conducted teaching/learning interviews with 20 students in a first-semester calculus-based physics course on several topics in introductory mechanics. These interviews helped identify the common difficulties students encountered when solving physics problems posed in multiple representations as well as the hints that help students overcome those difficulties. We found that most representational difficulties arise due to the lack of students' ability to associate physics knowledge with corresponding mathematical knowledge. Based on those findings, we developed, tested and refined a set of problem-solving exercises to help students learn to solve problems in graphical and equational representations. We present our findings on students' common difficulties with graphical and equational representations, the problem-solving exercises and their impact on students' problem solving abilities.

Keywords: problem solving, representation, teaching interview, focus group interview, physics education research
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INTRODUCTION

Research in physics education indicates that the use of multiple representations in teaching and learning helps students become better problem solvers. [1-3] We conducted a study to investigate students' difficulties when solving mechanics problems in graphical and equational representations and the hints that might help students over those difficulties. Based on the findings of that study, we created several sets of research-based exercises which could be used as instructional materials to improve students' ability in solving mechanics problems posed in graphical and equational representations.

PHASE 1 – SPRING 2009

In this phase of the study, we investigated the common difficulties students encountered when solving mechanics problems in graphical and equational representations and the hints that might help them overcome those difficulties. The research questions in this phase of the study were:

- What kinds of difficulties do students have when solving mechanics problems posed in graphical and equational representations?
- What kinds of hints may help students overcome those difficulties?

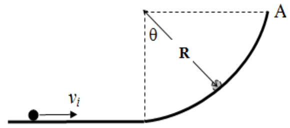
We conducted individual teaching/learning interviews [4] with 20 students randomly selected from a pool of 102 volunteers enrolled in a first-semester calculus-based physics course. Most participants were freshmen or sophomores majoring in engineering. Each participant was interviewed four times during the semester, each time after an exam in their physics course. In each interview, students were asked to solve three problems:

- Original problem: a problem from their most recent exam.
- Graphical problem: similar to the original problem but part of the information was given as a graph.
- Equational problem: similar to the original problem but part of the information was given as an equation.

Students were asked to think aloud when solving the problems and were given verbal hints whenever they made an error or were unable to proceed.

We found that students encountered a variety of difficulties when solving our interview problems.[5] These difficulties could be classified into those with the physics and those with the representation of the problems.

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown below. What is the launch speed of the sphere as it leaves the curve at point A?

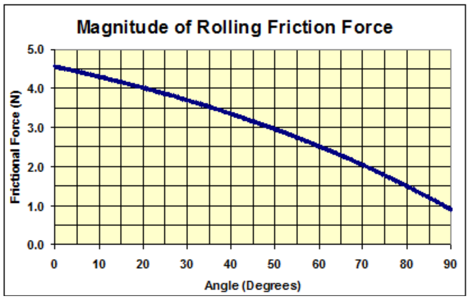
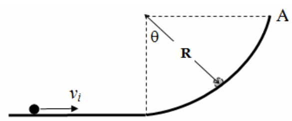


FIGURE 1. Graphical problem in interview 4.

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies as angle θ (radians) as per the following function

$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

What is the launch speed of the sphere as it leaves the curve at point A?

FIGURE 2. Equational problem in interview 4.

The difficulties with the physics of the problems (e.g. inappropriate principle, incorrect physics equations, misunderstood notations, incorrect units) were due to students' misunderstanding or misuse of the appropriate physical principles and concepts needed to solve the problems, while the difficulties with representations (e.g. extracting information and calculating physical quantities from graph/equation) could be attributed primarily to students' inability to activate appropriate mathematical knowledge in physics contexts. Hints asking students to rethink about the physics principle and concepts might help correct students' misunderstanding of the physics of the problems, and hints that guided students to think about the physical meaning of mathematical processes might trigger the activation of appropriate mathematical tools to calculate the desired physical quantity from a graph or an equation. These findings suggested that instructional material emphasizing on specific mathematical ideas and the application of those ideas into physics contexts might help students learn and activate the appropriate mathematical knowledge to solve physics problems.

PHASE 2 – SPRING 2010

In this phase of the study, we created several sets of problem-solving tasks targeting the common difficulties observed in phase 1 and tested their impacts on students' learning to solve mechanics problems in graphical and equational representations. Each of these problem sets included a sequence of matched math and physics problems focusing on specific mathematical ideas and its application to physics.

We conducted five focus group learning interviews (FOGLIs) [6] with two groups of students randomly selected from a pool of 88 volunteers enrolled in a first-semester calculus-based physics course. Most participants were freshmen or sophomores majoring in engineering, with high school physics background. The participants were randomly assigned into either a control group or a treatment group. The number of students in each group varied with each FOGLI session, ranging from eight to 10 students in the control group and from 12 to 14 students in the treatment group.

In each of these 90-minute FOGLI sessions, for the first 15-20 minutes students individually attempted a pre-test that was composed of the graphical and equational problems from the corresponding interview in phase 1. In the next 40–50 minutes, students worked in pairs on the problem sets prepared by us. Students in the treatment group worked on a problem set which included pairs of matched math and physics problems, a debate problem and one or two problem posing tasks. Students in the control group worked on isomorphic textbook problems covering the same concepts. Finally, in the last 15-20 minutes, students worked individually on the post-test which differed from the pre-test only in numerical values of physical quantities given in the problem statements.

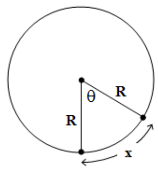
Students in both groups were encouraged to work with their partners while solving the problems. Students in the control group were provided with a printed solution of each problem before proceeding to the next problem. Students in the treatment group were required to check-in with a facilitator before proceeding to the next problem. The facilitator engaged in Socratic dialog with the students to elicit their ideas and facilitate them to solve the problems in the problem set.

Rubrics were created to grade the pre-test and post-test problems in each FOGLI session. Each problem was graded separately on the *physics* aspect and the *representation* aspect. The maximum score on the physics aspect was 10 points and on the representation aspect was 8 points.

Problem 1

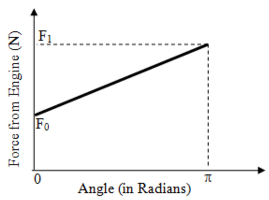
a. What is the length of the arc 'x' along a circle in terms of radius R and angle θ (in radians)?

b. A bug sits on the edge of the turn table of radius R = 2.0 m which is rotating around its center. What is the distance 'x' that the bug has traveled after the turn table has rotated by an angle $\theta = \pi/4$?



Problem 2

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R (in meters). The graph below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians).



a. Plot the graph of force of the engine (in Newtons) with respect to the distance 'x' (in meters) that the plane travels along the circular arc from its initial to its final point.

b. Find the work done by engine when the plane travels from its initial point to the final point.

Problem 3

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R = 3.0 m. The equation below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians)

$$F(\theta) = a\theta + b$$

where, a, b are constants; F is in Newtons, and θ is in radians.

a. Write down the equation of force of the engine as a function of the distance 'x' the plane travels along the circular arc from its initial to its final point.

b. Find the work done by the engine when the plane travels from its initial point to the final point in terms of a and b.

FIGURE 3. Three problems in the treatment problem set of interview 4 that facilitate students' representational skills.

The rubric for the ***physics*** aspect rated five dimensions: *approach* (i.e. was correct principle used?), *equations* (i.e. were correct equations used?), *values* (i.e. were correct values of quantities used?), *manipulation* (i.e. were the equations correctly manipulated?), and *units* (i.e. were the units correct?).

The rubric for the ***representational*** aspect also rated five dimensions: *gathering* (i.e. was the correct information gathered from the representation?), *mapping* (i.e. was the information correctly mapped on to the physics problem), *setting up* (i.e. was the information correctly used in the physics problem?), *manipulation* and *units* which are same as above.

In this paper, we discuss the problems and results of FOGLI session 4. In this session, the physics part of the transfer tasks (pre- and post-test) includes the application of conservation of energy in rotational motion, while the representation part involves the calculation of work done from the graph of force vs. angle or from the equation of force. Findings from phase 1 of the study had shown us that students have difficulties finding work in these problems although they know that work equals force times distance and are able to calculate an integral. Students also have difficulties figuring out the factor to convert units from degrees to meters. There seems to be a gap between their understanding of Work = Force x Displacement and the recognition that when the force is a function of angle, they need to use unit conversion to find force as a function of displacement and then integrate force over displacement. Based on this findings, we created a set of problems for the treatment group, which targeted three key ideas:

(i) Relation between angle and distance along a circle.

(ii) Work equals area under the curve of F(x) vs. x or radius of the circular track times area under the curve of F(θ) vs. θ.

(iii) Work equals $\int F(x)dx$ or $\int F(\theta)Rd\theta$ where x is the distance along the circle of radius R covering an angle θ.

Each of the first three problems of our problem set in this FOGLI session (Figure 3) targeted each of these key ideas with question a) being a math exercise and question b) a physics exercise that uses the mathematical tool mentioned in question a) to calculate a physical quantity.

Problem 4 is a debate problem in which fictitious students discuss the physics of the solution to the problem. The reasoning of these fictitious students contains common errors that students displayed in phase 1 of the study. The goal of this problem is to prepare students with the physics knowledge needed to solve the transfer tasks by recognizing the errors other students make. The debate aspect of this problem is supposed to foster reflection on various problem solving approaches.

Problem 5 contains two problem posing tasks which ask students to embed the idea they learn from previous problems into a physics context to pose more complex physics problems. The goal of this problem is to prepare students to integrate the math and physics ideas they had learned in previous problems in the problem set. The problem posing and solving aspect of this task is designed to foster metacognition.

We present the results of FOGLI session 4 in Tables 1 and 2. There were nine students in the control group and 13 students in the treatment group. The inter-rater reliability for scoring the physics part was 88%, while the inter-rater reliability for scoring the representational part was 95%. The means and standard deviations of the scores of each group in the pre-test and post-test are presented below.

TABLE 1. ***Physics*** score out of 10: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	4.89 (± 3.66)	7.00 (± 3.04)
	Treatment	6.54 (± 3.57)	8.77 (± 1.09)
Equation	Control	8.78 (± 3.31)	5.11 (± 4.31)
	Treatment	6.08 (± 3.95)	8.62 (± 1.39)

TABLE 2. ***Representation*** score out of 8: Mean (± S.D.)

Problem	Group	Pre-test	Post-test
Graph	Control	2.00 (± 2.45)	3.78 (± 2.68)
	Treatment	3.08 (± 2.56)	5.92 (± 2.81)
Equation	Control	3.22 (± 2.22)	4.56 (± 2.24)
	Treatment	3.54 (± 1.45)	7.00 (± 1.53)

Given the small number of participants in each group, the non-parametric Mann-Whitney U Test [7] was employed to test the significance of the difference between the scores of two groups on the pre-test and post-test. The null hypothesis is that the scores of the two groups are not statistically significantly different.

We present below the results of the Mann-Whitney U test for the physics part and the representation part of each of the problems in the pre-test and post-test.

(i) *Physics* scores: Table 3 indicates that the score on the physics aspect of the treatment group is not statistically significantly different from that of the control group, on both the pre-test and post-test.

TABLE 3. Mann-Whitney for ***physics*** scores.

Problem	Pre-test	Post-test
Graph	U = 40.5, p = 0.23, z = - 1.24, r = - 0.26	U = 36.0, p = 0.12, z = - 1.57, r = - 0.33
Equation	U = 39.0, p = 0.19, z = - 1.31, r = - 0.28	U = 32.0, p = 0.07, z = - 1.80, r = - 0.38

This implies that our treatment problem set does not appear to improve students' ability to solve work-energy problems compared to the control problem set. This result might suggest that the treatment should be refined to increase students' practice with the underlying physics knowledge of the problems.

(ii) *Representation* scores: Table 4 indicates a promising result. The score on the representation aspect of the treatment group is not statically significantly higher than that of the control group on the pre-test, but it is statistically significantly higher in the post-test. The effect sizes, r = -0.44 in the graph problem and r = -0.56 in the equation problem in the post-test suggest that these are strong effects.

TABLE 4. Mann-Whitney for ***representation*** scores

Problem	Pre-test	Post-test
Graph	U = 40.0, p = 0.20, z = -1.29, r = -0.28	U = 28.0, p = 0.04, z = -2.07, r = -0.44
Equation	U = 58.5, p = 1.00, z = -0.00, r = -0.00	U = 20.0, p = 0.01, z = -2.65, r = -0.56

This result implies that the treatment problem set significantly improves students' ability to work with graphical and equational representations more than the control problem set does.

CONCLUSIONS

We found that students encountered a variety of difficulties when solving mechanics problems in graphical and equational representations. The difficulties with the physics of the problems were due to students' misunderstanding or misuse of physics principles and concepts, while the difficulties with graphical and equational representations were due to students' inability to activate the appropriate mathematical knowledge in physics contexts. We also created problem sets targeting those difficulties. Initial results indicate that our problem sets help improve students' representation skills while they are not as effective in improving students' physics problem solving skills.

Pedagogically, the promising result on the representation aspect of problem solving appears to suggest a strategy to improve students' representational skills in physics. The proposed strategy leads students through a sequence of problems which is structured to emphasize the activation and application of mathematical knowledge and skills in physics contexts.

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