Longitudinal Development of Students' Representational Skills in Introductory Physics

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INTRODUCTION

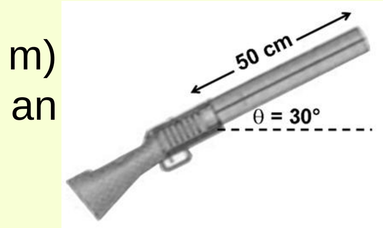
Motivation

Track problem solving with graphs & equations across two semesters of calc-based physics.

Methodology

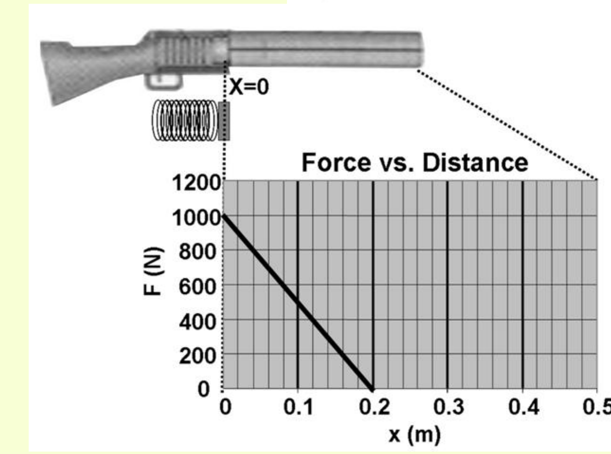
- Follow 15 volunteers across two semesters of calculus-based physics.
- 8 Individual teaching/learning interviews.
- In each interview, students:
  - solve problems in numerical, graphical & equational representations.
  - think aloud while solving problems.
  - are given verbal hints whenever unable to proceed.

INTERVIEW 2 – GRAPH PROBLEM



A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring as shown. The gun is then tilted at an angle of 30° and fired.

The only information you are given about the gun is that the barrel of the gun is frictionless and when the gun is held horizontally, the net force F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of its position x (m) in the barrel as shown in the graph. **What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?**

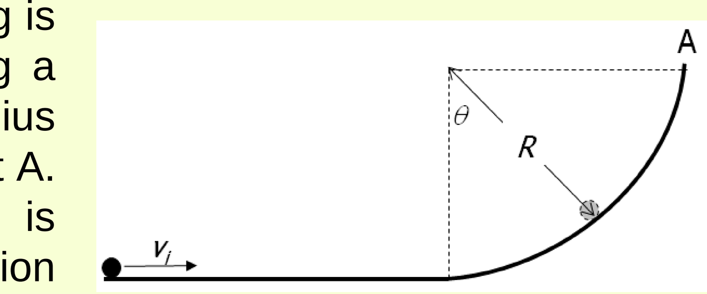


I: Can you think of a way to find work done without knowing 'k 'and 'x'?
S: I don't know.
I: What information can you extract from this graph of F vs. x?
S: Spring constant … I don't know … maybe the work.
I: How can you calculate work from this graph?
S: It's force times distance.
I: What value of force do you use?
S: I assume 1000 N.
I: Is the force 1000 N all the way?
S: … No.
I: So what should you do?
S: You have to integrate it through.
I: What does that integration mean on this graph?
S: It's the area under the curve thing.

> Student needed hints to activate the idea that work was area under the curve of force vs. displacement.

INTERVIEW 4 – EQUATION PROBLEM

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force F_{roll}(N) on the curved section varies as angle θ (radians) as per the equation shown. **What is the launch speed of the sphere as it leaves the curve at point A?**



$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

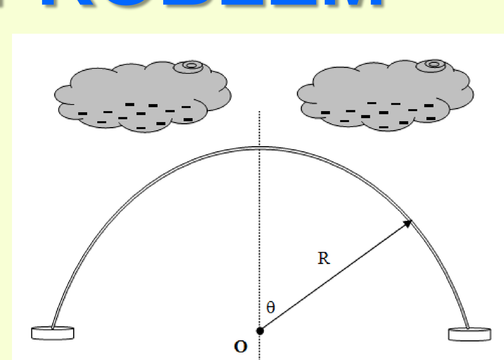
Student set up the equation: $\Delta K + \Delta U = W$

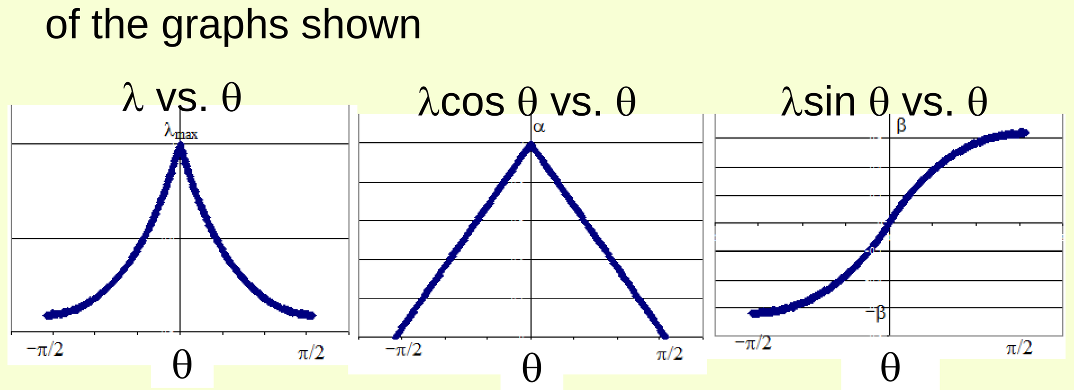
S: Work is the integral of force over distance.
I: Which distance?
S: The radius.
I: The distance in the work equation is the distance that the object travels.
S: I'm not sure.
I: It's the distance along the curve. How do you relate that distance to angle?
S: This is your starting distance (bottom of the ramp), this is your ending distance (point A), so … would you break it into a unit circle?
I: Suppose you want a small distance ds and the corresponding angle is dθ so how do those two relate?
S: It's R times dθ.

> Student easily figured out that work was integral of force, but had difficulty with the conversion of units e.g. angle to displacement.

INTERVIEW 5 – GRAPH PROBLEM

You are standing at the center of the arch as in problem 1 on a stormy day. There are negatively charged clouds over the arch. The charge distribution on the arch now depends on the angle θ as per one of the graphs shown





Find the magnitude and direction of the electric field at your feet (i.e. at point O on the ground).

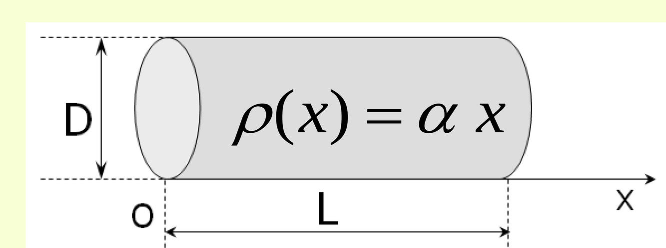
Student had set up the integral: $$E = \frac{1}{4\pi\varepsilon_0 R}\int_{-\pi/2}^{\pi/2} \lambda\cos\theta\, d\theta$$

S: You can just find the area instead.
I: What area?
S: The area under the curve.
I: You have several graphs…which do you choose to find the area under?
S: You better do it under the λcosθ versus θ.
I: How do you know you should use that graph?
S: Just assuming that because we're using cosθ in the integral.
I: Do you mean that because you see the cosine in the integral you should use the graph concerning cosθ?
S: That's what I thought.

> Student needed hints to figure out under which graph she should find the area under.

INTERVIEW 6 – EQUATION PROBLEM

A cylindrical conductor of length L, diameter D is shown. The resistivity ρ(x) is changing along the conductor as per the function shown, where x is the distance from the left end of the conductor. **Find the electrical resistance of the conductor.**



S: Do I need to set up an integral for this one?
I: Integral of what?
S: I'm not sure … I guess …
I: What's the meaning of integral?
S: I don't know how to explain meaning of integral.
I: Taking integral means you chop the resistor into thin slices, find the resistance of each slice and then add up the resistance of all slices.
S: Yeah.
I: So what is the resistance of each slice whose length is dx?
S: Like the integral?
I: The resistance dR of just this little resistor.
S: Okay, [Student writes equation]
I: Yes, this is the resistance of a thin resistor. now you want to have the resistance of the whole thing, so …
S: You need to do the integral then, from 0 to the length of the cylinder.

$$dR = \frac{4\alpha x\, dx}{\pi D^2}$$

> Student needed hints to apply the idea of "integration as accumulation"

CONCLUSIONS

As students progressed through the interviews

- their ability to calculate a physical quantity from a function by integration improved, but was limited to cases where only the function needed to be integrated.
- they were aware of the idea of "integration as accumulation," but they had difficulty applying it to physics contexts.
- their ability to calculate a physical quantity from graphs improved within each level of complexity of the task.
- when solving a problem at a higher level of complexity, students first encountered some difficulties but then adapted with hints.