Characterizing Students' Use of Graphs in Introductory Physics with a Graphical Analysis Epistemic Game

Elizabeth Gire[1], Dong-Hai Nguyen[2] & N. Sanjay Rebello[2]

[1]University of Memphis

[2]Kansas State University

Abstract

We use the framework of epistemic games to characterize students' use of graphs to solve physics problems and the hints given by an instructor to facilitate the students' problem solving. We describe interviews with five university-level students enrolled in a first semester calculus-based introductory physics course in which individual students solved a problem that provided information in the form of a graph. We introduce the epistemic game Graphical Analysis and characterize and describe the solutions and interactions between the student and interviewer in terms of the moves of the Graphical Analysis game. We discuss the level of student facility with the moves of the Graphical Analysis game, describe various hinting strategies employed by the interviewer, and comment of their effectiveness in helping students use graphs more effectively to solve physics problems.

Keywords: introductory physics, graphs, epistemic game, hinting strategies, representational competence

Introduction & Background

It is well documented that students in introductory physics courses have many difficulties working with graphs (McDermott, Rosenquist & van Zee, 1987; Beichner, 1994; Meltzer, 2005). These difficulties include, but are not limited to, interpreting information that is presented graphically, relating a graph to a physical situation, and making a computation with a graph (like finding a rate of change by calculating the slope or finding an accumulation by calculating the area under a curve). In light of the ubiquitous use of graphs by practicing scientists, helping students use graphs efficiently and effectively is a common instructional goal in the sciences. In this study, we use the framework of epistemic games to characterize students' use of graphs to solve physics problem and how hints help them to use graphs correctly.

Collins & Ferguson (1993) introduced the construct of epistemic games, or e-games, as a set of rules and strategies that guides inquiry. The e-games discussed by Collins & Ferguson are normative - identified based on general knowledge of how people think. The elements of an e-game include an epistemic form (i.e. a representation), entry conditions (that determine when it is appropriate to play the game), and moves (i.e. actions that can be taken at different points of the game). The e-game discussed by Collins & Ferguson most relevant to the use of graphs in science is the Trend Analysis game. In this game, a person plots a variable of interest with respect to time, identifies a general character of the trend (linear, exponential, cyclical, growth, etc), extrapolates the curve based on its general character, and makes revisions to the extrapolation by comparing the extrapolation to new data. In this Trend Analysis e-game, the moves of the game have a natural progressive order, but in general, the moves of e-games need not be. Although this e-game is certainly useful in physics and other sciences, it has limitations for use in this study. First, it only includes graphs that are plotted with respect to time. Science students must also learn to work with a broader range of graphs, including those that are plotted with respect to distance. The Trend Analysis game is also limited in its focus on extrapolation. Physics students must be able to extract other physically relevant information (like accumulations, turning points, resonances, phase changes, etc).

Tuminaro & Redish (2007) identified additional e-games, those used by students when doing physics, by observing small groups of introductory physics students solving problems. To identify these e-games, Tuminaro & Redish looked for coherent activities or routines that the students engaged in for the purpose of solving a problem or creating knowledge. In general, students need not be explicitly aware of the e-game or that they are playing one. One of the e-games identified by Tuminaro & Redish is the Pictorial Analysis game. The moves of this e-game include identifying a target concept, choosing an external representation (like a schematic or diagram generated by the student), creating a conceptual story based on spatial relations among the objects, and then filling in the "slots" in the representation (i.e. arranging the objects in the picture or diagram and adding other appropriate elements). In introductory physics, a Pictorial Analysis game might be played with a free-body diagram, a circuit diagram, or a schematic drawing.

In this study, we identify a new normative epistemic game, the Graphical Analysis game, and use it as a framework for analyzing students' solutions to a physics problem involving a graph. We aim to address the following research questions:

- RQ1: What moves in the Graphical Analysis e-game do introductory physics students use? What moves are difficult for students?

- RQ2: How do hints provided by an instructor help students overcome their difficulties? What is the role of these hints in playing the e-game?

Design / Procedure

We conducted teaching & learning interviews (Steffe, 1983; Steffe & Thompson, 2000) with five university students enrolled in a first semester calculus-based introductory physics course. Students were asked to solve a series of similar problems involving the concepts of work and energy. Students first considered a problem where information about the force exerted by a spring was given in the form of a spring constant. In the second problem, a graph of the spring force plotted against distance was provided (see Figure 1). During the interviews, students were asked to write a solution while thinking aloud and explaining their reasoning. The interviewer (co-author Nguyen) provided hints so that each student solved all three problems correctly by the end of the interview. Students were expected to discuss two solution paths: one path involving Hooke's law (F=-kx) to determine the spring constant of the spring and one path calculating work done by the spring by computing the area under the curve. The current analysis focuses only on the second (graphical) problem.

The interviews were video recorded and transcribed for analysis. We identified episodes where the discussion was centered on the graph or using information obtained from the graph. We then coded these episodes, identifying moves in the Graphical Analysis e-game (see Table 1 and Figure 2) being employed by the student or the interviewer. The unit of analysis are statements or groups of statements made by the student or the interviewer. We looked for patterns in the moves that students readily used or had difficulty using. We also looked for patterns in how the interviewer helped each student proceed to a correct solution.

Analysis / Findings

We begin by giving a detailed description of the analysis of one student's solution. We will then summarize the analysis of the five interview transcripts.

Analysis of Cici

Cici begins her solution of the graph problem by identifying the information in the problem statement and trying to match it to an equation (a statement of energy conservation) she used in the previous problem. She identifies the quantities needed for this equation that are not given in the problem statement (a spring constant k, and the distance of the bullet from the equilibrium position x). She begins the Graphical Analysis e-game by identifying that the problem has a force vs. distance graph and then tries to start connecting the information in the graph to the physical situation described in the problem. At this point, she has trouble figuring out how to use the graph to find k and the x in her equation. She does not initially recognize that the "x" plotted on the graph is not the "x" in her energy conservation equation. The "x" on the graph is distance of the bullet from the fully compressed position, and not the distance from the equilibrium position. The interviewer's initial hinting strategy is to make the Create a Story move himself and have the student contribute to the story with values from the graph (Read-out Value).

1	Int:	How can you find x and k?	
2	Cici:	Um… I'm thinking I might have to look at the graph. I'm just not sure how exactly. Um, right so, the graph is force vs. distance. Um so…ok, this is giving me the force that the spring exerts at a given compression of x. Um…I'm not sure where you'd start with that.	**Interpreting Lexical Info:** Identifies the graph as Force vs. Distance **Create a Story:** tries to start connecting the information in the graph to the physical situation although her attempt is not quite correct - she identifies x as the "compression" but in fact it is the displacement from the fully compressed position (this misunderstanding would likely lead to errors in her solution)

3	Int:	Ok. Let's analyze the graph a little. At x = 0, then the force is what?	
4	Cici:	1000 N.	**Read-out a Value**
5	Int:	Ok, that means at this point (points to the picture) the force is 1000N. And at x=0.2, like at this point (points to picture) what is the force then?	**Create a Story**: Interviewer makes a connection between the graph and the physical situation
6	Cici:	Ok, zero. Ok.	**Read-out a Value**
7	Int:	Zero. The force equals zero means the spring is compressed, expanded, or relaxed?	**Create a Story**: Student makes a connection between the graph and the physical situation that is prompted by the interviewer. Ends with Cici recognizing that she must translate the horizontal axis to compression and she indicates an understanding of how to do the translation.
8	Cici:	Uh, yeah it should be relaxed at zero.	
9	Int:	Ok, so the spring is relaxed at 0.2 meter.	
10	Cici:	Yes.	
11	Int:	No force.	
12	Cici:	Right.	
13	Int:	Ok, so at 0.2 it is not compressed, and when you put the bullet in, then it's compressed to zero, it has a force of 1000. So, do you know, can you figure out the compression and the force?	
14	Cici:	Ok, so when your x equals two meters (writes x=-0.2) – assuming, I put the 0.2 as zero. Um, and so then your force equals 1000 at that point. Um, I just, so I'd be able to find x… but I'm not sure how to do k from that.	

By the end of this interaction, the student is able to translate the variable x on the graph to the compression variable in her energy conservation equation. The critical exchange occurs at lines 7 and 8. Up until this point, Cici's contributions have involved the Read-out a Value move prompted by the interviewer. In line 7, the interviewer engages in a second hinting strategy. He prompts Cici to connect the value of force at x=0.2m that she just read-out ("zero"), to a description of the state of the spring ("relaxed"). By describing the state of the spring, she is herself engaging in the Create a Story move. In line 8, it seems that Cici recognizes she must make a translation from the x values on the graph to the compression variable she has in her equation. She describes this translation in line 14.

The interview continues with discussion between the interviewer and Cici about how to calculate the spring constant by recognizing that the linear graph displays a relationship between the force and compression that Cici can match a general algebraic form to: F=-kx. At this point, Cici can translate the value from the graph to the compression variable in her equation and has calculated a spring constant. She is ready to use her energy conservation equation to solve for the speed of the bullet. However, she expresses some confusion about what values of the compression to use.

19	Cici:	Um, we don't know what x the actual value is for this instance. Um… and we need to know that. Um…I'm not sure.	
20	Int:	Ok, so this picture and this graph are related to each other, so you can use information here to apply here. (Pauses) When there is no bullet, then the spring is relaxed at 0.2 meter. And when there is a bullet, then it is compressed to 0 meter. So, what is the compression?	**Create a Story:** Interviewer points out an explicit connection between the picture and the graph.
21	Cici:	0.2 meters.	**Read-out a Value**
22	Int:	0.2, ok. So let's apply that to this situation.	

In line 19, Cici indicates a lack of understanding that the graph contains information about the compression of the spring for the problem situation. This is somewhat surprising given the previous interaction. This comment reveals that although that exchange was highly interactive and ended with Cici being able to translate the variable x into a spring compression, Cici did not adequately recognize the connection between the graph and the problem situation. Making this connection is done through the Create a Story move. The interviewer helps Cici again by employing Create a Story himself and having Cici contribute values (Read-out a Value) useful in that story. At the end of this solution, although Cici obtains a correct answer, there is no evidence of whether she recognizes this graph-situation connection. She does, however, recognize that this connection is a difficult part of the problem and one of the main differences between this problem and the first problem of the interview. The interviewer asks Cici to reflect on this problem compared to the first problem she solved:

Int:	How is this solution, the way you solved this problem different from the way you solved the previous problem?
Cici:	This [problem] was more involved because you weren't given all of the variables in the question. Definitely the trickiest part is using both the diagrams (points to upper picture and lower picture+graph) together to find your x and to find your force which then you could find your k.

RQ1: What moves in the Graphical Analysis e-game do introductory physics students use? What moves are difficult for students?

In this episode, the student is readily able to use the Interpret Lexical Information and Read-out a Value moves. She has trouble employing the Create a Story move. This difficulty is consistent with other research (McDermott, Rosenquist & van Zee, 1987; Beichner, 1994; Meltzer, 2005) students' use of graphs, particularly connecting graphical information with problem situations and physics concepts.

RQ2: How do hints provided by an instructor help students overcome their difficulties? What is the role of these hints in playing the e-game?

In this episode, the interviewer employs two hinting strategies. The first strategy is to identify the connection between the graph and the physical situation for the student while prompting the student to contribute by identifying values from the graph. In this strategy, the interviewer is making the Create a Story move himself while the student is making the Read-out a Strategy move. There is some evidence in this episode that providing this graph-situation connection *for* the student does not lead to a full appreciation of this connection *by* the student. The second strategy is to prompt the student to provide information about the connection between the graph and the physical situation. This hint allows the student to make the Create a Story move herself (rather than the Read-out a Value move, as in the first hint). This hint leads the student to a productive understanding of how to proceed with her solution method.

This sequence of hinting strategies may itself be beneficial. The first hinting strategy involves the interviewer modeling the Create a Story move for the student. This modeling may communicate to the student that the Create a Story move is important for the solution of this problem and when it is appropriate to make that move. This process of modeling may also demonstrate to the student how to construct a story that connects the graph to the problem situation. This hinting strategy may be an important scaffold that enables the success of the second hinting strategy.

Analysis of Five Interviews

A histogram of codes for each interview is shown in Figure 3. The most commonly coded move for each interview is the Create a Story move. These codes identify statements made by the student or the interviewer that are related to connecting the graph to the problem situation or to concepts that lead to a solution. There are two reasons why this move was coded so frequently. First, this move is central to the problem solution because it connects the surface features of the problem (the spring inside the gun) to the deep structure (conservation of energy, including the elastic potential energy of the spring). Therefore, it accounts for a significant portion of the solution process. Second, this move is difficult for the students. Like Cici, many students had difficulty connecting the "x" plotted on the graph to the compression of the spring from the equilibrium position. Also, two of the five students thought the graph only characterized the spring in the graph and didn't correspond to the specific incident of the problem. In terms of conceptual connections, four of the five students failed to suggest a solution path that involved finding the work done by the spring by calculating the area under the curve; the interviewer needed to suggest this solution path. The number of Create a Story moves made by the interviewer is evidence of the students' difficulties. A majority of the hints provided by the interviewer were about the Create a Story move. Often, the interviewer needed to perform, model or prompt this move so that the solution could proceed. Students demonstrated facility with other moves, such as Read-out a Value or Interpret Lexical Information.

The second most common move employed by the interviewer is the Enter the Graphical Analysis E-Game move. The interviewer initiated the e-game in about half of the instances. In these cases, the interviewer was drawing the student's attention to the graph and its role a the solution of the problem. Four of the students first solved the problem by calculating the spring constant of the spring using Hooke's law. All but one of these students spontaneously recognized that the graph was needed to perform this solution; the interviewer initiated the graphical analysis e-game for the last student. The fifth student, Dustin, first solved the problem by calculating the work done by the spring, and he also spontaneously recognized the role of the graph in this solution. The interviewer most often initiated the Graphical Analysis e-game when prompting the students to perform a second solution, involving the work done by the spring and calculating the area under the curve. This move is performed by the interviewer in order to manage the attention of the student. Understandably, the students wanted to move on to the next problem after producing the first solution to the graphical problem.

Not all of the potentially relevant moves were used by the students. None of the students made an estimation of a value, calculated a slope, or translated the graph to a new representation. The Identify a Feature move was verbalized by only one student, who explicitly identified the graph as linear while planning how to perform an integral of the graph. The absence of these moves do not necessarily indicate a lack of proficiency - the nature of the problem did not require these moves to be used.

RQ1: What moves in the Graphical Analysis e-game do introductory physics students use? What moves are difficult for students?

For this problem, these five students demonstrated facility with the Interpret Lexical Information, Read-out a Value, and Calculate Area moves in the Graphical Analysis e-game. For the Calculate Area move, the interviewer usually had to explain that this would be a productive move, but once this hint was given, the students had no trouble making the calculation. Most students also knew to initiate a graphical analysis, although the interviewer often initiated the second instance of this e-game (for instructional purposes).

The Create a Story move was the most difficult move for these students, both in terms of connecting the graph to the story situation and connecting the graph to concepts leading to a solution. None of these student made the Make an Estimation, Calculate a Slope, or Translate to a New Representation moves, but they were not necessary to the solution of the problem.

RQ2: How do hints provided by an instructor help students overcome their difficulties? What is the role of these hints in playing the e-game?

The interviewer employed two hinting strategies: modeling a move for the student and asking questions that prompt the student to make a move. The interviewer also made moves to manage the attention of the student to productive elements of the problem, like initiating the Graphical Analysis e-game to bring the students attention to the graph and employing the Create a Story move to bring a student's attention to the connection between the graph and the physical situation. Although all the interviews ended with the students solving the problem correctly, it is not clear that the students obtained a deep understanding of these solutions despite receiving these hints.

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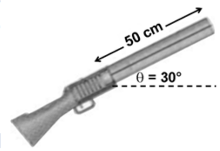
Table 1: Graphical Analysis E-Game

Graphical Analysis E-Game	
Epistemic Form	Graph
Constraints	Information cannot be changed on an existing graph
Moves	• Interpret Lexical Information *(e.g. axes, legend, titles, units)* • Create a Story *(i.e. match graph to physical situation or concepts)* • Read-out a Value • Compare Multiple Data Sets • Identify a Feature *(e.g. trends, extrema, intercepts)* • Extrapolate/Interpolate Data • Make an Estimation *(i.e. estimating a value of the curve at a specific point that may not line up exactly with gridlines or hash marks on the axes)* • Calculate Slope • Calculate Area • Translate to a New Representation

Figure 1. Graph Problem

Graphical Problem

A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring as shown. The gun is then tilted at an angle of 30° and fired.



The only information you are given about the gun is that the barrel of the gun is frictionless and when the gun is held horizontally, the net force F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of its position x (m) in the barrel as shown in the graph below.

What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

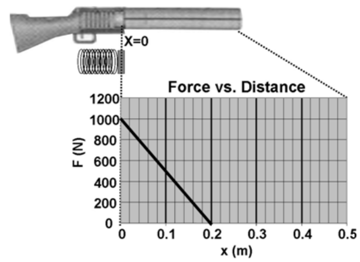
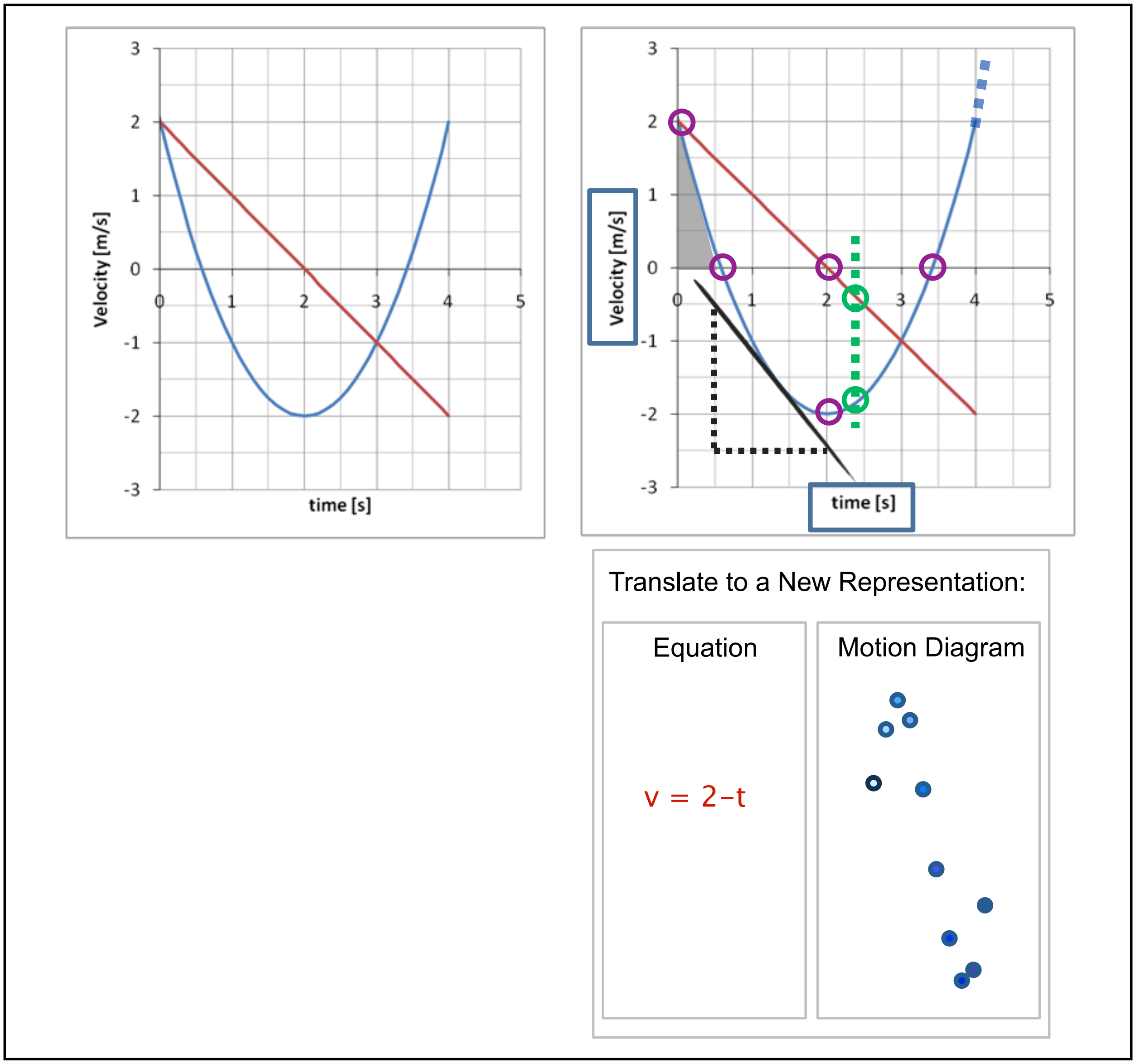
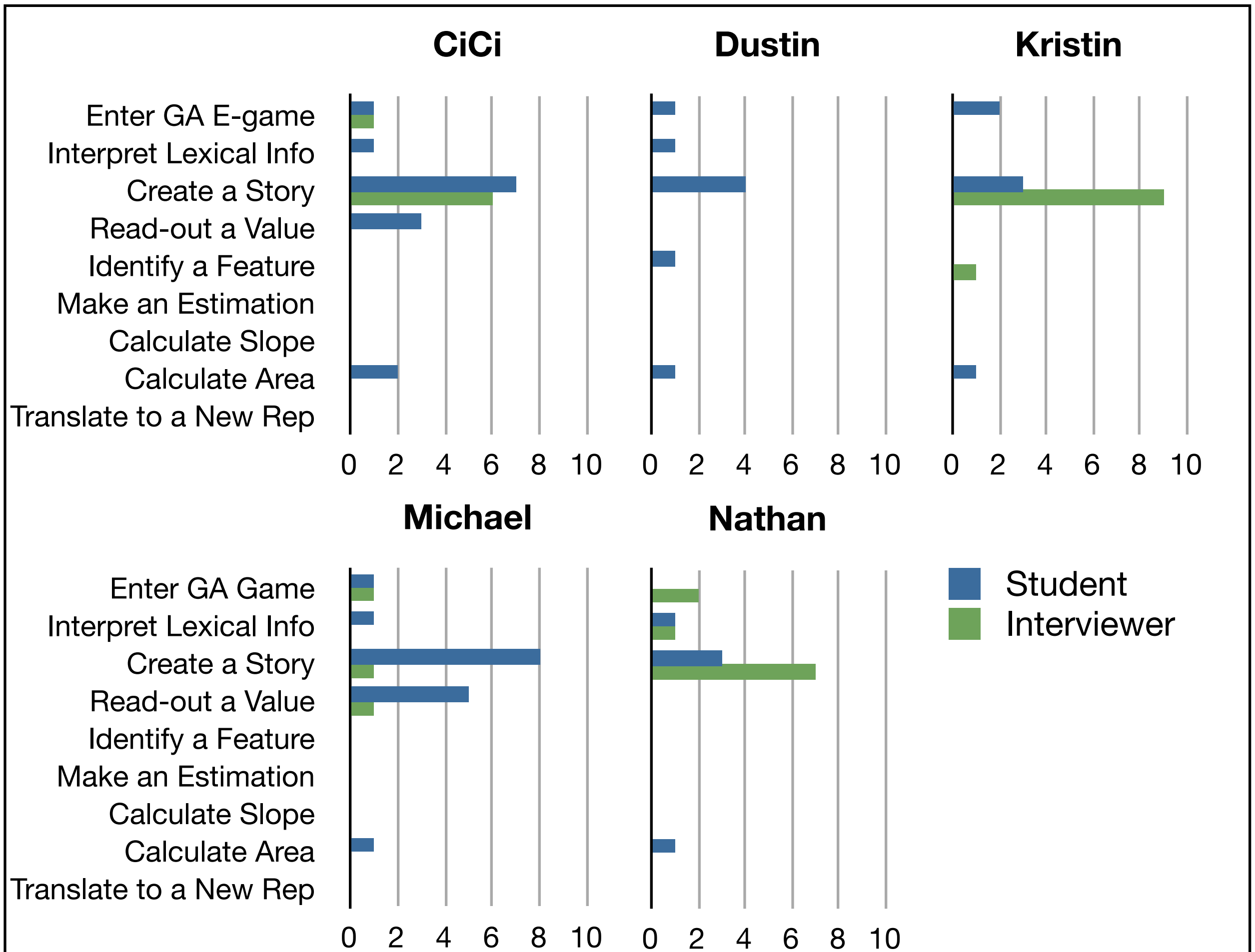


Figure 2: Illustration of Graphical Analysis Moves



On top right: An example of a graph that might be given as part of a physics problem of two curves describing the velocity of two objects as functions of time. *On top left:* Interpret Lexical Information (blue boxes); Read-Out a Value, Compare Multiple Data Sets, and Make an Estimation (green circles and green dotted line), Identify a Feature (purple circles), Extrapolate/Interpolate Data (dotted blue line), Calculate Slope (black dotted lines and slash), Calculate Area (grey fill). *On bottom:* Illustration of Translate to New Representation Move. The equation corresponds to red line and motion diagram corresponds to blue parabola.

Figure 3: Histograms of Graphical Analysis Codes for Each Interview



"Compare Multiple Data Sets" move is not included because the graph in this problem contained only one data set.