

# Teaching-Learning Interviews to Understand and Remediate Student Difficulties with Fourier Series Concepts

## Abstract

Teaching-learning interviews have been used in education to identify students' conceptual weaknesses and to improve learning. This effort utilized teaching-learning interviews in a Spring 2010 linear systems course to (a) identify the conceptual difficulties that students face when studying Fourier series and (b) improve students' understanding of this topic. These interviews attempted to focus more on higher-level Fourier series concepts (consistent with levels 4 through 6 in Bloom's taxonomy) and less on the procedural calculations and plotting (levels 1 through 3 in Bloom's taxonomy) in an effort to attempt to formalize assessment of this topic area relative to a well-established learning framework. Twenty eight students were interviewed for this study using a scripted protocol, where interview sessions lasted about one hour each. Conceptual struggles were identified in thematic areas such as (a) the definition of the mathematical integral and its connections to signal average/behavior, (b) properties of even/odd functions and their relationship to the trigonometric basis set, and (c) the links between time shifts/inversion and the resulting phases of the contributing coefficients.

## I. Introduction

Linear Systems is a required course in most electrical and computer engineering curricula that addresses subjects such as convolution, Fourier series, and continuous/discrete Fourier transforms. This course is widely perceived as useful but difficult, as the subjects tend to be higher-level concepts that rely on a well-developed understanding of lower-level mathematical constructs and procedures.<sup>1-5</sup> Students with an inadequate mathematical foundation and a poor sense of the underlying systems theory regularly struggle with such subjects, as they are typically able to perform sequences of the underlying calculations but cannot piece together the higher conceptual relationships that drive these procedures. As a result, many students are unable to address exam questions and analysis problems that deviate from a solution recipe described in the textbook, and they often cannot explain how slight changes in mathematical renderings will affect system or signal behavior.

In order to better understand how to address these issues, it is helpful to discuss them within the context of a cognition and learning framework. Bloom's taxonomy is a broadly accepted classification scheme for the cognitive learning domain that arranges intellectual behavior into six levels of increasing abstractness or complexity<sup>6</sup> (see Figure 1; Questions in parentheses are based on the updated Bloom's taxonomy<sup>7,8</sup>) Linear systems subjects often address multiple levels within Bloom's taxonomy simultaneously, which is in contrast to courses leading up to linear systems, many of which focus primarily on procedural calculations and plotting (Bloom's levels 1 through 3). As a result, students often struggle with, e.g., higher-level Fourier series concepts that are more consistent with levels 4 through 6, where they are asked to construct signals out of more rudimentary building blocks and assess changes in signal behavior due to changes in the associated coefficients.

In order to more clearly identify the higher-level linear systems concepts with which students struggle, the authors decided to conduct teaching-learning interviews similar to those that have

been used in the past to identify students' conceptual weaknesses and to improve learning<sup>9</sup>. To this end, twenty eight students in a Spring 2010 section of Linear Systems were interviewed using a scripted protocol, where interview sessions lasted about one hour each. The interview protocol, the sample questions, the conceptual misunderstandings related to Fourier series, and the learning assistance methods (i.e., interviewer help/prompts) utilized in the interviews are discussed in more detail in the paper.

1. **Knowledge:** arrange, define, duplicate, label, list, memorize, name, order, recognize, relate, recall, repeat, reproduce, state (**Remembering:** *Can the student recall or remember the information?*)
2. **Comprehension:** classify, describe, discuss, explain, express, identify, indicate, locate, recognize, report, restate, review, select, translate (**Understanding:** *Can the student explain ideas or concepts?*)
3. **Application:** apply, choose, demonstrate, dramatize, employ, illustrate, interpret, operate, practice, schedule, sketch, solve, use, write (**Applying:** *Can the student use the information in a new way?*)
4. **Analysis:** analyze, appraise, calculate, categorize, compare, contrast, criticize, differentiate, discriminate, distinguish, examine, experiment, question, test (**Analyzing:** *Can the student distinguish between the different parts?*)
5. **Synthesis:** arrange, assemble, collect, compose, construct, create, design, develop, formulate, manage, organize, plan, prepare, propose, set up, write (**Creating:** *Can the student create a new product or point of view?*)
6. **Evaluation:** appraise, argue, assess, attach, choose, compare, defend, estimate, judge, predict, rate, core, select, support, value, evaluate (**Evaluating:** *Can the student justify a stand or decision?*)

**Figure 1. Classification levels in Bloom's taxonomy.**

## II. Theory

A focus on Fourier series is common in linear systems courses, as the subject provides a conceptual bridge between the time and frequency domains. In other words, a Fourier series has the time-domain character that is familiar to students, yet its coefficients convey frequency-dependent information. The role and distribution of these coefficients makes for interesting classroom discussions because the students must get past the actions required to compute the coefficients and think more deeply about the coefficients' effects on the character of the time-dependent waveform. When introducing this subject, one can begin with the thought that any periodic signal,  $f(t)$ , can be decomposed into a sum of sinusoids, each with a different magnitude, phase, and frequency. A **trigonometric Fourier series (TFS)**,  $f_{TFS}(t)$ , can be expressed as

$$f_{TFS}(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t)$$

Here,

$$a_0 = \frac{1}{T_0} \int_{t_1}^{t_1+T_0} f(t) dt$$

is the DC, or average, value of the signal over a given time interval of duration  $T_0 = 1/f_0$  seconds ( $f_0 = \omega_0/2\pi$  is referred to as the “fundamental” frequency). The coefficients  $a_n$  and  $b_n$  represent the magnitudes of the cosines (even functions) and sines (odd functions) that constitute the signal. These coefficients are determined using the following expressions

$$a_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \cos(n\omega_0 t) dt, n = 1, 2, 3, \dots$$

and

$$b_n = \frac{2}{T_0} \int_{t_1}^{t_1+T_0} f(t) \sin(n\omega_0 t) dt, n = 1, 2, 3, \dots,$$

where  $n$  is an integer that represents the number of harmonics used to reconstruct the signal. The coefficients  $a_n$  and  $b_n$  are positive or negative real numbers. Also, note that if the original signal,  $f(t)$ , is not periodic, the Fourier series approximation assumes periodicity outside of the original time range (e.g., for  $t < t_1$  and  $t > t_1 + T_0$ ).

The information in a trigonometric Fourier series can be encapsulated in a set of coefficients,  $C_n$  and  $\theta_n$ , that represent the magnitudes and phases of these sinusoidal components. This is known as a **compact trigonometric Fourier series (CTFS)**, where the signal  $f(t)$  is expressed as

$$f_{CTFS}(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(2\pi n f_0 t + \theta_n).$$

These compact coefficients are related to the original Fourier series coefficients through the following relationships:

$$C_0 = a_0, C_n = \sqrt{a_n^2 + b_n^2}, \text{ and } \theta_n = \tan^{-1}\left(\frac{-b_n}{a_n}\right).$$

### III. Methods

#### A. Overall Approach

**Student Population and Interview Timing.** The authors interviewed twenty four students in the Spring 2010 section of *ECE 512 – Linear Systems*, a course offered by the Kansas State University (KSU) Department of Electrical & Computer Engineering. These students were predominantly undergraduate seniors in Electrical Engineering or Computer Engineering. The interviews were conducted (a) after the students had submitted Fourier series handwritten assignments, used the online linear systems modules, and taken exams on these same subjects but (b) before the final exam, implying that the students had absorbed the material to a level of understanding that would be typical at the end of a semester. All interviews were conducted over a period of two weeks just prior to the final exam for the course.

**Interview Protocol.** Each interview was conducted as a one on one, teaching-learning interview and was videotaped for follow-on analysis, where the camera was directed over the shoulder of the student so that it recorded video of only the work surface in front of them. Prior to an interview, a student signed a consent form (KSU IRB protocol #4691) stating their willingness to participate in this research exercise within the context of the overarching course experience. Four separate Fourier series problems were provided to each student (see the next section), where the student was asked to describe their work out loud as they progressed through each problem. Note that these interview components were not provided in “question form” but rather in “exercise form,” as if the student was thinking out loud as they sat down to work a sequence of homework problems. When a student reached a point where their response was incorrect or they could not continue, the interviewer provided help/prompts in comment/question format. On average, the interview process took about an hour per student. Areas of conceptual misunderstanding were recorded both during the interview and during follow-on analyses of the video recordings.

**Motivation for the Focus on Coefficient Roles.** These interview problems were chosen to specifically address the roles of the Fourier series coefficients with respect to the shapes or behaviors of the reconstructed Fourier series. An understanding of these roles is an indication that a student has learned Fourier series concepts at a higher conceptual level, and past experience with exams that address Fourier series has taught the authors that describing these coefficient roles is a task where students begin to falter, even if they are adept at performing the calculations to determine the coefficients.

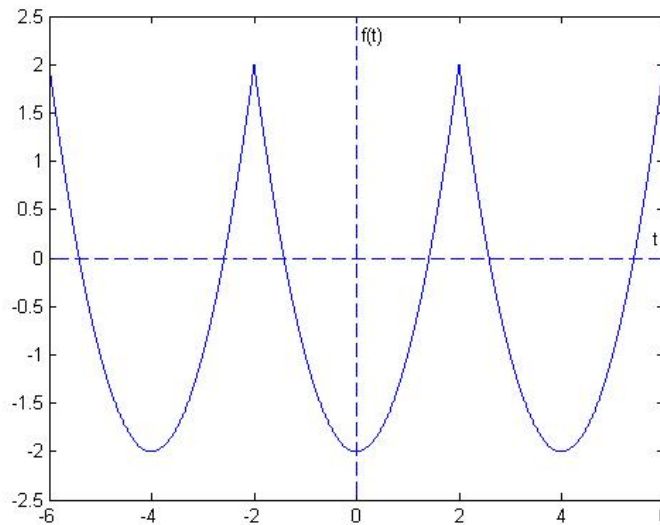
**B. Interview Problems**

**Problem #1:** A trigonometric Fourier series is used to describe the signal  $f(t) = t^2 - 2$  (see Figure 2) over the time range of  $t = [-2, 2]$  seconds. Determine the trigonometric Fourier series,  $f_{TFS}(t)$  for this signal. (Answers: Table 1)

- Before you start to solve the problem, estimate the sign of  $a_0$ .
- Can you describe how you solved the problem? (Use of even/odd symmetry or neither; the overall process; other details)
- Given  $f_{TFS}(t)$ , what is the value of  $f(t)$  at  $t = 0$ ?

**Table 1. Answers for interview problem #1.**

<ul style="list-style-type: none"> <li>• Sign of <math>a_0</math>: negative</li> </ul>	<ul style="list-style-type: none"> <li>• <math>a_0 = -2/3</math></li> <li>• <math>a_n = \frac{16}{n^2 \pi^2} \cos(n\pi)</math></li> <li>• <math>b_n = 0</math>.</li> </ul>	<ul style="list-style-type: none"> <li>• <math>f(0) = -2</math>. Note: <math>f(0) = -\frac{2}{3} + \sum_{n=1}^{\infty} a_n</math> does not provide a direct result, so the student must understand that they need to consult the plot rather than the Fourier series.</li> </ul>
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**Figure 2. The parabolic function,  $f(t) = t^2 - 2$ , used in interview problem #1.**

**Problem #2:** The parameters for  $f_{TFS}(t)$  ( $a_0$ ,  $a_n$ , and  $b_n$ ) are known for the original signal in Figure 3a). Identify how the parameters  $\omega_0$ ,  $a_0$ ,  $a_n$ , and  $b_n$  change if the original signal changes to each of the signals in Figure 3b, Figure 3c, and Figure 3d. (Answers: Table 2)

**Table 2. Answers for interview problem #2.**

Figure 3b	Figure 3c	Figure 3d
<ul style="list-style-type: none"> <li>• <math>\omega'_0 = \omega_0/2</math></li> <li>• <math>a'_0 = a_0</math></li> <li>• <math>a'_n = a_n</math></li> <li>• <math>b'_n = b_n</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>\omega'_0 = \omega_0</math></li> <li>• <math>a'_0 = a_0</math></li> <li>• <math>a'_n = a_n</math></li> <li>• <math>b'_n = -b_n</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>\omega'_0 = \omega_0/2</math></li> <li>• <math>a'_0 = a_0 + 1</math></li> <li>• <math>a'_n = a_n</math></li> <li>• <math>b'_n = b_n</math></li> </ul>

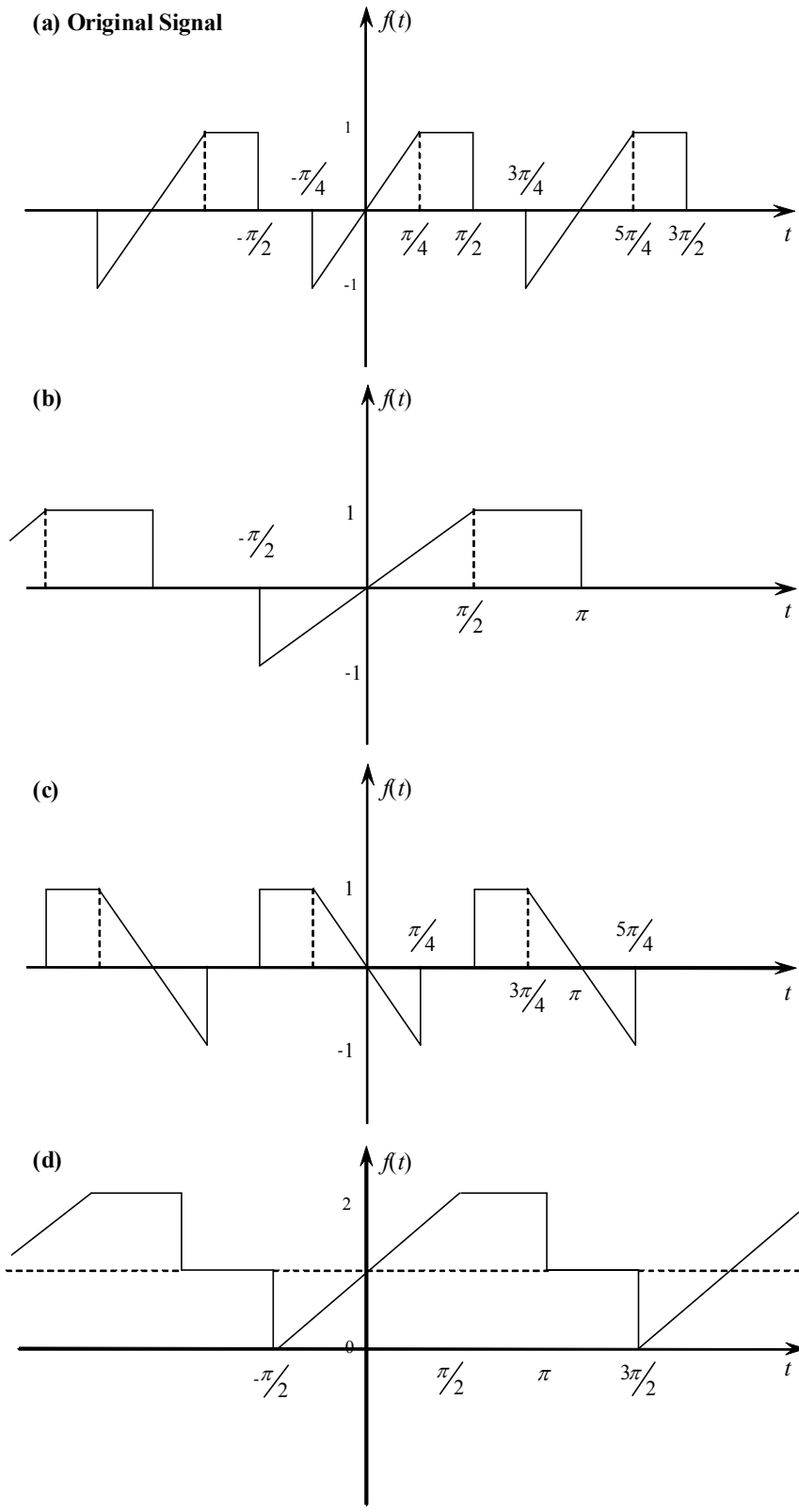
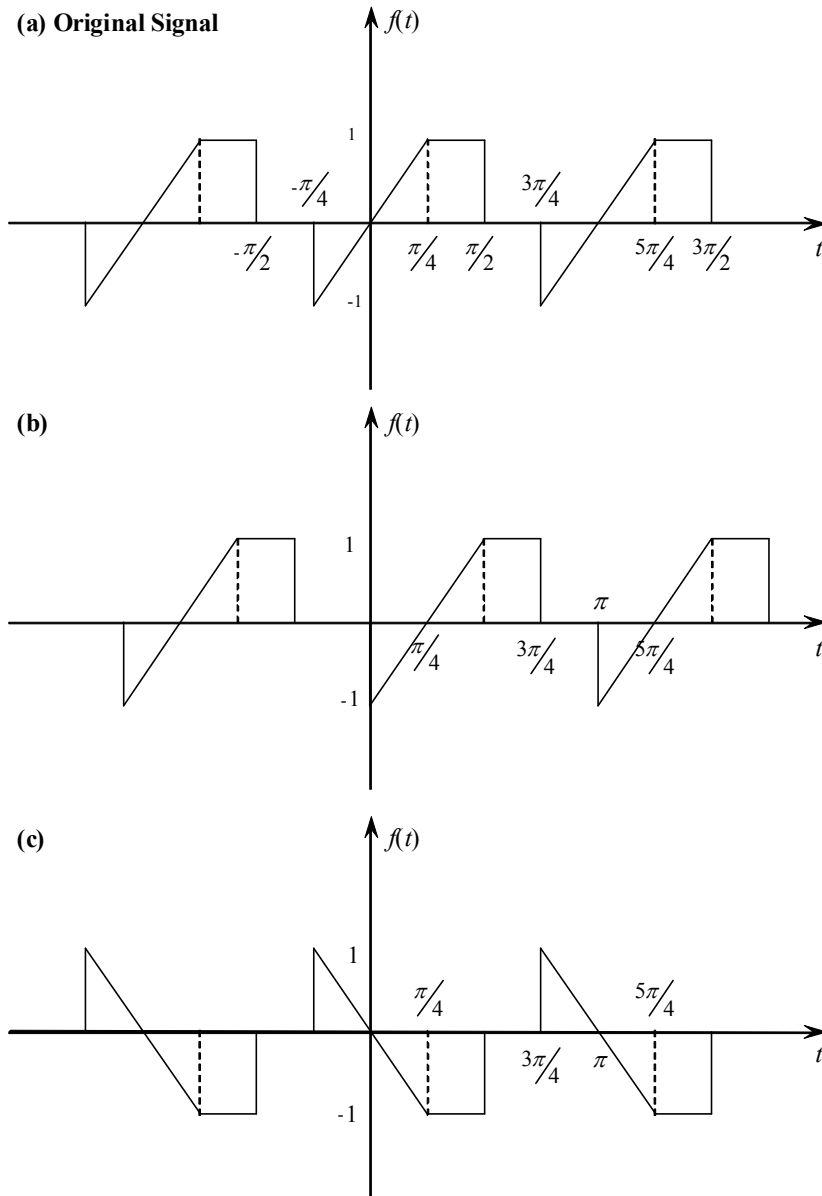


Figure 3. Signals for interview problem #2.

**Problem #3:** The parameters for  $f_{CTFS}(t)$  ( $C_0$ ,  $C_n$ , and  $\theta_n$ ) are known for the original signal in Figure 4a. Identify how the parameters  $\omega_0$ ,  $C_0$ ,  $C_n$ , and  $\theta_n$  change given the signals in Figure 4b and Figure 4c. (Answers: Table 3)

**Table 3. Answers for interview problem #3.**

Figure 4b	Figure 4c
<ul style="list-style-type: none"> <li>• <math>\omega'_0 = \omega_0</math></li> <li>• <math>C'_0 = C_0</math></li> <li>• <math>C'_n = C_n</math></li> <li>• <math>\theta'_n = \theta_n - \frac{\pi}{4} n \omega_0</math></li> </ul>	<ul style="list-style-type: none"> <li>• <math>\omega'_0 = \omega_0</math></li> <li>• <math>C'_0 = -C_0</math></li> <li>• <math>C'_n = C_n</math></li> <li>• <math>\theta'_n = \theta_n + \pi</math></li> </ul>



**Figure 4. Signals for interview problem #3.**

**Problem #4:** The parameters for  $f_{CTFS}(t)$  ( $C_0$ ,  $C_n$ , and  $\theta_n$ ) are known for the three original signals in Figure 5. If we wish to use these signals as building blocks to construct the signals in Figure 6, which signal(s) should we use? What changes in the respective  $f_{CTFS}(t)$  parameters would be needed to make that happen?

**One Acceptable Answer for Figure 6a.** We choose the signal in Figure 5b to generate the signal in Figure 6a. In this case, the signal in Figure 5b will be used twice. First, an instance of the signal,  $f_1$ , can be flipped about the  $t$  axis, yielding  $C'_{n1} = -C_{n1}$  (here  $C_{01} = 0$ ). Then, the result will be delayed by half of the period ( $\pi/2$  seconds in this case), which means the new phase  $\theta'_{n1} = \theta_{n1} - \frac{\pi}{2}n\omega_0$ . Another instance of the signal in Figure 5b, called  $f_2$ , can be added to  $f_1$  to obtain signal  $f_3$ . Further, the amplitude of  $f_3$  will be multiplied by  $1/2$ , which means  $C'_{n3} = \frac{1}{2}C_{n3}$ . The final step is to raise the entire signal by  $1/2$ , which means  $C'_{03} = C_{03} + 1/2$ .

**One Acceptable Answer for Figure 6b.** The signals in Figure 5a and Figure 5c can be used to generate the signal in Figure 6b given their period and duty cycle. The dashed lines in Figure 6b are drawn to assist the reader. The procedure is similar to that used for Figure 6a, only a bit more complex.



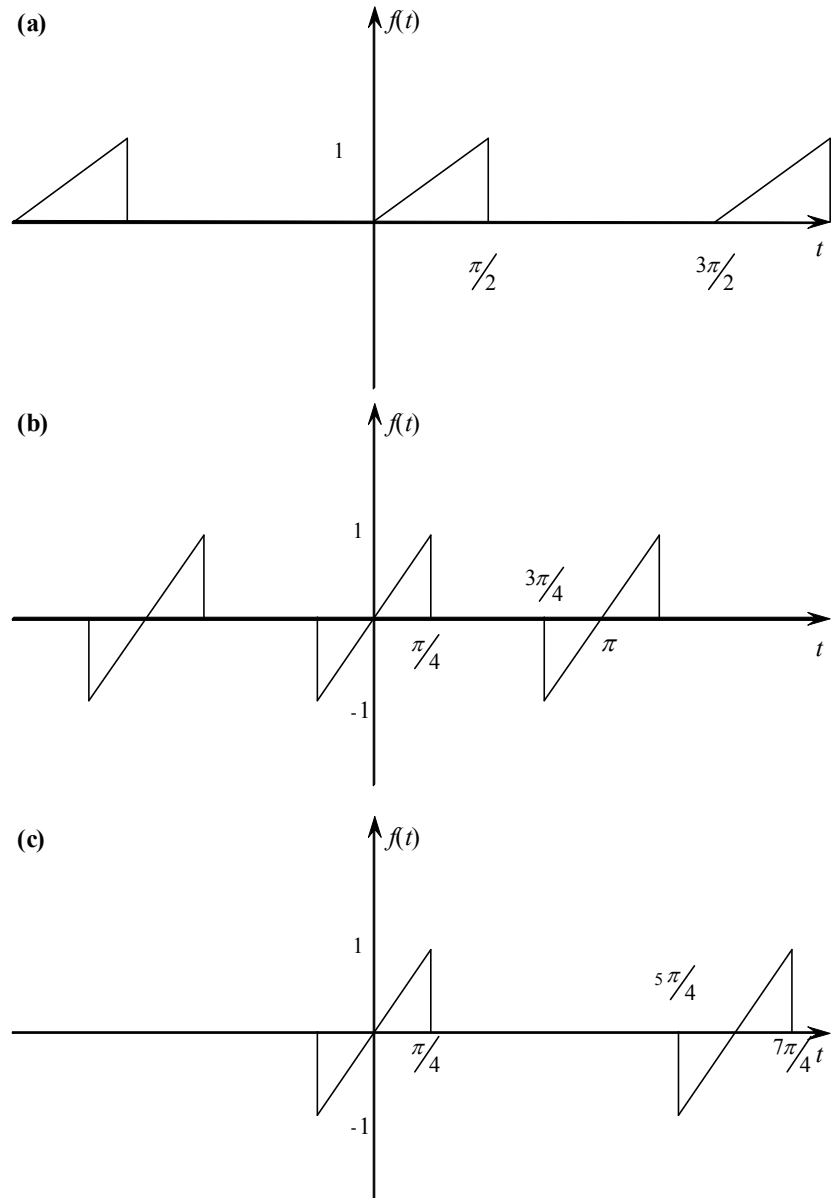
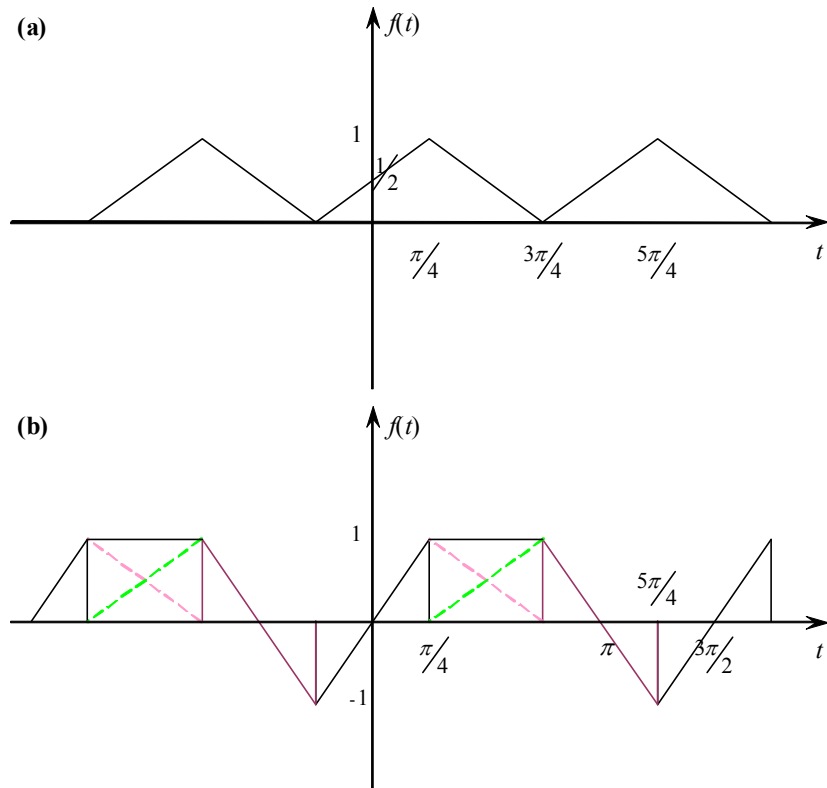


Figure 5. Building block signals for interview problem #4.



**Figure 6. Target signals for interview problem #4.**

## IV. Results and Discussion

### A. Overall Themes: Concepts that Cause Students to Struggle

Notes taken during the individual interviews were analyzed to summarize the types of concepts that caused students to struggle and the types of hints that were often necessary in order to help students make progress on certain types of problems. Videos acquired during each of these sessions were also analyzed to corroborate and supplement these findings. This section provides an annotated listing of the types of concepts that were problematic for the students and the types of hints that were supplied to help them move forward.

**1. Physical meaning of the term ‘integral.’** For a one-dimensional function, the meaning of the term ‘integral’ is often defined functionally as accumulation but visually as ‘the area under the curve’, meaning the area between the curve and the independent axis. This is one concept that all students in an upper-level linear systems course should understand, as they have experienced it multiple times in various contexts. The first question in the first problem sought to assess this understanding when the interviewer asked students to estimate the sign of  $a_0$  by looking at the  $f(t)$  curve. Some of the students (5 out of 24) had forgotten the meaning of ‘integral’ altogether, and another group of students (5 out of 24) understood the concept but either did not know how to apply it or applied it in the wrong way for this problem, such as visualizing the area between the curve and minus infinity as a literal interpretation of ‘area under the curve.’

**2. Properties of even and odd functions.** For a function with even symmetry,  $f_e(t) = f_e(-t)$ , whereas a function with odd symmetry has the property  $f_o(t) = -f_o(-t)$ . In a trigonometric Fourier series formulation, cosine (even) and sine (odd) functions specify the building blocks of the series and are paired with the coefficients  $a_n$  and  $b_n$  with the understanding that these coefficients specify the amplitudes of these basis functions. Students are instructed that if a function,  $f(t)$ , is even, then its Fourier series will only require  $a_n$  coefficients; if it is odd, only  $b_n$  coefficients are required. Even so, these interviews indicated that 10 of 24 students still had trouble understanding the even or odd character of cosine and sine functions. For example, if  $t$  is changed to  $-t$ , these students had difficulty understanding the commensurate change in  $\sin(n\pi\omega_0 t)$  and  $\cos(n\pi\omega_0 t)$  behavior and therefore the related changes to  $a_n$  and  $b_n$ . Regarding integration of an even function, as in problem #1, the definite integral from  $-t$  to  $t$  should be twice the integral from 0 to  $t$ , while the definite integral of an odd function from  $-t$  to  $t$  yields 0. When addressing integrals in these interviews, 7 of 24 students did not use this concept to save time or used it in the wrong way, such as choosing wrong integration limits.

**3. Properties of the sine and cosine functions.** Other sine and cosine properties were also used in the interview problems. For example, when working with a compact trigonometric Fourier series representation, the  $C_n$  coefficient is defined as a magnitude (positive number), so when a term such as  $C_n \cos(n\pi\omega_0 t)$  is negated, the minus sign must be addressed through the angle of the cosine by changing  $\cos(n\pi\omega_0 t)$  to  $\cos(n\pi\omega_0 t \pm \pi)$ . Seven students struggled with this idea.

**4. Inverse frequency/period relationships.** One of the problems required each student to find the period and then the fundamental frequency (problem #1, question 2). Others addressed changes in period or frequency, such as problem #2 question 1 (see Figure 3b), which asked how the frequency would change if the signal was stretched to be doubly wide. The equations for the relationship between frequency and period, such as  $\omega_0 = 2\pi f_0$  and  $T_0 = \frac{2\pi}{\omega_0}$ , were given on a

formula sheet, yet 5 of 24 students still had a little trouble, where some students were inclined to say the frequency also doubled.

**5. Math-to-plot versus plot-to-math disconnect.** Students seem uncomfortable establishing relationships between mathematical equations and plots and explaining changes in one given changes in the other. In problem #2, question 2 and problem #3, question 2, the plots were flipped about the vertical axis and horizontal axis, respectively. In the first case, most of the students could reason that the function changed from  $f(t)$  to  $f(-t)$ , whereas a few students (3/24) misunderstood. In the second case, some of the students described the result as  $f(-t)$  rather than the correct response,  $-f(t)$ . If they needed a hint, students were asked to compare the previous  $f(-t)$  plot with the plot in front of them and consider the differences. Eventually, most of the students came up with the correct answer, but 20 students struggled with this concept at some level.

**6. The mistaken equivalence between phase shift and time shift.** In problem #3, question 1, the plot was moved  $\pi/4$  seconds to the right in the time domain, so the CTFS function would become

$$f_{CTFS}(t - \pi/4) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0(t - \pi/4) + \theta_n) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n - \pi n\omega_0/4),$$

where the phase representation has changed to  $\theta_n - \pi n\omega_0/4$ . Most students specified a phase of  $\theta_n - \pi/4$  as the answer, which implies their willingness to accept ‘phase shift’ as equal to ‘time shift,’ which is incorrect. This issue was arguably the most common mistake in the interview, as 22 students did not get the right answer without a hint, and 2 students made an initial mistake but soon corrected themselves.

**7. Incorrect use of lookup aids.** A few students have trouble using lookup aids such as integral tables and Fourier series conversion tables. Both types of tables were made available during these interviews, yet two students stumbled by using the wrong integral table or wrong Fourier series equations, such as the use of  $\int \sin x \, dx$  instead of  $\int x \sin x \, dx$ .

**8. Inability to start in the middle.** When addressing CTFS problems, some of the students feel the need to start back at the TFS representation and then move those coefficients into the CTFS domain, which inevitably adds calculations and therefore time. This usually leads to the correct answers but also implies a reliance on calculations and recipes rather than an understanding of the concepts of magnitude and phase.

### ***B. Additional Notes***

**Unforeseen Benefits of Tutoring Sessions.** One unforeseen benefit of this interview process was that, in some cases, the interview as planned turned into more of a personal tutoring session. This led to unsolicited feedback from many of the participants that indicated the hour-long interview was worth their time from that viewpoint alone, irrespective of the fact that they received course credit for participating in the interviews. More specifically, some students mentioned that the pace and feel of the interview were different from in-class learning (which would be expected), since they could work with the instructor individually and spend their interaction time on issues directly related to their areas of misunderstanding.

**Findings Regarding Interviewer Help/Prompts.** From these interviews, it is clear that most of the students have reached a satisfactory level of capability with regard to the types of mathematical calculations that one must perform in order to calculate Fourier series representations of signals.

**Generalizations of Areas Where Students Struggle.** Section IV.A. noted specific areas where students struggled within the context of the Fourier series problems presented in the interviews. The following listing seeks to generalize and expand upon these areas of struggle with the thought that more overarching changes in pedagogy might be applied to address them.

- Students often have difficulty ‘seeing’ the relationship between (a) mathematical representations and signal features and (b) changes in mathematical representations as they relate to changes in the visual appearance of a signal.
- The general issue of performing the mathematical operations versus understanding their impact or purpose is an important discussion point. For signals constructed from basis functions, students have trouble getting past the details of a mathematical process that employs basis functions so that they can visualize the way in which these signals are constructed from those fundamental building blocks. In the case of Fourier series, these basis functions are cosines and sines, but other good basis sets exist (e.g.,  $t^n$  for polynomial functions).
- Visually pulling a signal apart can be a struggle for some students. For example, in a Fourier series context, it is hard for some students to visually remove the baseline (even component) of a signal and realize that the remaining signal may actually have odd symmetry on its own.
- Presenting a student with a mathematical shortcut does not ensure that they will understand when its use is or is not justified. This is demonstrated in Fourier series calculations by the use of symmetry to shorten the coefficient calculation process.
- The inverse relationship between time and frequency is always an issue. This issue not only relates to the misperception that a wider sinusoid means a higher frequency, but it includes misperceptions such as making a signal longer increases its bandwidth as represented by its TFS coefficients, or (in the discrete domain), sampling a signal faster improves the resolution of the coefficients in the frequency domain.
- The mistaken equivalence between time shift and phase shift speaks to students’ fundamental misunderstandings about Fourier series. If a waveform is shifted, then all of the sinusoids that comprise that waveform must also be shifted. Since these sinusoids are at different frequencies, yet they must retain alignment relative to one another in order to retain the overall signal shape, then each sinusoid (building block) experiences a different phase shift, even though the time shifts are all equal.
- When students do not know quite how to proceed, they fall back on process and recipe rather than think about the problem at a high level. For example, to describe the change in phase due to a time shift, most would be more comfortable recalculating the Fourier  $C_n$  and  $\theta_n$  values from scratch rather than reason through the change in coefficient values.

### ***C. Future Work***

In response to these lessons learned, it is clear that more future instruction (in class and in homework) needs to concentrate on the relationship between signal changes and their corresponding mathematical representations. This instruction cannot, however, be at the expense

of the procedural calculations that students already make as they learn Fourier series, since it takes time for students to get these calculations ‘under their belts.’ Additionally, this assessment is offered in light of the fact that current instruction \*does\* concentrate on some of these issues, yet students are not properly absorbing and retaining the concepts. Therefore, the authors plan to take a two-pronged approach, where higher-level concepts are emphasized in class and additional practice exercises are made available through online learning module updates as discussed in the following paragraph. Once these changes have been implemented, the interview process will be completed with a new set of students in order to ascertain whether statistically significant improvements have taken place in learning. Exam questions will also be applied, where the results can be correlated with results from questions asked in previous semesters.

**Online Module Updates.** For several years, the authors have used online learning modules in these linear systems courses<sup>2, 10-13</sup>. These online tools utilize PHP, HTML, Java, and PostgreSQL to generate and assess homework problems in the areas of complex numbers, signals, transient response, Fourier series, and Fourier transforms. Features and benefits of this approach include a visually appealing user interface, custom problem sets for each student, online help, immediate score feedback, problem solutions, practice problems, and the opportunity for a student to rework categories of problems until they receive their desired score. Currently, the students work on three separate Fourier series modules that address trigonometric Fourier series (TFS), compact trigonometric Fourier series (CTFS), and exponential Fourier series (EFS). The authors are developing a higher-level-concept module that will be between the CTFS module and the EFS module, where the module will concentrate on the kinds of higher-level issues addressed in these interviews (e.g., If a signal changes, what happens to the related TFS coefficients?). Further, additions will be made to the transient response modules (zero-input response and unit impulse response) that plot the time-domain signals addressed in these modules and label their transition points.

## V. Concluding Remarks

In a class like linear systems, students often demonstrate the ability to perform mathematical calculations but struggle when it comes to the relationship between those calculations and the plots of the signals that those calculations represent. This is especially true for subjects such as Fourier series, where changes in coefficient values influence the shape and character of the associated signal. In this study, the authors sought to better understand this lack of higher-level thinking by hosting technical interviews where students solved Fourier series problems and relayed their thoughts and methods to the interviewer while they worked. While many small areas of need were identified, general conceptual struggles were identified in broader thematic areas such as (a) the definition of the mathematical integral and its connections to signal average/behavior, (b) properties of even/odd functions and their relationship to the trigonometric basis set, and (c) the links between time shifts/inversion and the resulting phases of the contributing coefficients. Note that these struggles are generally understood by an instructor that teaches this subject frequently. The added value of this work is therefore a more formal means to quantify the prevalence of these misconceptions so that the benefits of curricular adjustments can be more effectively assessed. In response to these lessons learned, the authors are making changes to this course in the form of lecture and homework updates, including a new online learning module that addressed these higher-level subjects directly. Follow-on exams and interviews will determine whether these updates were successful.

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