Investigating Trajectories of Learning & Transfer of Problem Solving Expertise from Mathematics to Physics to Engineering

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Annual Report for Year II: 2009-2010

PROJECT ACTIVITIES

During this second year we continued a longitudinal study of students by conducting individual semi-structured interviews to capture fine grained data about individual student's problem solving.

Based on these results and those from the interviews conducted in the first year of the project, we created classroom materials to address student difficulties with representations and conducted focus group interviews with students using these materials.

Research Questions: During the second year of this grant we addressed the following research questions

1. What kinds of conceptual understanding do students develop and what kinds of difficulties do they encounter when transferring their problem solving skills across problems in different representations in different domains?
2. How can we facilitate students' transfer of conceptual understanding and problem solving skills across problems in different representations in different domains?

OVERALL SUMMARY OF ACTIVITIES

We completed the following kinds of activities to address the two research questions above

- Semi-Structured Clinical Interviews: The object of these interviews is to probe students' conceptual understanding and problem solving skills. There were primarily conducted in mathematics and engineering.
- Teaching/Learning Interviews to investigate how people learn. The object of these interviews is not merely to probe students' current state of knowledge, skills or understanding. Rather, the intent is to provide students cues, hints and other kinds of scaffolding that would enable students to learn.
- Focus Group Learning Interviews: The object of these interviews is to scale up the interaction model used in the Teaching/Learning Interview. So rather than a facilitator interacting with an individual student, we have a situation which is akin to a classroom. The students work in small groups of two or three students each. They are provided with written materials to scaffold their learning and the facilitator floats around the room interacting with these groups. The written materials has specific stopping points built in that require the students to interact with the facilitator
- Online Tutorials: The object of the online system is to further scale-up the interaction model, but now to a large enrollment class. In the Engineering courses we took advantage of an online homework delivery and tutorial system to collect data on student performance on various kinds of problems as well as provide students feedback about their own learning.

We describe below activities completed this year in Mathematics, Physics and Engineering courses separately.

MATHEMATICS

Three rounds of interviews to establish level of conceptual understanding of students in mathematics.

1. We interviewed 19 Calculus 1 students to determine their understanding of the concept of function. Interviews took place during October and November of 2009. We attempted to find interviewees who were likely to take Physics in the future so we could track growth longitudinally across classes.

2. We interviewed 11 Calculus 2 students to determine their understanding of accumulation (especially in the context of integration). Interviews took place during February 2010. We attempted to interview the same students as in the previous semester, but because some students changed their minds either about being interviewed or about taking Calculus 2, we ended up with 8 students interviewed before and 3 new students.

3. We interviewed 11 Calculus 2 students to determine their understanding of both function and accumulation. Interviews took place during April 2010. 10 of these students had been interviewed in February. In addition to measuring growth in their conceptual understanding, these interviews had an additional focus on the students' beliefs about mathematics and its role in physics. This additional focus was developed after observing that certain concepts seemed available to students when interviewed in mathematics but did not transfer when working problems in physics.

PHYSICS

Methodology: Our questions were addressed by two sets of interviews with introductory physics students. The first set of interviews included four think-aloud problem-solving interviews with individual students. Each interview was conducted after an exam in the class. In each interview, students were asked four questions in the following formats:

– Original problem: a problem from a recent homework topic related to the content of the recent exam
– Graphic problem: modified version of original problem in which part of info is given as graph
– Equation problem: modified version of original problem in which part of info is given as an equation
– Modified-Geometry problem: modified version of original or equation problem in which the geometry of the physical situation is changed.

The fourth interview included an additional modified geometry problem.

In the second set of interviews, students met in focus groups five times during the semester and solved problems in pairs. Two groups were formed: a treatment group that solved problems developed from the results of the Spring 2009 interviews conducted in the first year of this project and a control group who spent the same amount of time solving textbook problems on the same topics. Starting with the second interview, each student also took a pre/post assessment at each interview. Also starting with the second interview, the set of problems completed by the treatment group included the following:

- Math problem: a problem posed in an abstract math context focusing on a math concept that is helpful for subsequent physics problems.
- Physics problem: a simple problem focusing on a featured physics topic.
- Debate problem: a related physics problem where students must evaluate lines of reasoning put forward by fictitious students.
- Problem Posing problem: students must create a more complex physics problem by combining elements of the previous problems.

The first interview used a slightly different format that was substantially modified in the subsequent interviews. The fifth interview included a summative assessment covering all the topics of the previous interviews.

Participants

Individual Interviews The participants were 15 students who were currently enrolled in a second semester calculus-based physics. These students previously participated in the interviews we conducted in Fall 2009. They included the following:

- 3 students in electrical engineering
- 5 students in mechanical engineering
- 3 students in chemical engineering
- 1 students in civil engineering
- 1 student in architectural engineering
- 1 student in environmental engineering
- 1 student in chemistry

Focus Group Interviews The participants were 28 students who were currently enrolled in a first semester calculus-based physics. These students previously participated in the interviews we conducted in Spring 2009. They included the following:

- 3 students in electrical engineering
- 19 students in mechanical engineering
- 2 students in chemical engineering
- 1 students in civil engineering
- 1 student in nuclear engineering
- 2 student in industrial engineering

Interview Topics and Problems:

Individual Interviews, Fall 2009 - The interview questions were based on the following topics : Coulomb's Law, resistance, Ampere's Law, and LRC circuits. The attached documents include the problems that were administered in these interviews.

Focus Group Interviews, Spring 2010 - The focus group interviews questions were based on the following topics: kinematics, Newton's 2nd Law, work & energy, and work & energy with rotational motion. The attached documents include the pre-tests, the problems solved by the control group, the problems solved by the treatment group, and the post-tests for each of the interviews.

Data Collection and Organization: All individual and focus group interviews were video and audio taped. Student work on problem sheets was also collected. Full transcripts were created for each of the individual interviews conducted in the fall.

Additionally, for each problem in these interviews, we wrote down in our own words a description of what happened based on the video/audio recordings and students work which we refer to as the 'pseudo-transcript'.

Data Analysis: We used a phenomenographic approach to analyze the pseudo transcripts. The pseudo-transcript was coded for the difficulties that students expressed while solving the problem. The pseudo-transcript was also coded for the hints provided by the researcher.

Student work was also analyzed using a phenomenographic approach. The approaches students used to solve the problem were identified and categorized. Additionally, we created rubrics to rate students' solutions.

ENGINEERING

Automatic homework generation modules were offered to Electrical Engineering students in the Fall 2009 and Spring 2010 Linear Systems courses.

We upgraded the zero-input response and unit-impulse response modules to incorporate detailed plots of system responses that illustrate higher-level mathematical concepts associated with these types of homework problems. For example, the plots illustrate the relationships between the graphical depiction of a response signal and its constituent parts: the separate modes that contribute to these responses and their relationships to the initial conditions for these second-order systems (for three types of damping and bounded versus unbounded responses). We fixed a number of bugs in these software modules, which address complex numbers, signals, unforced response, unit impulse response, Fourier series, and discrete Fourier transforms. The modules are written in Java and PHP, and they utilize a PostGreSQL database.

We are also completing a new Fourier series module that will be offered for the first time in Fall 2010. This learning module addresses higher-level abstractions that relate to trigonometric and compact trigonometric Fourier series. These abstractions specify the linkages between the mathematical parameters in the Fourier series and their effects on the visual appearance of the resulting time-domain signals and frequency-domain coefficients. In other words, we hypothesize that a student's level of understanding of Fourier series concepts is related to their ability to tie together the mathematical and graphical representations of a Fourier series in multiple domains.

In Spring 2010, we conducted interviews ith 24 students enrolled in ECE 512 – Linear Systems. Each interview was conducted as a one-on-one teaching/learning experience, where students were given a sequence of consecutively more difficult problems related to Fourier series. Each student was asked to explain their reasoning out loud as they progressed so that the research time could archive their thought process and attempt to categorize the level of abstraction available to the student. The types of hints required by the interviewer, as well as the individual problems faced by each student, should help to inform this categorization process. These sessions were videotaped.

At the end of each semester, module surveys have archived student reactions to the use of these modules as learning experiences. Abstracts in support of this work were submitted to the 2010 conference of the American Society of Engineering Education and the 2010 Frontiers in Education conference.

INTERDISCIPLINARY ACTIVITIES

In addition to the activities described above, the project team met for one hour bi-weekly during the entire academic year to discuss progress. In addition members of the project team regularly attended and participated in the Physics Education Research Seminar.

Finally, we also had a videoconference between our entire project staff and our consultant Dr. David Jonassen. This meeting was completed in December 2009. The meeting with Dr. Jonassen provided us with useful feedback about future steps in our research project. We also had three individual meetings between the PI and Dr. Jonassen to keen him updated on the progress of our project.

APPENDICES

Several documents have been attached below. These include protocols used for the individual interviews, focus group learning interviews.

Math 220 Interview Protocol F09

November 2, 2009

1. *Prepare for the interview at least 5 minutes before the scheduled time. Unlock the conference room (CW 023, Andy has key) and leave the door open. Set out the IC Recorder, two copies of the Informed Consent form and a pad of paper for students to write or draw on as needed when they answer the questions.*

2. *When the student arrives, introduce yourself and welcome the student by name. Close the door to the conference room. Ask for permission to record the interview. If permission is granted, start the recorder.*

3. *Explain the purpose of the interview:*
   We are interviewing students in Math 220 to better understand what EE students are learning about math. This is prompted by a desire to better prepare students for later courses. I will ask you some questions about your knowledge of functions, inverse of functions, limits and derivatives so we can better understand what and how students have learned from the course. Some of these questions have right and wrong answers while other questions may not have a particular right answer, but are intended just to give an idea of how you think about the material. This interview should take approximately 45 minutes. Your participation is completely voluntary and your grade will not be affected by your decision to participate or not. You will receive $10 for your time for participating in this interview and you may also benefit by improvements in instruction in mathematics and by having a chance to go over some mathematical ideas with an instructor. In the event we include any of your comments in any discussion or publication, your privacy will be maintained by the use of a pseudonym. We have two copies of an Informed Consent Form for you to sign; one for our records and one for you to keep.

4. *Have them read and sign the form. If they decline to sign the form, thank them for their time and terminate the interview. Otherwise sign and date the form as witness and then proceed to the questions below.*

5. *Mathematical questions. The purpose is to get a sense of different understandings of functions that different students have, rather than to classify each particular student's views. If a student gets stuck answering a question, you may offer a hint if appropriate to get them unstuck. Don't let the interview stretch over an hour.If students make mistakes, please correct them either at the end of the problem or at the end of the interview as appropriate.*

Protocol

(a) *Try to get a sense on how students relate a function and its derivative - Approximately 15 minutes*

Open the exam at problem 6 (find the tangent line to $y = x^2 - 2x + 1$ *at the point* $(0, 1)$*)*

- Can you describe to me how you solved the problem?

Give them the graph of the function (with the equation of the function)

- Can you tell me if the derivative at $x = 3$ is positive or negative?
- Can you estimate the derivative at $x = 3$ from the graph?
- Can you show me the tangent line at $(0, 1)$?

Ask as many (or as few) of the following questions as you deem necessary in order to situate the student's understanding within the APOS theory.

- How does $y(0) = 1$ relate to the graph?
- How does $m = -2$ relate to the graph?
- How does the derivative $f' = 2x - 2$ relate to the graph?
- What are we computing in this limit (in the definition of the derivative)?
- How would the tangent line to the function $y = x^2 - 2x + 5$ at $x = 0$ relate to the previous tangent line?

(b) *Questions about functions and their inverse - Approximately 15 minutes*

Show the student the table of values

- What is $f(2)$?
- What is $f \circ g(0)$?
- Is $f \circ g$ a function?
- What is $f^{-1}(-1)$?
- Can you explain to me what the inverse of a function is?
- Is f^{-1} a function?
- If we use the notation $h = f \circ g$, what is $h^{-1}(5)$? How would you find it?

(c) *Basic concept of function - Approximately 5 minutes*

- Can you talk about functions? How do you think about functions?
- What is a function? How would you explain it to a smart 10 year old?
- *if they can't get the definition right* Can you give an example of something that is not a function?
- Can you give an example of a function?
- Does a function always take numbers to numbers?

(d) *Concept of limits - Approximately 15 minutes*

Open the exam at problem 5

- Can you explain how you solved this problem?

Suppose we know the value of $f(x)$ everywhere except at 1

- Can you find $\lim_{x \to 0} f(x)$?
- Can you find $\lim_{x \to 1} f(x)$?
- How do you think about limits?
- Is the limit always the value of the function at the point?
- *ask appropriate probing questions to get students to talk about limits, functions, values of functions, continuity....*

(e) *If you have time... which probably won't happen*

Question about solving an equation - do students see a relation with the procedure of solving an equation and what it means for the (graph of the) function? - Approximately 5 minutes

- Can you solve $x^2 - 5x + 4 = 0$?
- What does it mean to solve an equation?
- How does the answer relate to the function $f(x) = x^2 - 5x + 4$?

End of the Interview

(a) *Other comments.* Are there any other comments or questions you would like to make about any aspect of the course? *(Ask follow-up questions or provide answers (if you know the answers) as appropriate. You may tell the student you will refer questions to Prof. Bennett and he will get back to them if you dont feel you can adequately address a question (say about why we are doing labs in the course))*

Final Questions

i. What year are you in school?

ii. What grade do you expect from Calculus?

iii. Did you take Calculus before?

(b) *Thank the student for participating. Gently correct any errors they made that havent been addressed already. Let them know they are always welcome to email any additional comments or suggestions about the course to bennettmath.ksu.edu.*

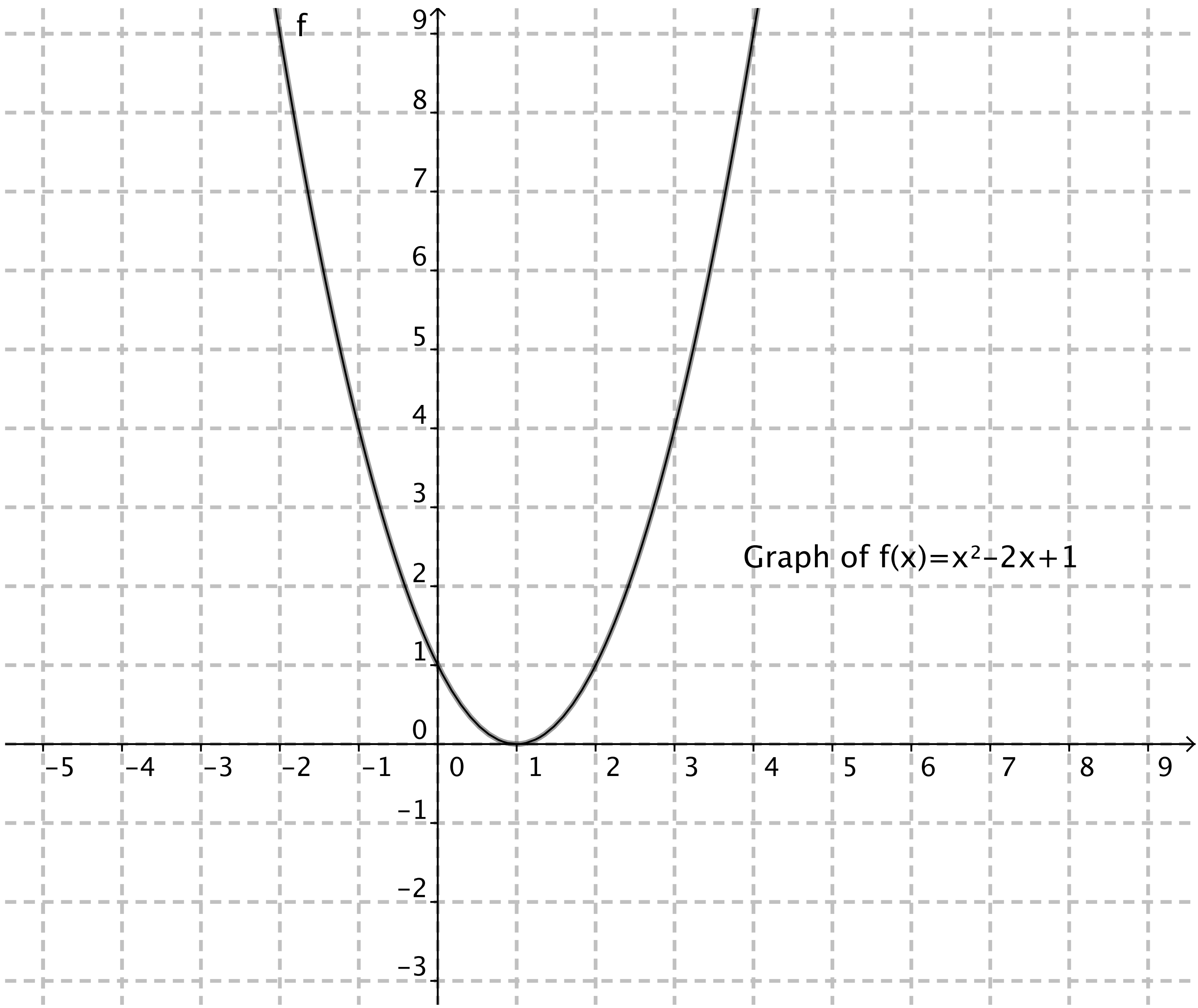
(c) *Stop the recorder. Have the student fill out a receipt form with their*

- Name
- Address
- Social Security Number
- Date
- Signature

Once you have the completed receipt, give the student $10 and thank them again. Place the white copy of the completed receipt in the money envelope and leave the yellow copy in the receipt book.

(d) *As soon as you have time (since sometimes there are two interviews back to back), please write up notes on the interview, transfer the recording to the computer system and erase the session from the IC Recorder.*

(e) *Write the name of the student/interviewee on each piece of scratch paper used and file the paper with his/her consent form.*


f
Graph of f(x)=x²−2x+1
9
8
7
6
5
4
3
2
1
0
−1
−2
−3
−5
−4
−3
−2
−1
0
1
2
3
4
5
6
7
8
9

x	$f(x)$	$g(x)$
-2	0	7
-1	5	2
0	-1	-2
1	2	-1
2	1	5

MATH 220 FALL 2009
EXAM 1

YOUR NAME:

RECITATION INSTRUCTOR NAME:

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Problem 1. (a) – (e): Find the limit (if it is infinite, determine the sign of ∞).

(a) (8 pts) $\displaystyle\lim_{x\to 3} \frac{x^2-5x+6}{x^2-9}$

(b) (8 pts) $\displaystyle\lim_{x\to 2} \frac{x-2}{\sqrt{x+2}-2}$

(c) (8 pts) $\displaystyle\lim_{x\to 1^+} \frac{x^2-3x+2}{x^2-2x+1}$

(d) (8 pts) $\lim_{x\to+\infty}\left(\frac{x^2}{x-1}-\frac{x^2}{x+1}\right)$

(e) (8 pts) $\lim_{x\to 0}\frac{\sin(x^2-x)}{x^2+x}$

Problem 2. (a) – (d): Find the derivative.

(a) (8 pts) $\left[\frac{\sqrt[3]{x^2}+\sqrt{x}}{x}\right]'$

(b) (8 pts) $\left[x^3 e^x\right]'$

(c) (8pts) $\left[\dfrac{x^2+1}{x+2}\right]'$

(d) (8pts) $[\sin(x)\cos(x)]'$

Problem 3. (7 pts) Let a function $f(x)$ be defined as follows:

$$f(x) = \begin{cases} (x-1)^2 & \text{if} \quad x < 0, \\ e^{x^2-x} & \text{if} \quad x \geq 0. \end{cases}$$

Is the function $f(x)$ continuous at the point $x = 0$? Explain your answer using the definition of continuity.

Problem 4. (7 pts) Use the Intermediate Value Theorem to show that the equation

$$\ln x = \frac{1}{x}$$

has a solution in the interval $(1, e)$.

Problem 5. (7 pts) Suppose that a function $f(x)$ satisfies the inequality

$$\frac{1}{x} < f(x) < x^2 \quad \text{for all} \quad x > 1.$$

What can you say about the limit $\lim_{x \to 1^+} f(x)$?

Problem 6. (7 pts) Use the definition of the derivative to find an equation of the tangent line to the parabola $y = x^2 - 2x + 1$ at the point $(0, 1)$.

Accumulation Interview Protocol S10

February 14, 2010

1. *Prepare for the interview at least 5 minutes before the scheduled time. Unlock the conference room (CW 023, Andy has key) and leave the door open. Set out the IC Recorder, two copies of the Informed Consent form and a pad of paper for students to write or draw on as needed when they answer the questions.*

2. *When the student arrives, introduce yourself and welcome the student by name. Close the door to the conference room. Ask for permission to record the interview. If permission is granted, start the recorder.*

3. *Explain the purpose of the interview:*
   We are interviewing students to better understand what students are learning about math. This is prompted by a desire to better prepare students for later courses. I will ask you some questions about your knowledge of functions and integrals so we can better understand what and how students have learned from math courses. Some of these questions have right and wrong answers while other questions may not have a particular right answer, but are intended just to give an idea of how you think about the material. This interview should take approximately 45 minutes. Your participation is completely voluntary and your grade will not be affected by your decision to participate or not. You will receive $10 for your time for participating in this interview and you may also benefit by improvements in instruction in mathematics and by having a chance to go over some mathematical ideas with an instructor. In the event we include any of your comments in any discussion or publication, your privacy will be maintained by the use of a pseudonym. We have two copies of an Informed Consent Form for you to sign; one for our records and one for you to keep.

4. *Have them read and sign the form. If they decline to sign the form, thank them for their time and terminate the interview. Otherwise sign and date the form as witness and then proceed to the questions below.*

5. *Mathematical questions. The purpose is to get a sense of different understandings of accumulations (integrals) that different students have, rather than to classify each particular student's views. If a student gets stuck answering a question, you may offer a hint if appropriate to get them unstuck. Don't let the interview stretch over an hour.If students make mistakes, please correct them either at the end of the problem or at the end of the interview as appropriate.*

Protocol

(a) *Questions about integrals as area under the graph.*

Show the student the first graph (the function is $f(x) = -x(x-3.8)(x+3.8)/5$ *but do not give the equation to the student)*

- Given that $\int_0^1 f(t)dt = 1.4$, can you find $\int_{-1}^0 f(t)dt$?
- Can you describe how you would do it?
- Is $\int_0^1 f(t)dt$ larger or smaller than $\int_1^2 f(t)dt$?
- What about $\int_{-1}^2 f(t)dt$, is it larger or smaller than $\int_0^1 f(t)dt$?

(b) *Questions about integral as area under the graph.*

- What is the area of a semicircle?
- Can you find $\int_0^4 f(t)dt$?
- Can you find $\int_0^2 f(t)dt$?

(c) *One more question about integrals as area...*

Show the "triangle" picture

- Can you find $\int_0^1 f(t)dt$?

  if they can not answer the question above,
  - Can you find the area $\int_0^2 f(t)dt$?
  - $\int_0^1 f(t)dt$ is the area of the trapezoid; can you find its area?

    see if the student tries to get the area of the trapezoid or if the student tries to find the equation of the line and the antiderivative of the function.
- Can you draw the area from 0 to x as a function of x?

(d) *Questions about differentiation and integration as inverse operations*

Show the graph of g.

- Can you draw the graph of $g'(x)$?
- Can you draw the graph of $\int_0^x g'(t)dt$?
- Can you draw the graph of $G(x) = \int_0^x g(t)dt$?
- Can you draw the graph of $G'(x)$?
- *if the student does well, you can modify* $g(x)$ *to be zero for* $x \geq 5$ *and ask what happens to* $G(x)$ *for this new function.*

(e) *Questions about product of integrals*

Show the student the graphs of $f(x)$ *and* $g(x)$

- Can you tell me if $\int_{-1}^1 f(t)g(t)dt$ is positive, zero, or negative?
- Actually, the integral is not zero but negative, do you see why?

End of the Interview

(a) *Other comments.* Are there any other comments or questions you would like to make about any aspect of the course? *(Ask follow-up questions or provide answers (if you know the answers) as appropriate. You may tell the student you will refer questions to Prof. Bennett and he will get back to them if you dont feel you can adequately address a question (say about why we are doing labs in the course))*

Final Questions

i. What year are you in school?
ii. What grade do you expect from Calculus 2?
iii. Did you take Calculus 2 in high school?

(b) *Thank the student for participating. Gently correct any errors they made that havent been addressed already. Let them know they are always welcome to email any additional comments or suggestions about the course to bennettmath.ksu.edu.*

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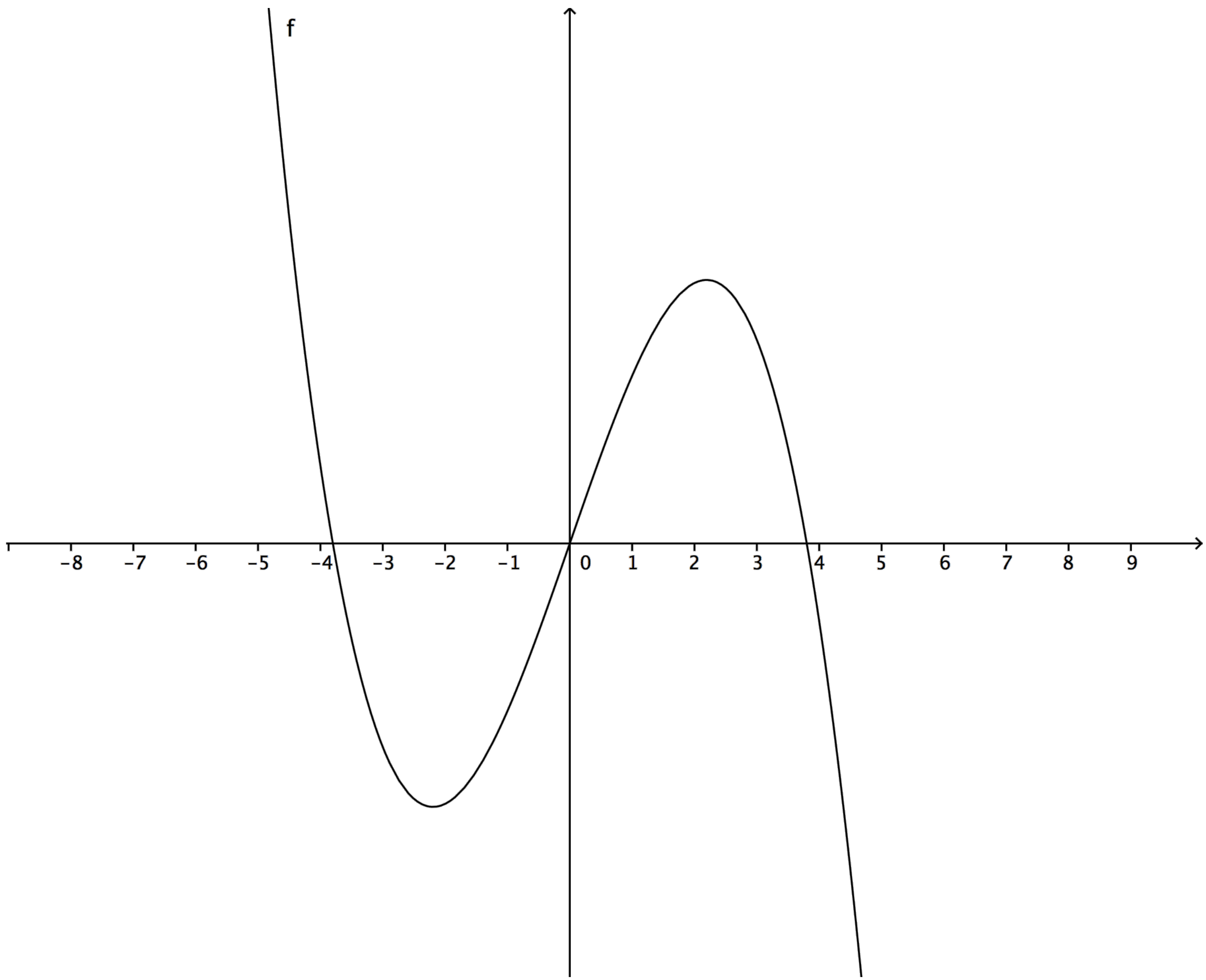

FIGURE 1. Question (a): Graph of f

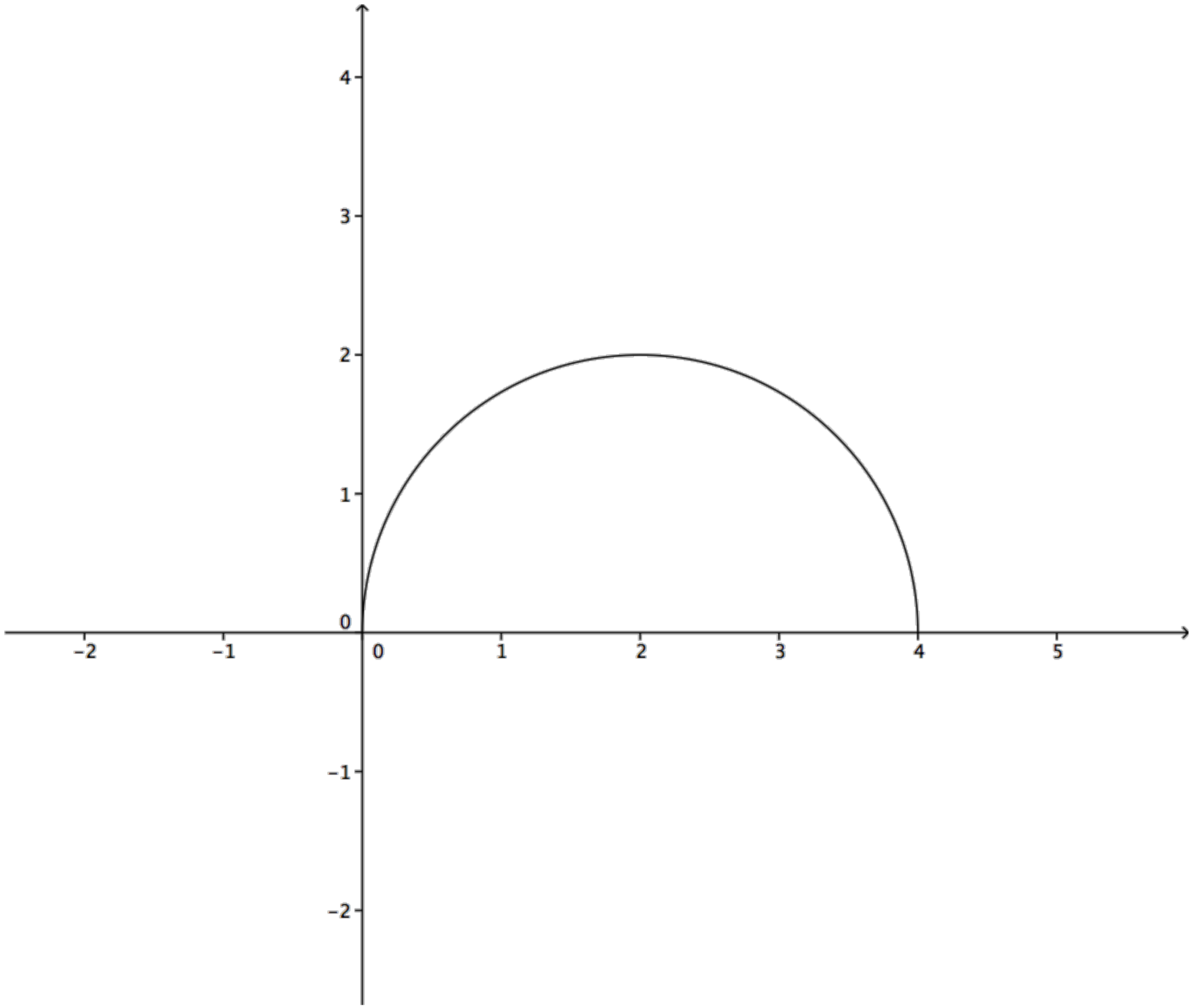

FIGURE 2. Question (b): Graph of f

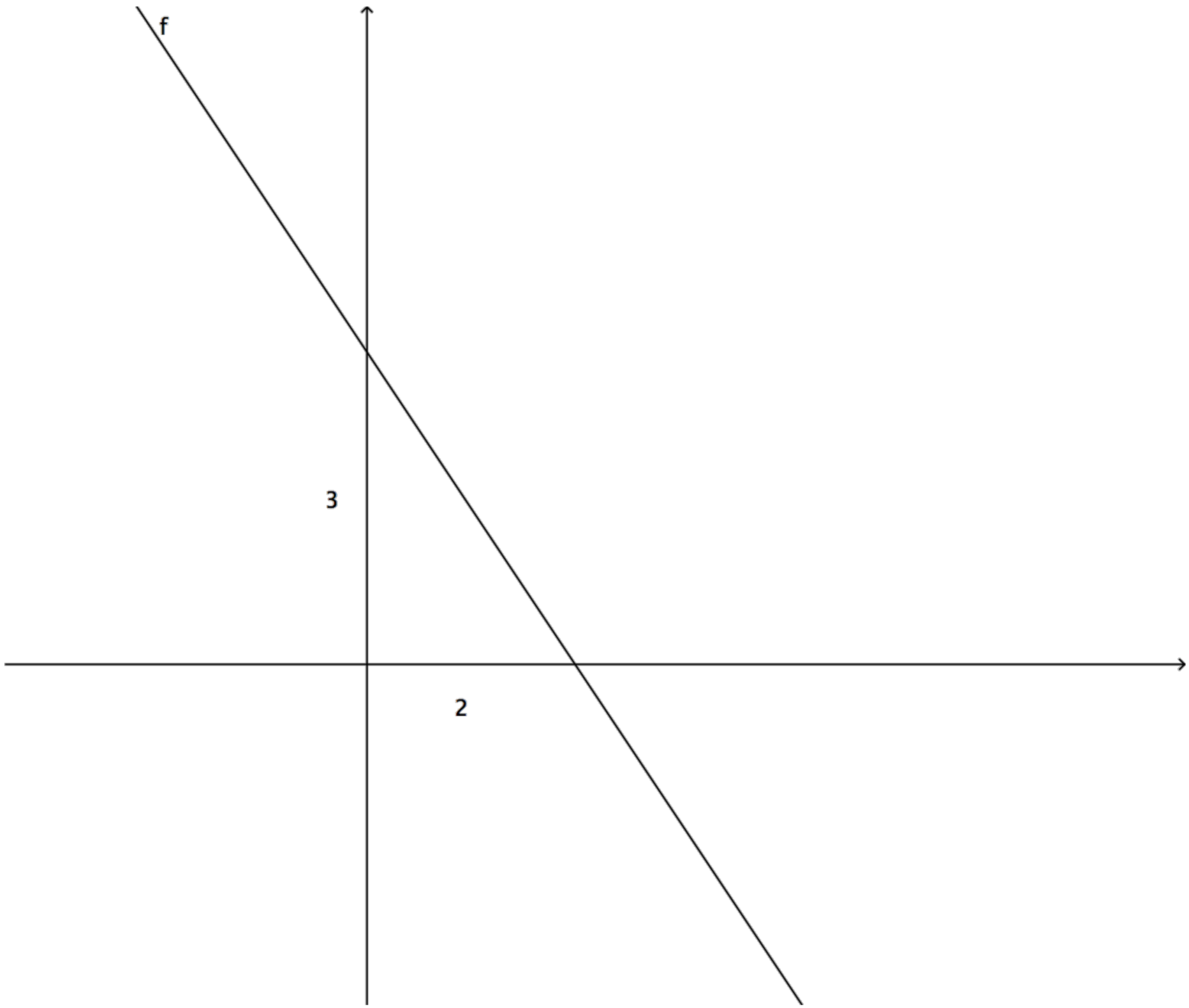

FIGURE 3. Question (c): Graph of f

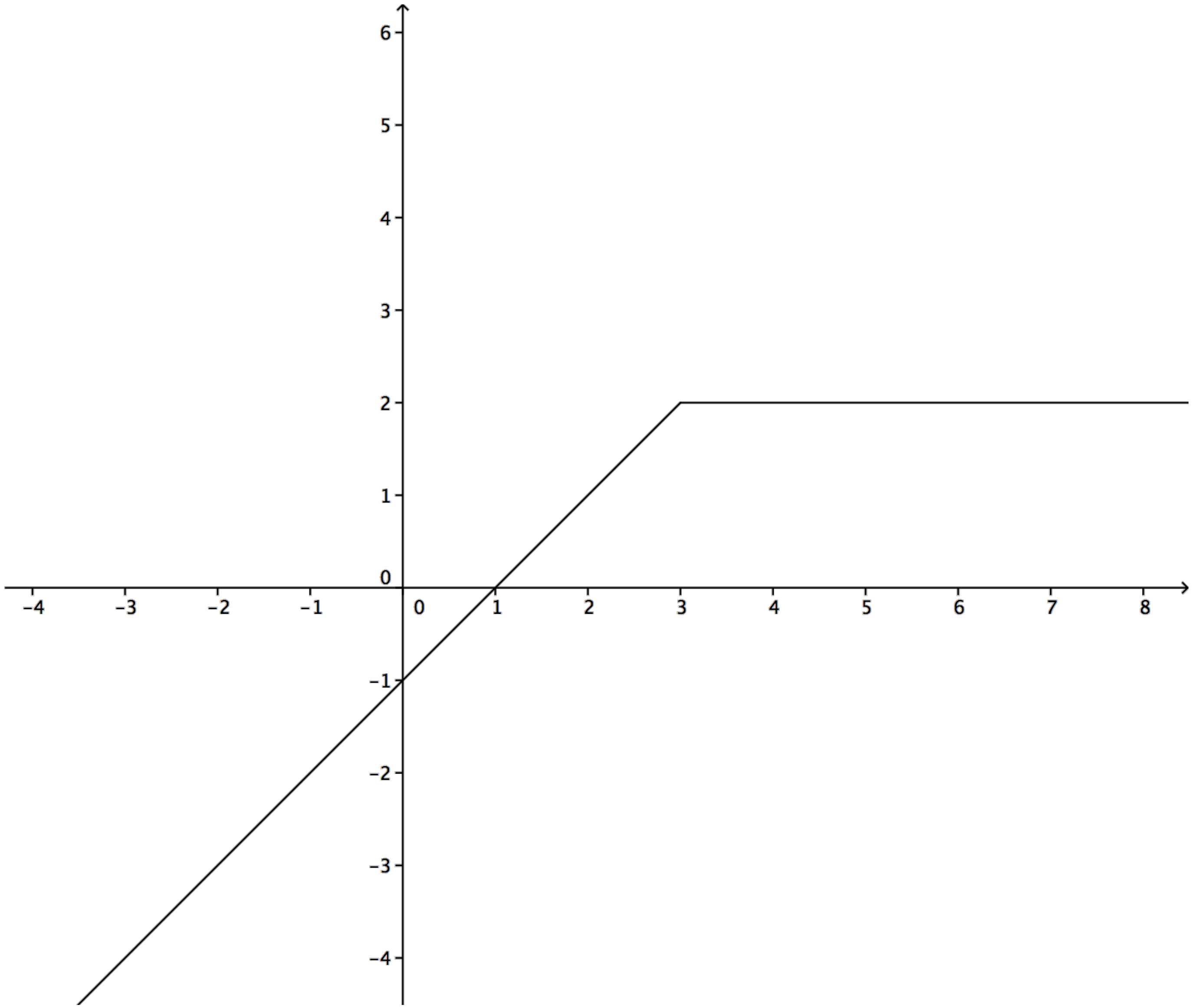

FIGURE 4. Question (d): Graph of g

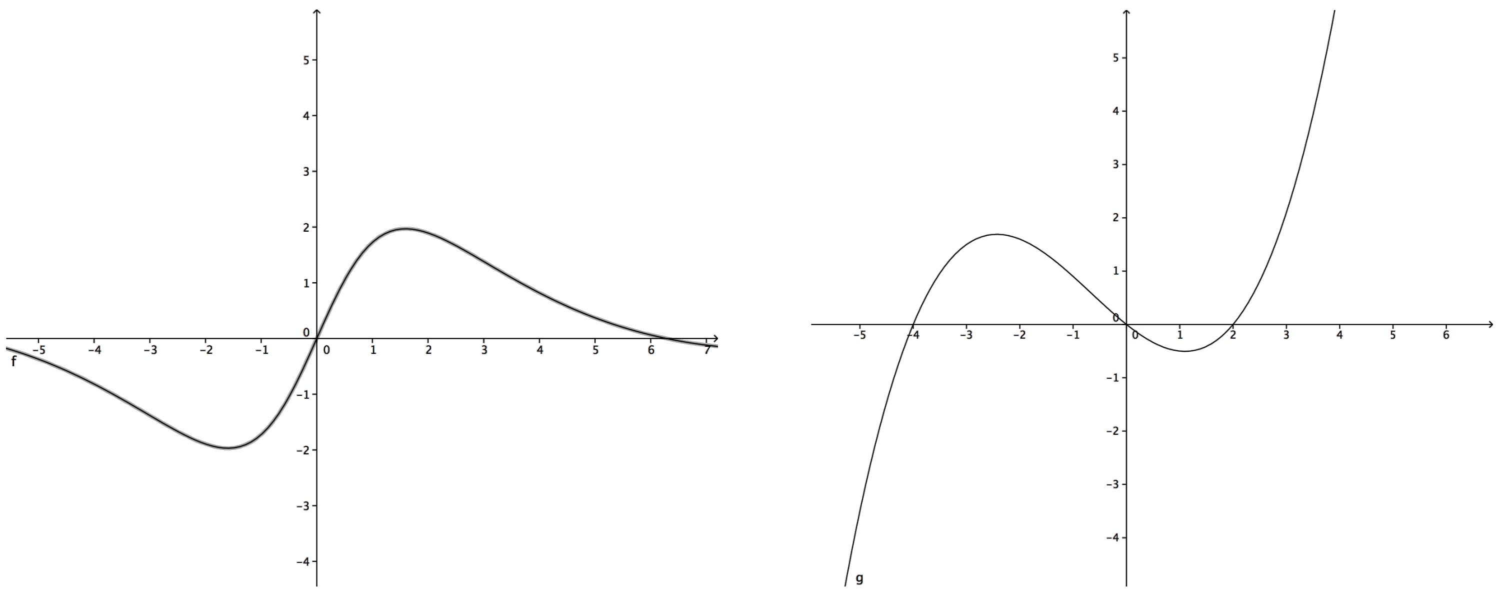

FIGURE 5. Question (e): graphs of f and g

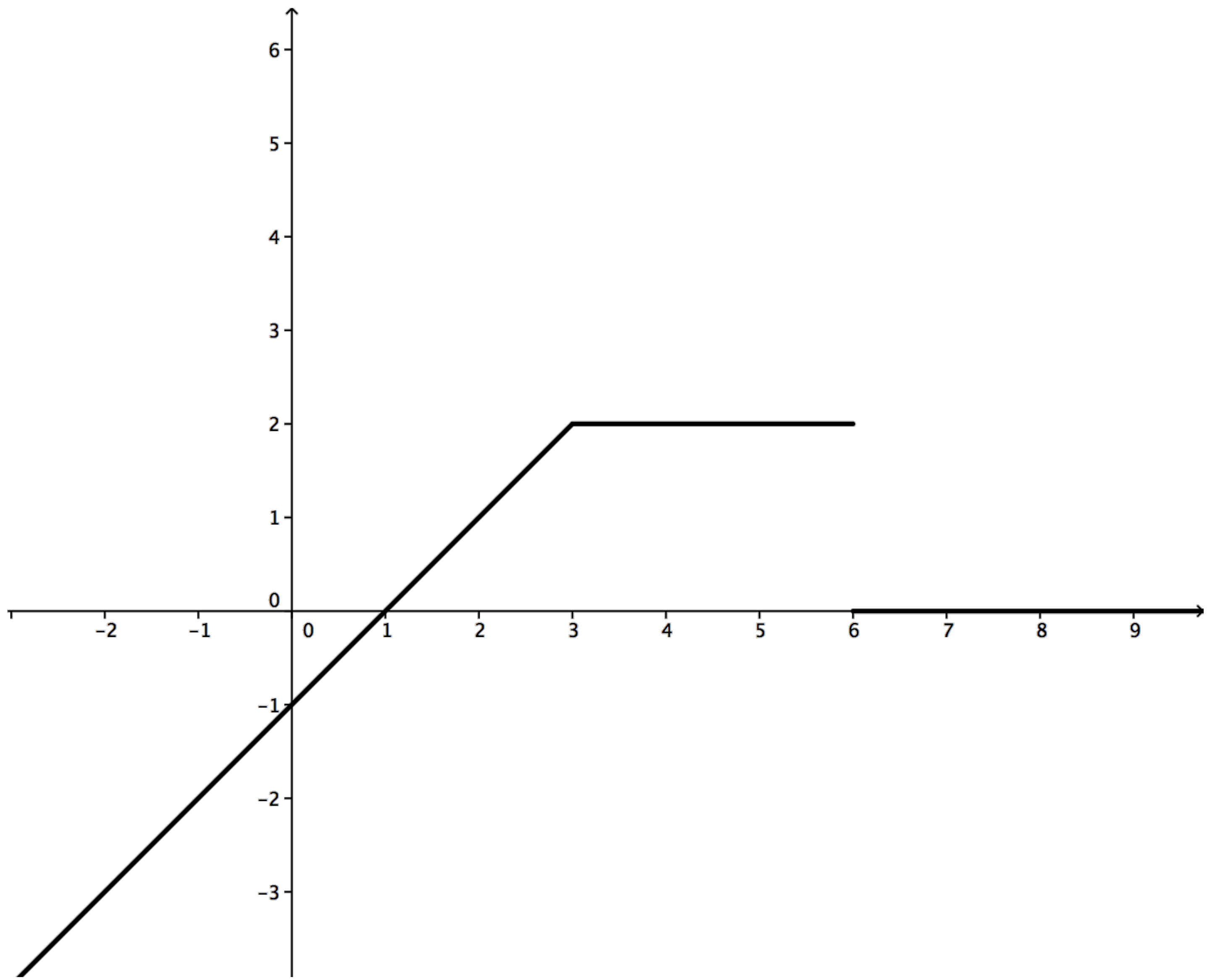

FIGURE 6. Optional question: Graph of g

Accumulation Interview Protocol S10

April 17, 2010

1. *Prepare for the interview at least 5 minutes before the scheduled time. Unlock the conference room (CW 023, Andy has key) and leave the door open. Set out the IC Recorder, two copies of the Informed Consent form and a pad of paper for students to write or draw on as needed when they answer the questions.*

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3. *Explain the purpose of the interview:*
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4. *Have them read and sign the form. If they decline to sign the form, thank them for their time and terminate the interview. Otherwise sign and date the form as witness and then proceed to the questions below.*

5. *Mathematical questions. The purpose is to get a sense of different understandings of accumulations (integrals) that different students have, rather than to classify each particular student's views. If a student gets stuck answering a question, you may offer a hint if appropriate to get them unstuck. Don't let the interview stretch over an hour.If students make mistakes, please correct them either at the end of the problem or at the end of the interview as appropriate.*

Protocol

(a) *Questions about function.*

- Given that $f(x, y) = x^2 + y^2$, can you find $f(r, \theta)$?
- What is a function?
- Is the derivative operation d/dx a function? Explain.
- Is $\int_0^x f(t)dt$ a function of x? Explain.

(b) *Questions about parametric curves.*

- Graph $x(t) = \cos t$, $y(t) = \sin t$, $0 \leq t \leq 2\pi$.
- Graph $x(t) = \cos 2t$, $y(t) = \sin 2t$, $0 \leq t \leq 2\pi$.
- Are these the same functions?

(c) *Questions about integral as accumulation.*

Give the student the graph with the two shapes.

- If we know that any horizontal line intersects both figures with the same length, what can we say about the area of the two shapes?

Give the student the graph of $x = sin\ y$

- Can you find the area of the shaded region?

(d) *Questions about belief.*

- What is math?
- What is the role of theorems in math?
- What is the role of logic/deductive reasoning in math?
- What is the role of math in physics?

End of the Interview

(a) *Other comments.* Are there any other comments or questions you would like to make about any aspect of the course? *(Ask follow-up questions or provide answers (if you know the answers) as appropriate. You may tell the student you will refer questions to Prof. Bennett and he will get back to them if you dont feel you can adequately address a question (say about why we are doing labs in the course))*

Final Questions

i. What year are you in school?

ii. What grade do you expect from Calculus 2?

iii. Did you take Calculus 2 in high school?

(b) *Thank the student for participating. Gently correct any errors they made that havent been addressed already. Let them know they are always welcome to email any additional comments or suggestions about the course to bennettmath.ksu.edu.*

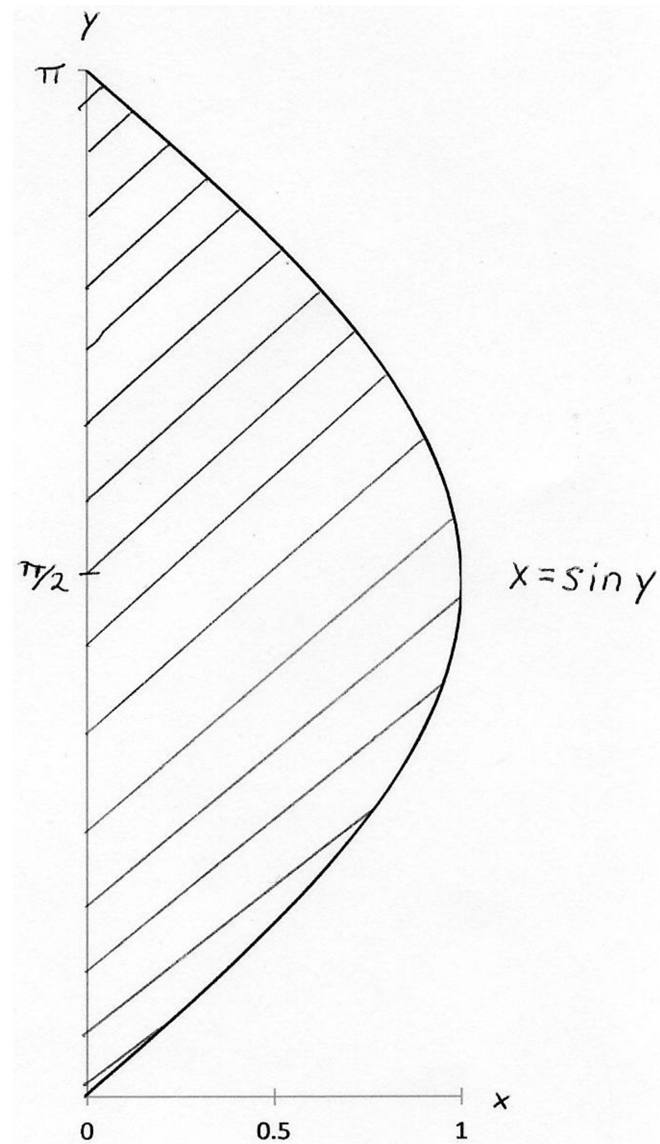
(c) *Stop the recorder. Have the student fill out a receipt form with their*

- Name
- Address
- Social Security Number
- Date
- Signature

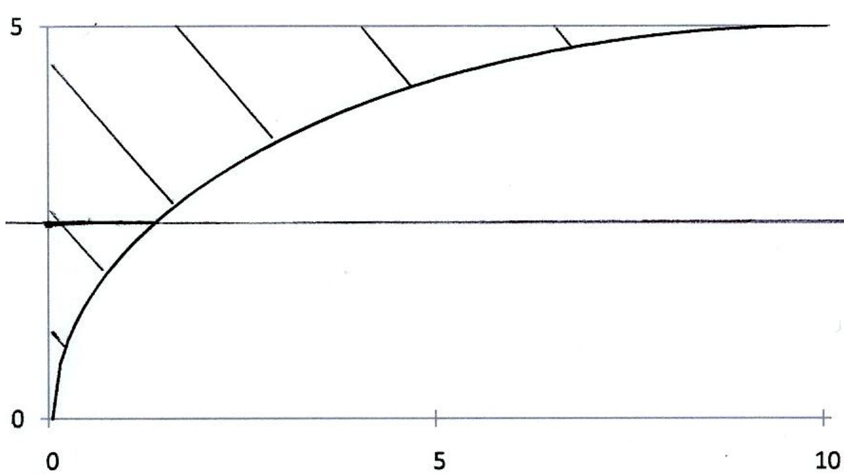
Once you have the completed receipt, give the student $10 and thank them again. Place the white copy of the completed receipt in the money envelope and leave the yellow copy in the receipt book.

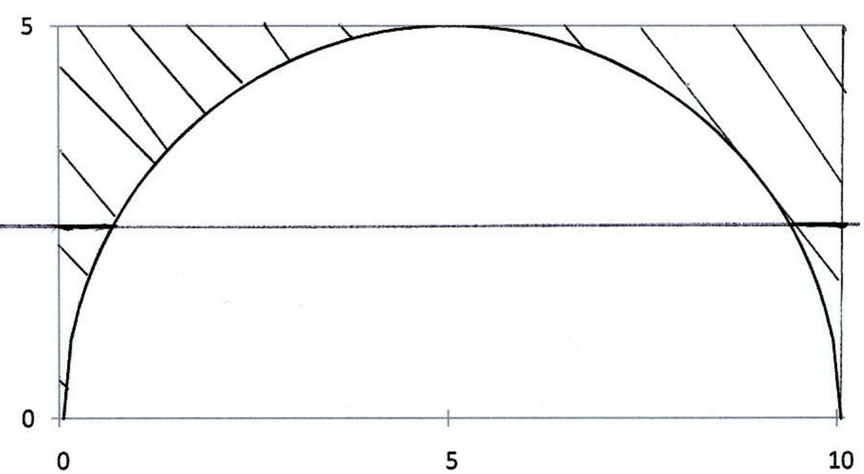
(d) *As soon as you have time (since sometimes there are two interviews back to back), please write up notes on the interview, transfer the recording to the computer system and erase the session from the IC Recorder.*

(e) *Write the name of the student/interviewee on each piece of scratch paper used and file the paper with his/her consent form.*



y
π
π/2
x = sin y
x
0
0.5
1

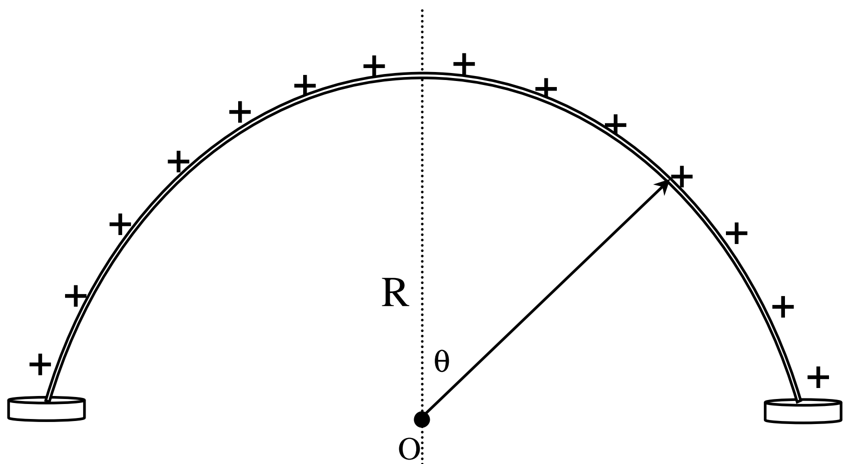

5
0
0
5
10


5
0
0
5
10

Problem 1

You are standing at the center O of a circular arch which has radius R and is electrically insulated from the ground, as shown in the figure below. The arch is positively charged with a constant charge density of λ along the arch.

Find the magnitude and direction of the electric field at your feet (i.e. at a point O on the ground directly below the top of the arch).



Problem 2

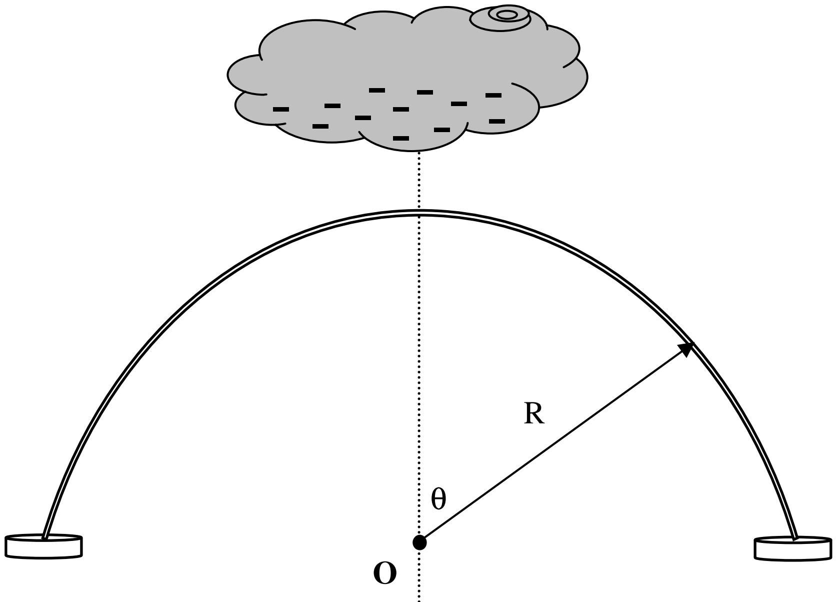
You are standing at the center of the arch as in problem 1 in a stormy day. There are negatively charged clouds over the arch. The charge distribution λ on the arch now depends on the angle θ as per the function:

$$\lambda(\theta) = \lambda_0 \cos\theta$$

where λ_0 is a positive constant.

(a) Indicate the charge distribution on the figure below.

(b) Find the magnitude and direction of the electric field at your feet (i.e. at a point O on the ground directly below the top of the arch).

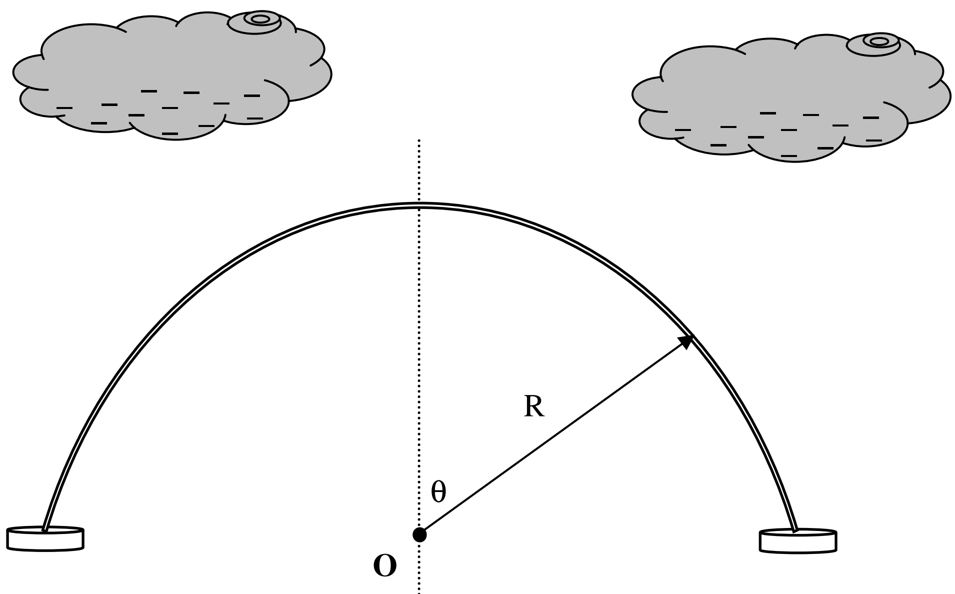

R
θ
O

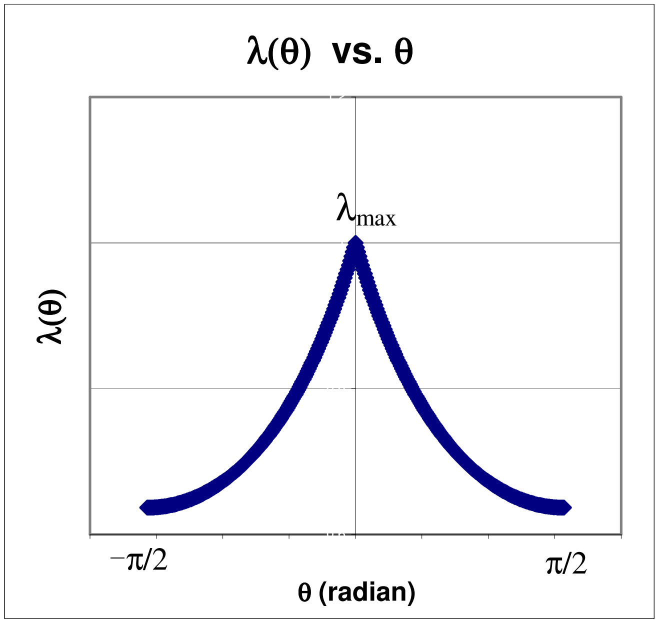
Problem 3

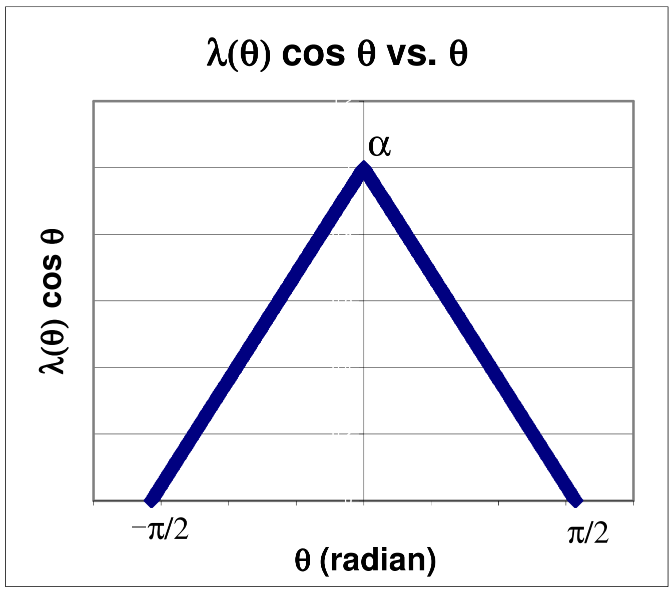
You are standing at the center of the arch as in problem 1 in a stormy day. There are negatively charged clouds over the arch. The charge distribution on the arch now depends on the angle θ as per one of the graphs shown.

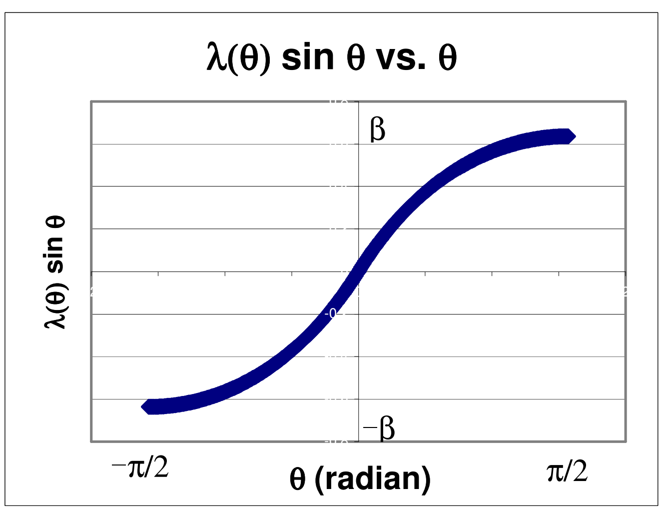
(a) Indicate the charge distribution on the figure below.

(b) Find the magnitude and direction of the electric field at your feet (i.e. at a point O on the ground directly below the top of the arch).




λ(θ) vs. θ
λ(θ)
λ_{max}
−π/2
π/2
θ (radian)


λ(θ) cos θ vs. θ
λ(θ) cos θ
α
−π/2
π/2
θ (radian)


λ(θ) sin θ vs. θ
λ(θ) sin θ
β
−β
−π/2
π/2
θ (radian)

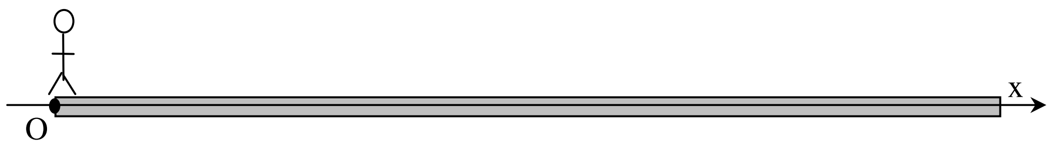
Problem 4

A straight metal rod of length L is lying on the ground but is insulated from the ground. The charge on the rod is distributed with charge density given as per the following function:

$$\lambda(x) = \alpha\, x^2$$

where: α is a positive constant
'x' is the position on the x-axis relative to the origin O as shown in the figure below.

(a) Indicate the charge distribution on the figure below.

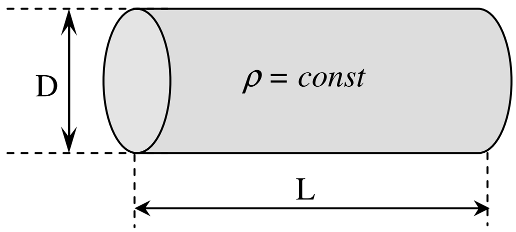


(b) Find the magnitude and direction of the electric field at your feet, located at x = 0.

PROBLEMS FOR INTERVIEW 2

Problem 1

Find the resistance of a cylindrical conductor of length L, diameter D and resistivity ρ (ρ is constant along the conductor).

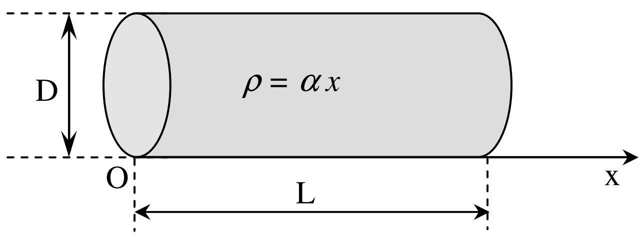


Problem 2

Find the resistance of a cylindrical conductor of length L, diameter D. The resistivity $\rho(x)$ is changing along the conductor as per the following function:

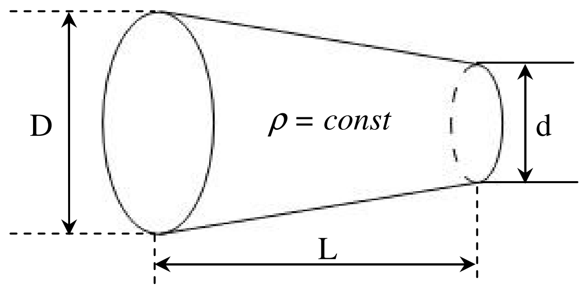
$$\rho(x) = \alpha\, x$$

where x is the distance from the left end of the conductor.



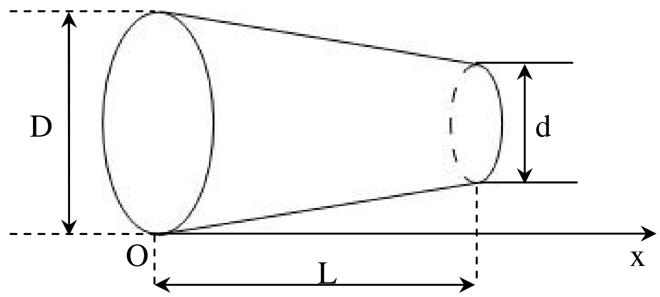
Problem 3

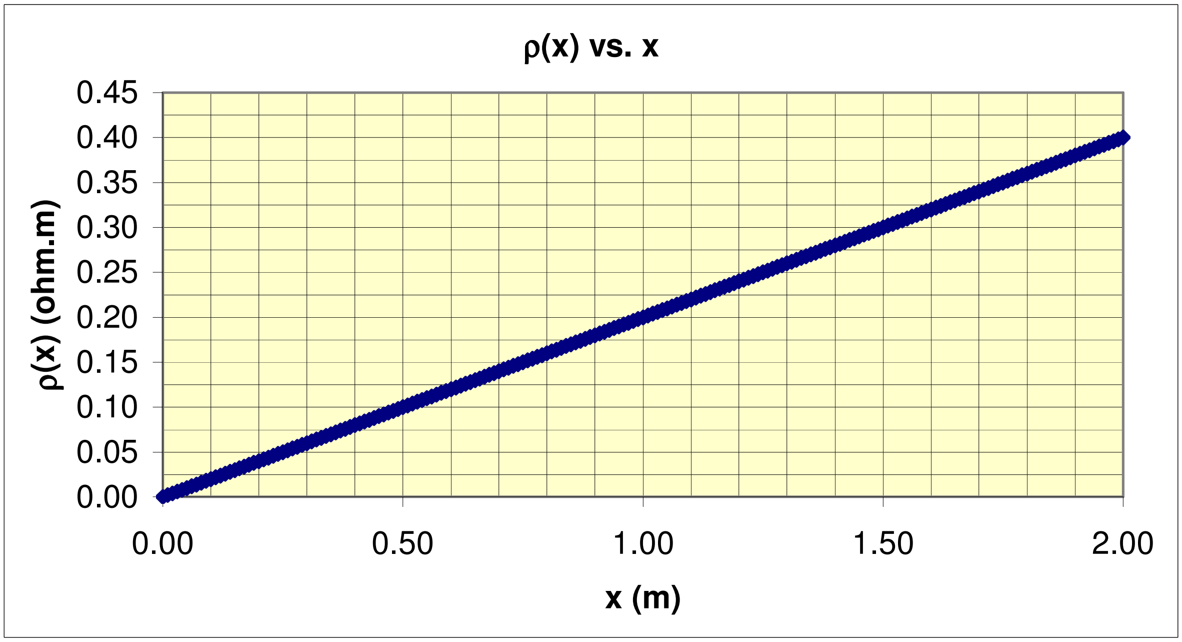
A conductor has diameter decreasing from D to d over its length L. The resistivity ρ is constant along the length of this conductor. Find the resistance of this conductor.

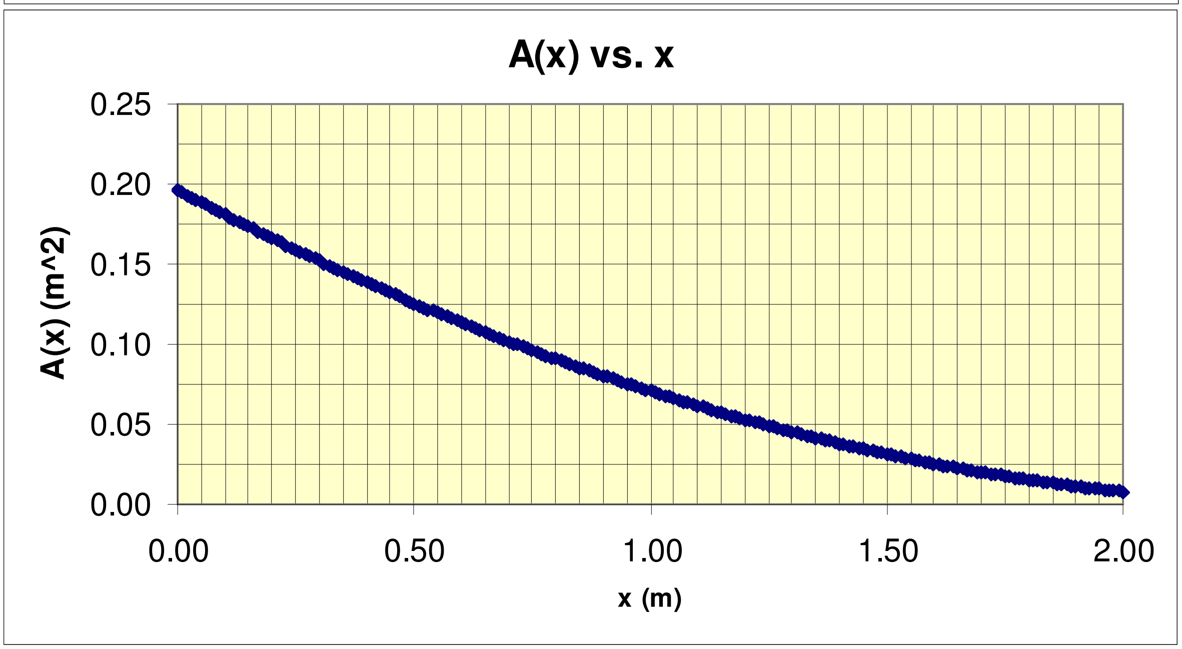


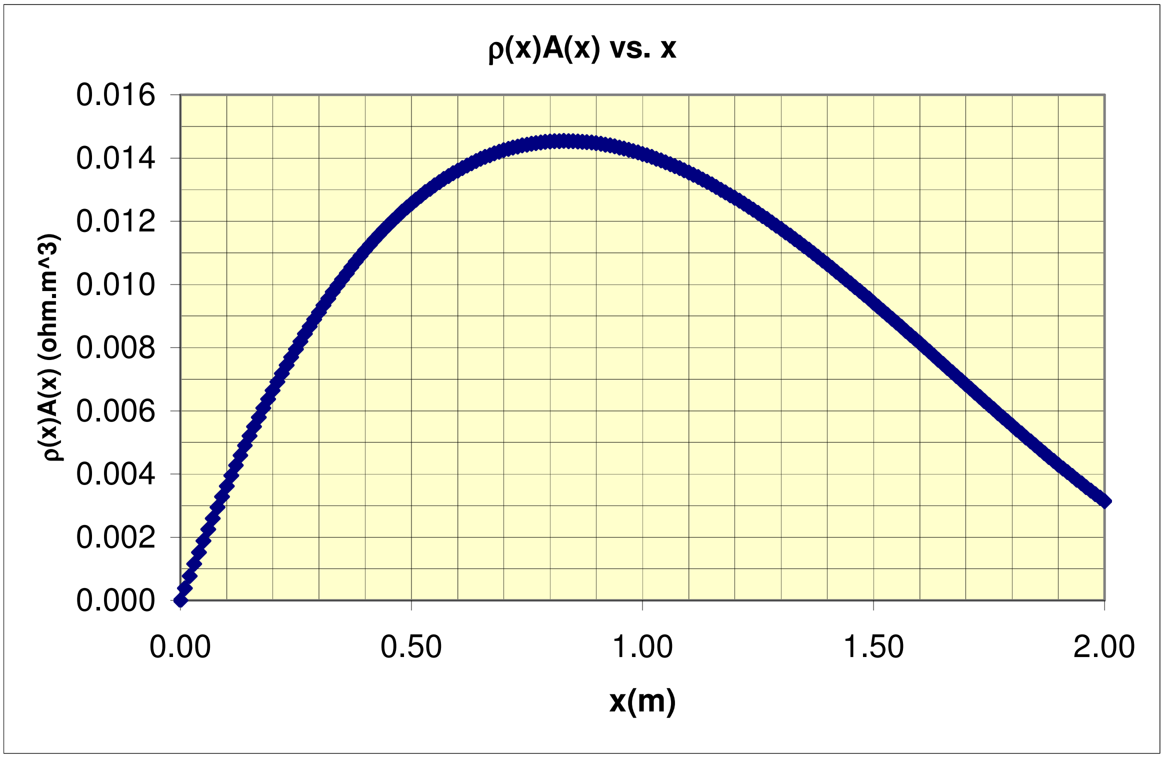
Problem 4

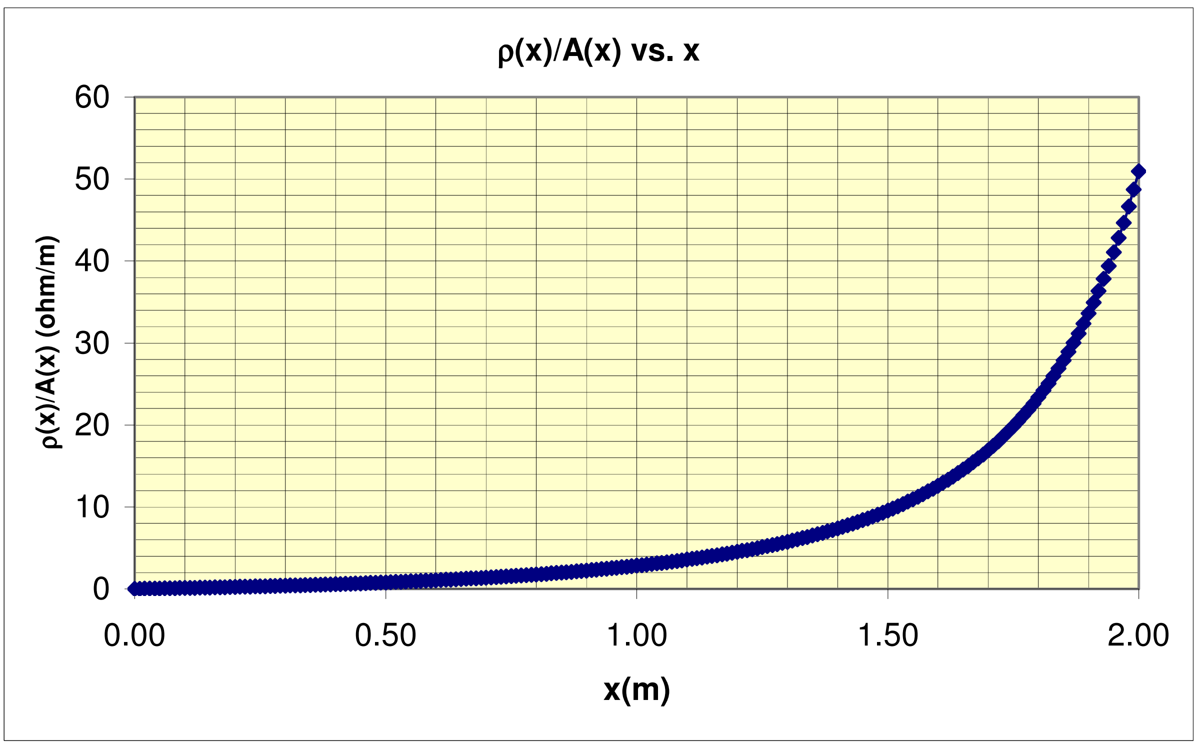
A conductor has diameter decreasing from D to d over its length L. The resistivity of this conductor along the x axis is $\rho(x)$ and its cross-sectional area is $A(x)$. The graphs of $\rho(x)$ vs. x, $A(x)$ vs. x, $\rho(x).A(x)$ vs. x, and $\rho(x)/A(x)$ vs. x are given. Find the resistance of this conductor.




ρ(x) vs. x
ρ(x) (ohm.m)
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0.00
0.50
1.00
1.50
2.00
x (m)

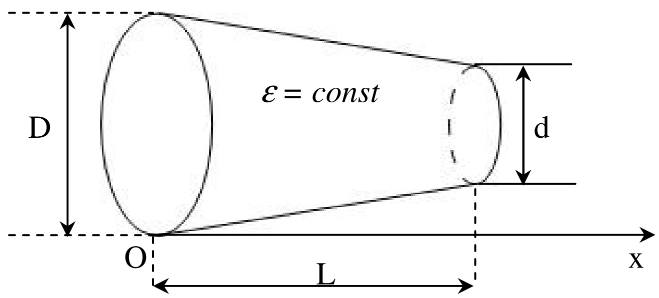

A(x) vs. x
A(x) (m^2)
0.25
0.20
0.15
0.10
0.05
0.00
0.00
0.50
1.00
1.50
2.00
x (m)


ρ(x)A(x) vs. x
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
ρ(x)A(x) (ohm.m^3)
0.00
0.50
1.00
1.50
2.00
x(m)


ρ(x)/A(x) vs. x
60
50
40
30
20
10
0
ρ(x)/A(x) (ohm/m)
0.00
0.50
1.00
1.50
2.00
x(m)

Problem 5

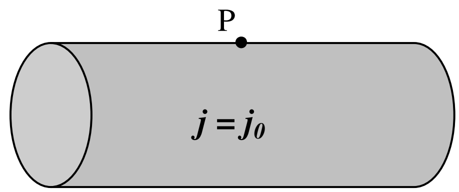
A capacitor is made of two circular conducting plates of diameter D and d. The permitivity ε of the material filled between the plates is constant. Find the capacitance of this capacitor.



PROBLEMS FOR INTERVIEW 3

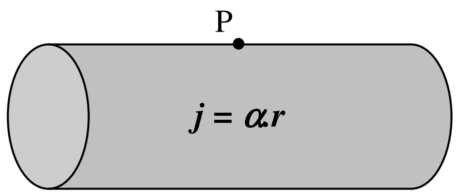
Problem 1

A cylindrical wire of radius R is carrying a current of density $j = j_0$ (j_0 is a constant). Find the magnitude of the magnetic field caused by the wire at a point P on its surface.



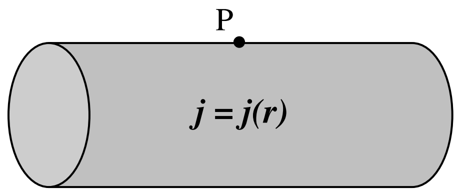
Problem 2

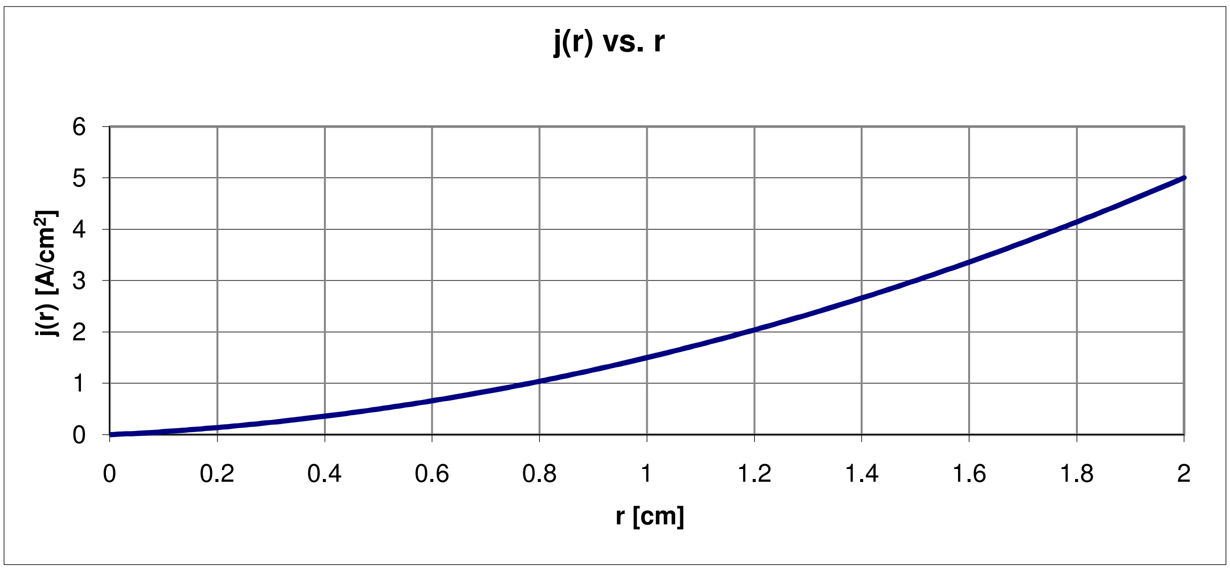
A cylindrical wire of radius R is carrying a current of density $j = \alpha . r$ (α is a constant, r is the distance from the center of the wire). Find the magnitude of the magnetic field caused by the wire at a point P on its surface.

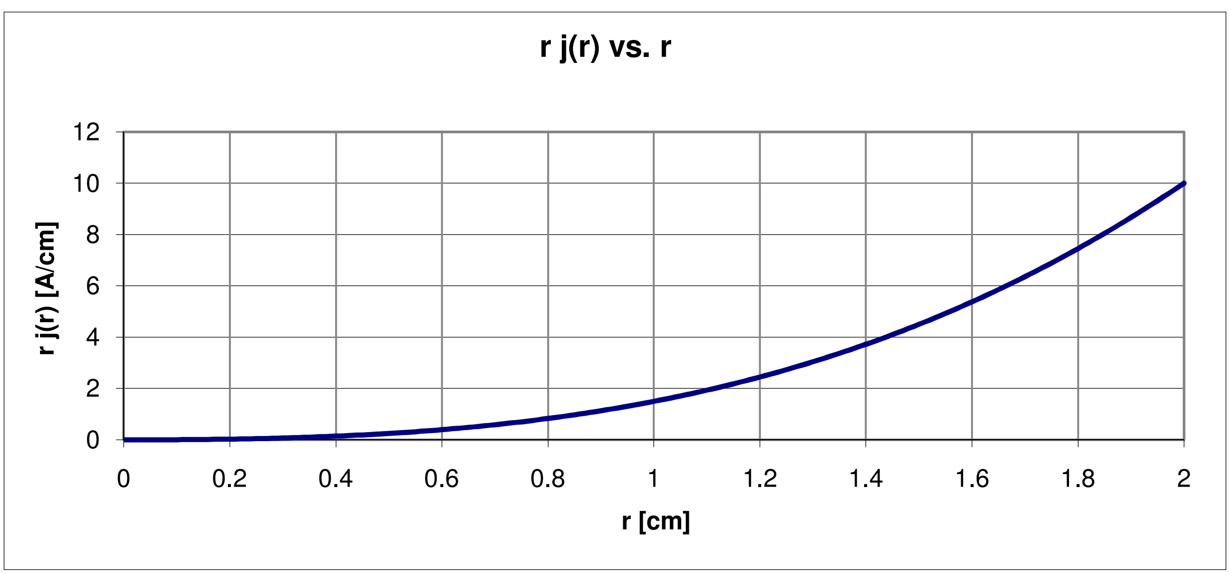


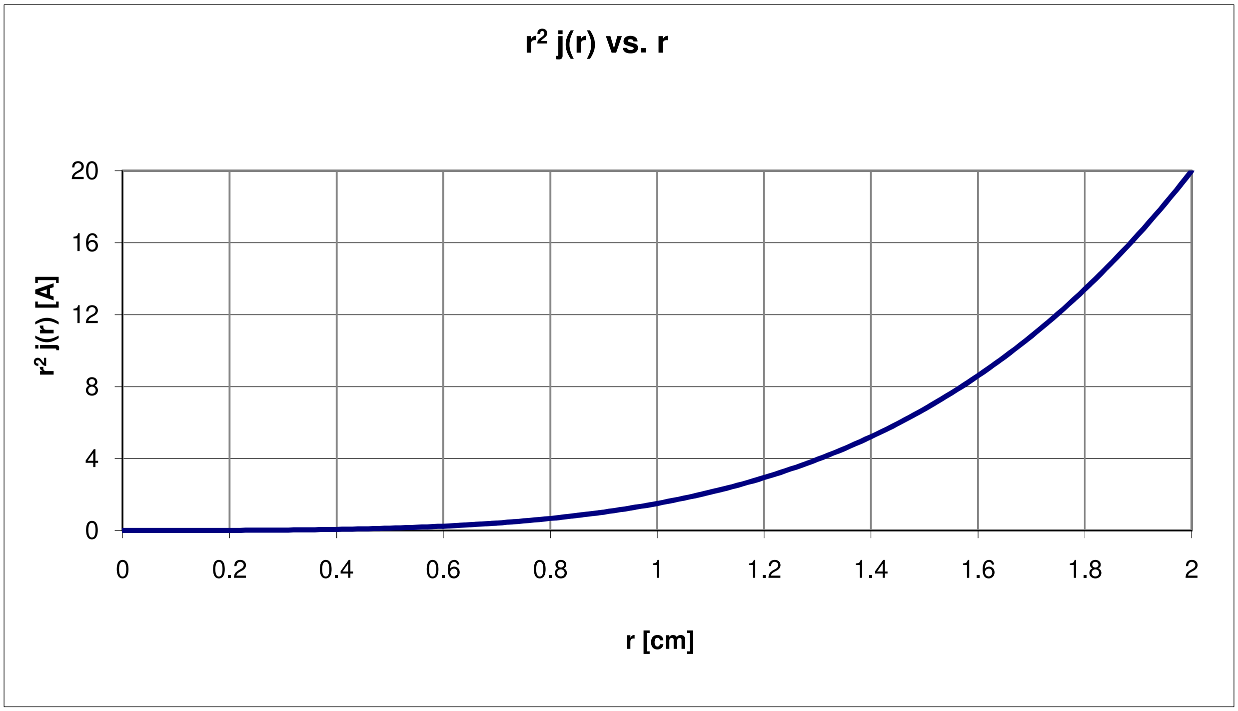
Problem 3

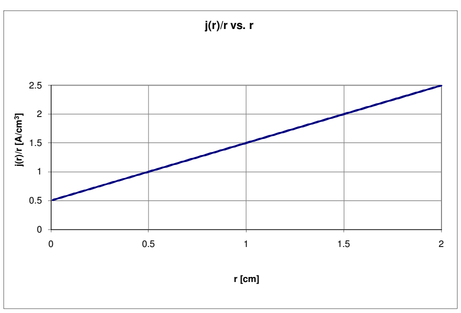
A cylindrical wire of radius R = 2 cm is carrying a current of density j which depends on the distance r from the center of the wire as per the graphs given. Find the magnitude of the magnetic field caused by the wire at a point P on its surface.





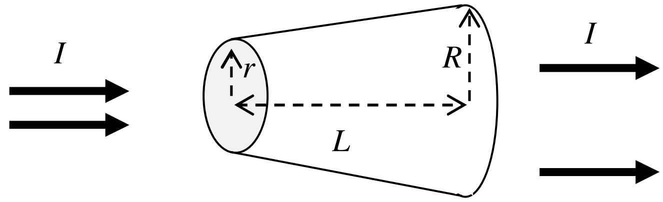



r² j(r) vs. r
r² j(r) [A]
20
16
12
8
4
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
r [cm]


j(r)/r vs. r
j(r)/r [A/cm³]
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
r [cm]

Problem 4

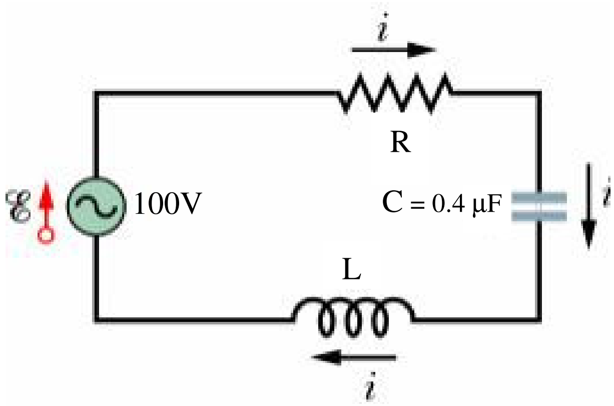
A tube carrying electric current expands uniformly over a distance L. The radius at the beginning of the tube is r, and at the end of the tube the radius is R. If the total current going through the tube is I, what is the average current density at location a quarter of the way down the tube (closer to the smaller end)?



PROBLEMS FOR INTERVIEW 4

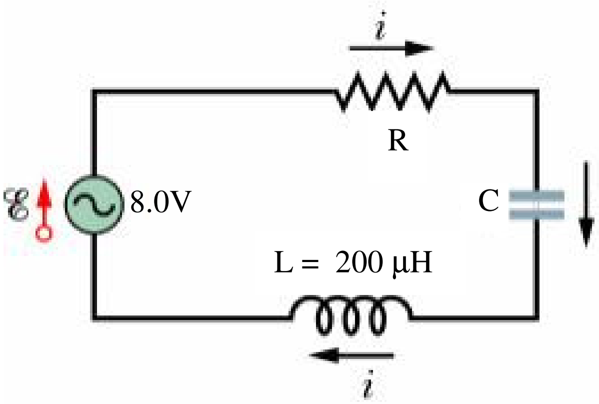
Problem 1

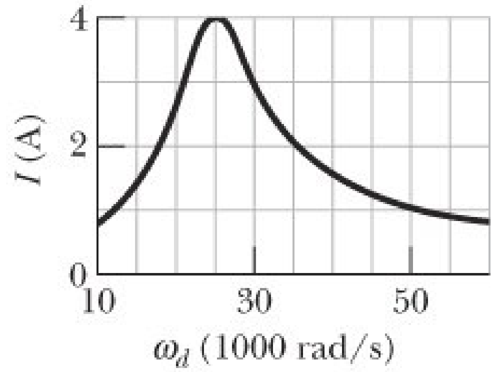
The current in a series RLC circuit reaches its maximum amplitude of $I_{max} = 2$ A when the driven angular frequency is $\omega_0 = 5\times10^4$ rad/s. The emf amplitude is 100V and the capacitance is 0.4 μF. Find R and L.



Problem 2

The current amplitude I versus driving angular frequency ω_d for a driven series RLC circuit is given in the graph below. The inductance is 200 μH and the emf amplitude is 8.0 V. Find C and R.



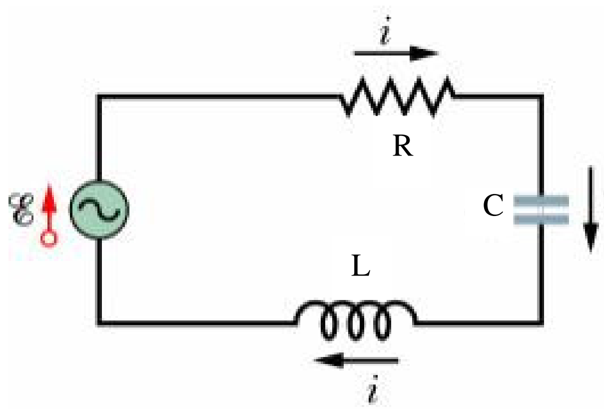


Problem 3

The current amplitude $I(\omega)$ (in Amperes) of a series RLC circuit depending on the driving angular frequency ω (in radian/second) is given as follow:

$$I(\omega)=\frac{30\,\mathrm{V}}{\sqrt{(30\,\Omega)^2+\left(\left(5\times10^{-4}\,\mathrm{H}\right)\times\omega-\dfrac{1}{\left(2\times10^{-7}\,\mathrm{F}\right)\times\omega}\right)^2}}$$

Find the resistance R, inductance L, capacitance C, resonance frequency ω_0, and maximum current amplitude I_{max}.

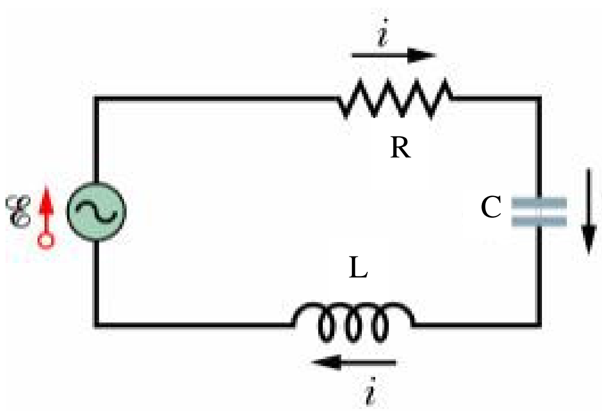


Problem 4

The current amplitude $I(\omega)$ (in Amperes) of a series RLC circuit depending on the driving angular frequency ω (in radian per second) is given as per the following function:

$$I(\omega) = 2\times10^{-6}\times\left(60000\omega - 450\omega^2 + \omega^3\right)$$

Find the resonance angular frequency ω_0 and the maximum current amplitude I_{max}.

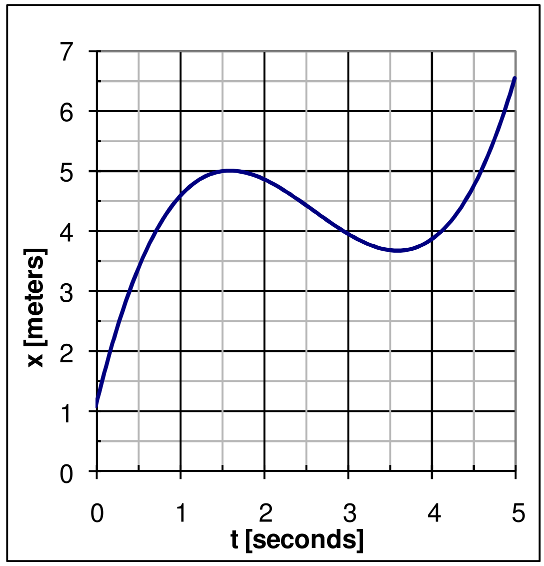


Name:____ **Date:**________________

FOCUS GROUP INTERVIEW 1

Problem 1

Consider the position of a particle as shown in the graph. Use the graph to answer the questions below:



(a) When is the particle farthest away from the origin? What is closest distance of the particle?

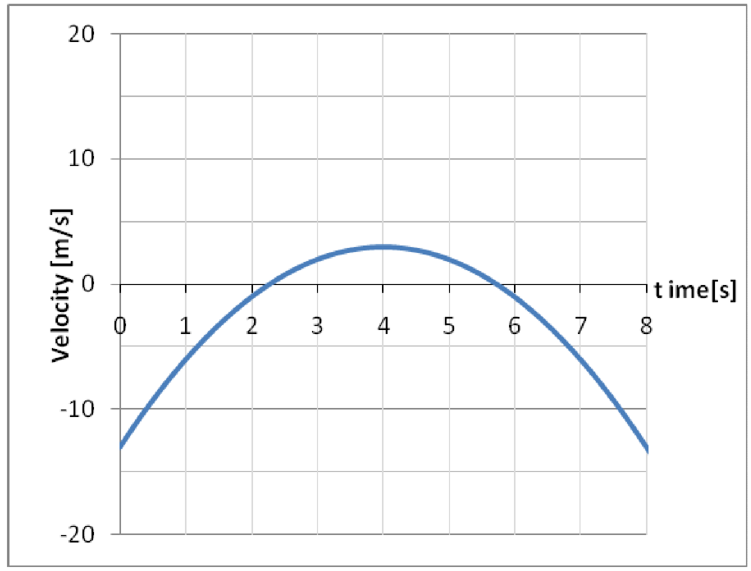
(b) What are the positions of the particle when it momentarily stops moving?

(c) What is the particle's speed at $t = 2.5$ s?

(d) What is the average speed of the particle between 0.5s and 2.5s?

Problem 2

Consider the velocity of a particle as shown in the graph. Use the graph to answer the questions below:



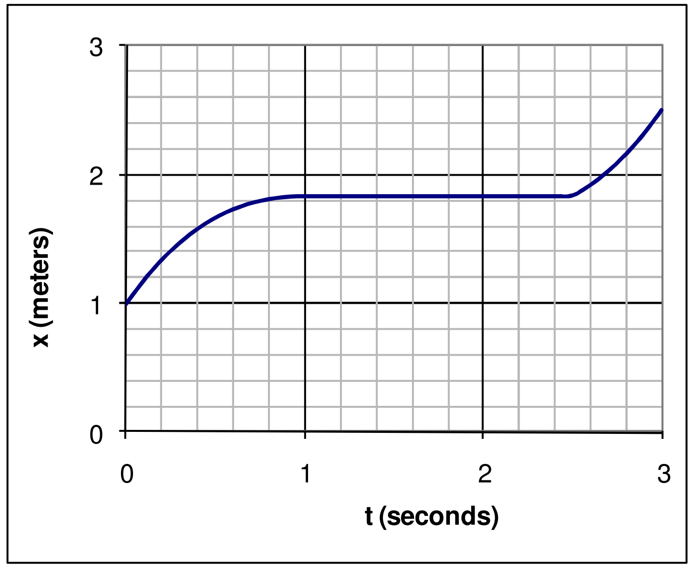
(a) When does the particle have its largest positive velocity?

(b) What is the particle's acceleration at t = 1.5 s?

(c) Describe the motion of the particle at t = 4s.

(d) What is the average acceleration of the particle between 1.5s and 2.5s?

Problem 3



The graph shows the distance of a child from her mother.

a. What is the child's distance when she's closest to her mother?

b. What is the child's distance when she's farthest from her mother?

c. During what time is the velocity of the child:

   i. positive?

   ii. negative?

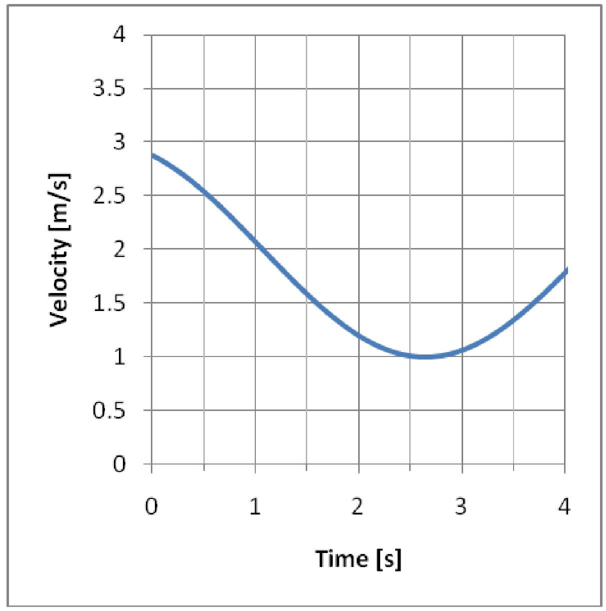
   iii. zero?

d. What is the average velocity of the child between 0s and 1.5s?

e. What is the velocity of the child at t = 0.5s?

Problem 4

Consider a particle that has a velocity describe by the graph.



(a) At what time is the particle moving slowest? What is its slowest speed?

(b) At what time is the particle moving fastest? What is its fastest speed?

(c) What is the velocity of the particle at t=1s?

(d) What is the acceleration of the particle at t=1s?

(e) What is the average acceleration of the particle between 1s and 3s?

Problem 5

The position of a particle moving along the x axis is given by $x(t)=12t^2-2t^3$ where x is in meters and t is in seconds.

(a) What is the position of the particle at t=3s?

(b) What is the velocity of the particle at t=3s?

(c) What is the most positive position reached by the particle?

(d) At what time is the most positive position reached?

Problem 6

The velocity of a particle is given by $v(t) = 5t - t^2$, where t is measured in seconds and v has units of m/s.

(a) What is the acceleration of the particle at the instant the particle is not moving (other than at t=0)?

(b) Determine the average acceleration of the particle between t=0s and t=3s.

Problem 7

Consider a particle that's position is given by $x(t) = 4 - 12t + 3t^2$.

(a) What is the position of the particle at t=3 seconds?

(b) When is the particle as its most negative position ?

(c) What is the particle's velocity as a function of time?

(d) What is the particle's average velocity between 1s and 3s?

Problem 8

Consider a particle that has a velocity $v(t) = 10+10t-t^3$ where t is measured in seconds and velocity is in m/s.

(a) What is the particle's acceleration as a function of time?

(b) When does the particle change direction?

(c) What is the average acceleration of the particle between t=0s and t=2.5s?

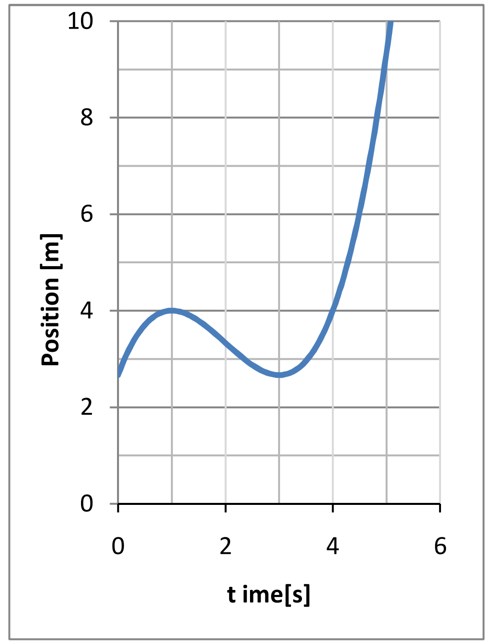
Name:____ Date:__________________________

FOCUS GROUP INTERVIEW 1

Problem 1 Partner:___________________________________

A function is shown in the graph. Use the graph to answer the following questions:

a. What is the value of **f** at x=2?



b. If **f** has units of gallons and **x** has units of years, what are the units of $\frac{df}{dx}$?

c. What is the value of $\frac{df}{dx}$ at x = 2.5?

d. What is the average value of $\frac{df}{dx}$ between 0 and 2?

e. For what value(s) of x is $\frac{df}{dx}$

i. positive?

ii. negative?

iii. zero?

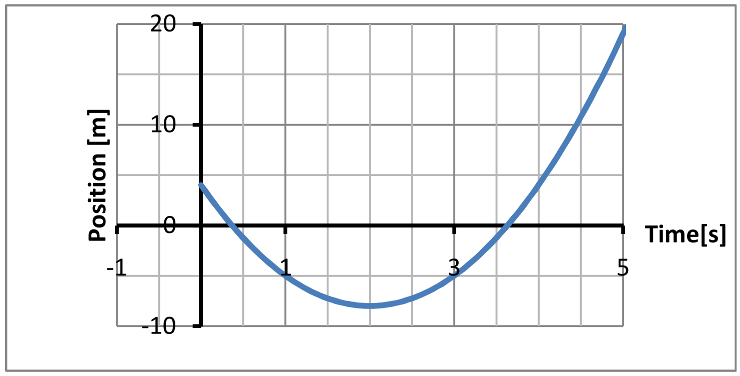
Name:____ **Date:**__________________________

Problem 2 Partner:_____________________________________

The position of a particle is given by the graph, where position is measured in meters and time is measured in seconds.

a. What is the position of the particle at t = 3s?

b. When does the particle momentarily stop moving (between 0s and 5s)?



c. Where is the particle when it momentarily stops moving?

d. What is the particle's average velocity between 1s and 3s?

e. What is the particle's velocity at t = 4s?

f. During what time is the velocity of the particle:

i. positive?

ii. negative?

iii. zero

Name:____ **Date:**_________________________

Reflection Questions Partner:___________________________________

It's important when learning physics to be able to explain concepts and procedures. Research shows that students who are reflective about their learning, think conceptually about mathematics and physics, and are able to generalize procedures to similar problems are more successful physics learners. The following two problems are aimed at helping you be reflective about your learning.

a. Briefly describe in words how you calculate the ***average velocity*** during an interval of time from a ***position vs. time equation***.

b. What similarities and differences do you see among Problems 1 & 2?

Similarities	Differences

c. What do you want to remember about these problems that might be helpful for solving problems in the future?

Central Ideas	Computational Tips/Tricks

Name:____ **Date:**___________________________

Compare Graphs & Equations Partner:________________________________

Find a partner who did the first two problems with *equations*.

a. **Explain** to your partner how you got your answers for Problems 1 & 2, including your reasoning and how you made any computations. Make sure you understand what your partner did.

b. **Compare** your answers. Are they the same? Resolve any discrepancies you find between your answers and your partner's.

c. What similarities and differences do you see for using ***graphs*** and ***equations*** to solve physics problems?

Similarities	Differences

Name:____ **Date:**____________________________

FOCUS GROUP INTERVIEW 1

Go back to and work with your original partner. Partner:______________________________

Problem 3

A function is described by the equation $f(x) = 10 + 10x - x^3$. Use the equation to answer the following questions:

a. What is f(-3)?

b. If **f** has units of centimeters and **x** has units of years, what are the units of $\frac{df}{dx}$?

c. What is the value of $\frac{df}{dx}$ at x = -3?

d. What is the average value of $\frac{df}{dx}$ between -3 and 4?

e. Where does the function cross the x-axis?

f. What is the derivative of the function?

Name:____ **Date:**___________________________

Problem 4 Partner:______________________________________

The velocity of a car is described by the equation $v(t) = -13 + 8t - t^2$. Use the equation to answer the following questions:

a. When does the car stop moving?

b. What is the average acceleration of the car between 5s and 8s?

c. What is the acceleration of the car at t = 2s?

d. What is the acceleration of the car as a function of time?

Name:____ **Date:**____________________________

Reflection Questions 2 Partner:______________________________________

(a) Briefly describe in words how you calculate the ***instantaneous acceleration*** at a point in time from a ***velocity vs. time graph.***

(b) What similarities and differences do you see among Problems 3 & 4?

Similarities	Differences

Name:____ **Date:**____________________________

(c) What do you want to remember about these problems that might be helpful for solving problems in the future?

Central Ideas	Computational Tips/Tricks

(d) Are there problems that are easier to solve with ***equations***? Which problems? Explain.

(e) Are there problems that are easier to solve with ***graphs***? Which problems? Explain.

Name:____ **Date:**____________________________

FOCUS GROUP INTERVIEW 1

Problem 1 Partner:_______________________________________

A function is described by the equation $f(x) = \frac{1}{3} x^3 - 2x^2 + 3x + 8/3$. Use the equation to answer the following questions:

a. What is f(2)?

b. If **f** has units of gallons and **x** has units of years, what are the units of $\frac{df}{dx}$?

c. What is the value of $\frac{df}{dx}$ at x = 2.5?

d. What is the average value of $\frac{df}{dx}$ between 0 and 2?

e. What is the derivative of the function?

Name:____ **Date:**___________________________

Problem 2 Partner:_____________________________________

The position of a particle is given by the equation $x(t) = 4 - 12t + 3t^2$, where x is measured in meters and t is measured in seconds.

a. What is the position of the particle at t = 3s?

b. When does the particle momentarily stop moving?

c. Where is the particle when it momentarily stops moving?

d. What is the particle's average velocity between 1s and 3s?

e. What is the particle's velocity at t = 4s?

f. What is the particle's velocity as a function of time?

Name:____ **Date:**__________________________

Reflection Questions Partner:____________________________________

It's important when learning physics to be able to explain concepts and procedures. Research shows that students who are reflective about their learning, think conceptually about mathematics and physics, and are able to generalize procedures to similar problems are more successful physics learners. The following two problems are aimed at helping you be reflective about your learning.

a. Briefly describe in words how you calculate the ***average velocity*** during an interval of time from a ***position vs. time graph***.

b. What similarities and differences do you see among Problems 1 & 2?

Similarities	Differences

c. What do you want to remember about these problems that might be helpful for solving problems in the future?

Central Ideas	Computational Tips/Tricks

Name:____ **Date:**____________________________

Compare Graphs & Equations Partner:_______________________________________

Find a partner who did the first two problems with *graphs*.

a. **Explain** to your partner how you got your answers for Problems 1 & 2, including your reasoning and how you made any computations. Make sure you understand what your partner did.

b. **Compare** your answers. Are they the same? Resolve any discrepancies you find between your answers and your partner's.

c. What similarities and differences do you see for using ***graphs*** and ***equations*** to solve physics problems?

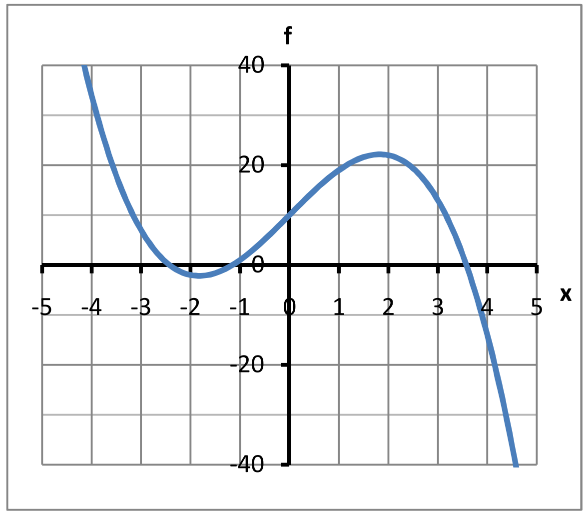
Similarities	Differences

Name:____ **Date:**____________________________

FOCUS GROUP INTERVIEW 1

Go back to and work with your original partner. Partner:__________________________________

Problem 3



A function is shown in the graph. Use the graph to answer the following questions:

a. What is the value of **f** at x=-3?

b. If **f** has units of centimeters and **x** has units of years, what are the units of $\frac{df}{dx}$?

c. What is the value of $\frac{df}{dx}$ at x = -3?

d. What is the average value of $\frac{df}{dx}$ between -3 and 4?

e. Where is the function zero?

f. For what value(s) of x is $\frac{df}{dx}$

   i. positive?

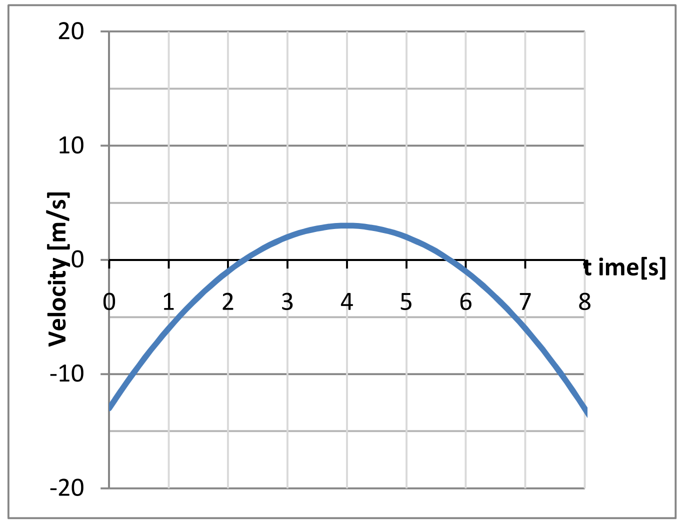
   ii. negative?

   iii. zero?

Name:____ **Date:**____________________________

Problem 4 Partner:_______________________________________

The graph shows the velocity of a car as a function of time. Use the graph to answer the questions below:



a. When does the car stop moving?

b. What is the average acceleration of the car between 5s and 8s?

c. What is the acceleration of the car at t = 2s?

d. During what time is the acceleration of the car:

i. positive?

ii. negative?

iii. zero

Name:____ **Date:___________________________**

Reflection Questions 2 Partner:_____________________________________

(a) Briefly describe in words how you calculate the ***instantaneous acceleration*** at a point in time from a ***velocity vs. time equation.***

(b) What similarities and differences do you see among Problems 3 & 4?

Similarities	Differences

Name:____ **Date:**____________________________

(c) What do you want to remember about these problems that might be helpful for solving problems in the future?

Central Ideas	Computational Tips/Tricks

(d) Are there problems that are easier to solve with ***equations***? Which problems? Explain.

(e) Are there problems that are easier to solve with ***graphs***? Which problems? Explain.

Name:____ **Date:**____________________________

FOCUS GROUP INTERVIEW 1

Problem 1 Partner:______________________________________

Imagine that you have a summer job painting fences. On your first day of work, you paint two fences. For the first fence, it takes you 30 minutes to mow and you make $20. For the second fence, you also make $20 but it takes you 90 minutes to paint.

a. What is your average wage (dollars per hour)?

b. What is the mathematical relationship between the total amount of money made and a wage?

Name:____ **Date:**__________________________

Problem 2 Partner:____________________________________

The position of a particle is given by the equation $x(t)=4-12t+3t^2$, where x is measured in meters and t is measured in seconds.

a. What is the position of the particle at t = 3s?

b. When does the particle momentarily stop moving?

c. Where is the particle when it momentarily stops moving?

d. What is the particle’s average velocity between 1s and 3s?

e. What is the particle’s velocity at t = 4s?

f. What is the particle’s velocity as a function of time?

Name:____ **Date:**___________________________

Problem 3 Partner:______________________________________

The position of a particle is given by the equation $x(t) = 4 - 12t + 3t^2$, where x is measured in meters and t is measured in seconds.

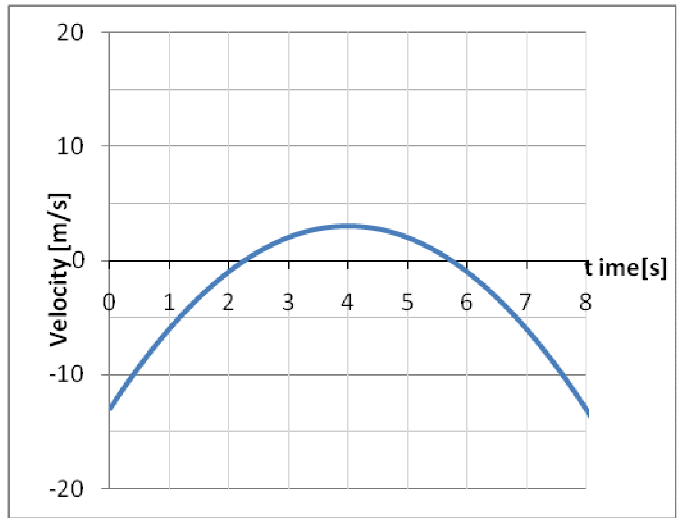
a. Using some of the information you calculated in the previous problem, sketch a graph of position as a function of time.

b. Using the graph above, sketch the graph of velocity as a function of time

Name:____ Date:_____________________________

Problem 4 Partner:____

The graph shows the velocity of a car as a function of time. Use the graph to answer the questions below:



a. When does the car stop moving?

b. What is the average acceleration of the car between 5s and 8s?

c. What is the acceleration of the car at t = 2s?

d. During what time is the acceleration of the car:

i. positive?

ii. negative?

iii. zero

Name:____ **Date:**____________________________

Problem 5 Partner:____

a. What similarities and differences do you see for between ***graph*** and ***equations*** physics problems in terms of the *information given in the problem*?

Similarities	Differences

b. What similarities and differences do you see for between ***graph*** and ***equations*** physics problems in terms of the *procedure needed to solve the problem*?

Similarities	Differences

Name:____ **Date:**___________________________

<u>Problem 6</u> Partner:______________________________________

a. Three students discuss how you *calculate the derivative of a function from a graph*.

Amy:	You take the value of the function and divide by the time.
Bob:	You draw a line between the value of the function at t=0 and t and calculate its slope.
Chloe:	You draw a line that is tangent to the function at and calculate its slope.

- Which of these students do you agree with?
    - Why?

- Explain why the other students are wrong.

Name:____ **Date:**_________________________

Problem 6 Partner:___________________________________

b. Four students discuss how you *calculate the average velocity* from a position equation.

Adam:	Take the final position, subtract the initial position, and divide by the difference in final and initial time.
Beth:	Calculate the final position and divide the total time.
Charles:	To find the average velocity, I'd take the average position and divide by the time.
Diane:	Take the derivative of the position function to get the velocity function. Plug in the starting and ending times to get the initial and final velocity. Subtract to get the change in velocity and divide by two.

- Which of these students do you agree with?
    - Why?

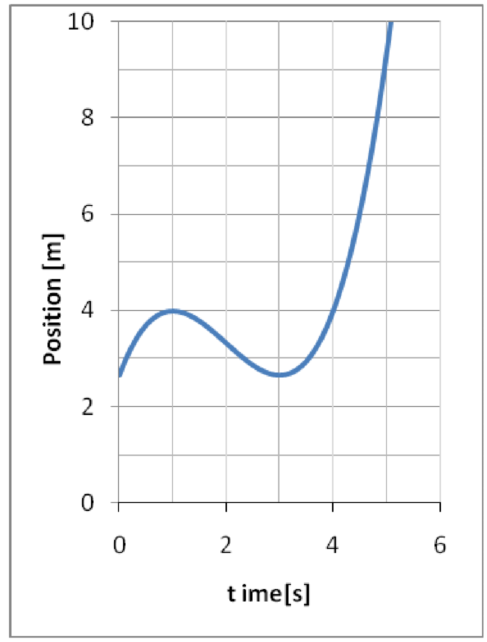
- Explain why the other students are wrong.

Name:____ **Date:**____________________________

Problem 6 Partner:_______________________________________

c. Five students consider the graph shown and discuss how to find the acceleration at t=1s.

Alyssa:	The acceleration is zero because the slope is zero.
Brian:	The acceleration is zero because the velocity is zero.
Charlotte:	The acceleration is positive because the position is moving in the positive direction.
Dave:	The acceleration is negative because the graph's shape is concave down.
Ellen:	The acceleration is negative because the object is turning around.



- Which of these students do you agree with?
    - Why?

- Explain why the other students are wrong.

Name:____ **Date:**____________________________

Problem 7 Partner:_______________________________________

The problem below is *incomplete*. Only starting statement of a problem is given below. The question is missing.

A particle follows a one-dimensional path, such that its displacement from the origin as a function of time is given by $x(t) = 7t^2 - 5t + 9$. _______________________________________?

A student works out the solution to the problem, but you only find the last part of the solution worked out by the student, which is shown below. Unfortunately, the answer is also missing the units, so you have to look at what the student worked out in this last step and work backward to figure out more about the problem.

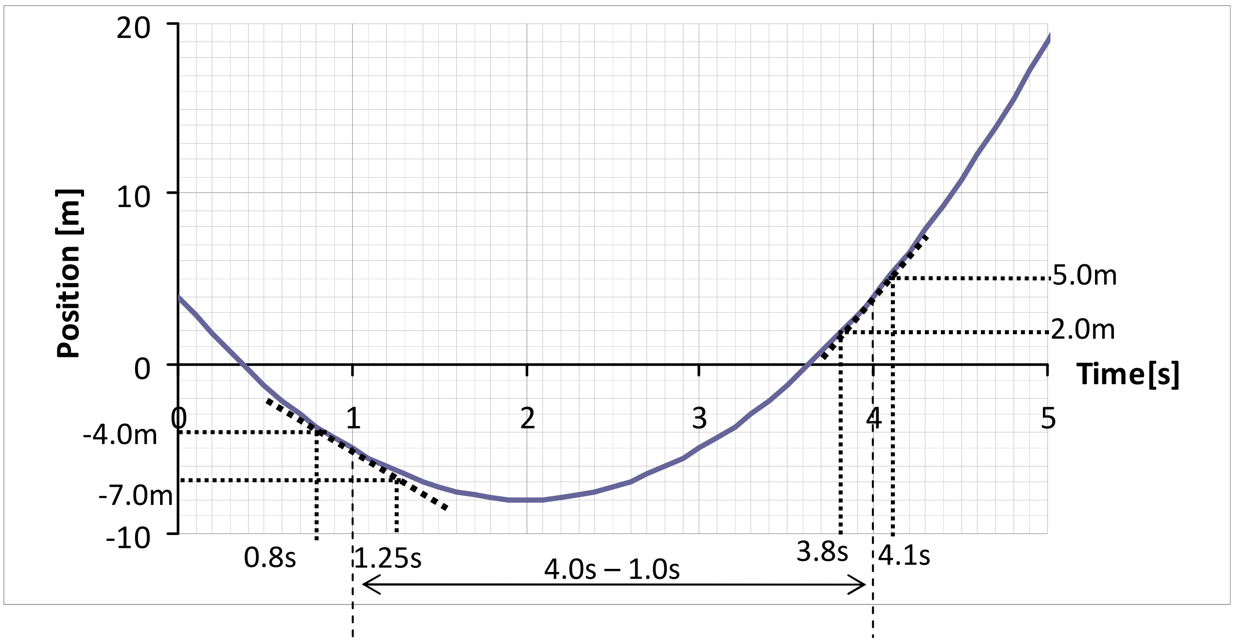
So, the answer is: $\frac{[2(7)(8) - 5] - [2(7)(3) - 5]}{(8-3)} = 14$

- Using the fragment of the solution provided in the second box above, find out what the problem was asking the student to determine?

Name:____ **Date:**____________________________

Problem 8 Partner:_______________________________________

A student is given a given a problem that asks her to use the graph shown below.



The student makes the following annotations on the graph and does the calculations shown below. Again, she forgets to write the units of the quantity that she calculated.

So, the answer is: $\dfrac{\dfrac{(5.0-2.0)}{(4.1-3.8)}-\dfrac{(-7.0)-(-4.0)}{(1.25-0.8)}}{(4.0-1.0)}=5.67$

- Using the fragment of the solution provided in the second box above, find out what the problem was asking the student to determine from the graph?

Name:____ **Date:**______________________________

FOCUS GROUP INTERVIEW 1
Please work on these problems individually

Problem 9

The position of an object moving along an x axis is given by $x = 3t^3 - 2t + 4$, where x is in meters and t in seconds.

a) Find at least one time when the velocity is zero.

b) What is the average acceleration between 0 and 3 seconds?

c) What is the acceleration at $t = 3$ seconds?

SOLUTION

a) Velocity is derivative of position : $v(t) = dx/dt = 9t^2 - 2$.

Velocity is zero when: $9t^2 - 2 = 0$, or $t = \sqrt{2/9}$ s.

b) $v(t) = dx/dt = 9t^2 - 2$

- At $t = 0$: $v(t) = -2$ m/s;

- At $t = 3$ s: $v(t) = 79$ m/s.

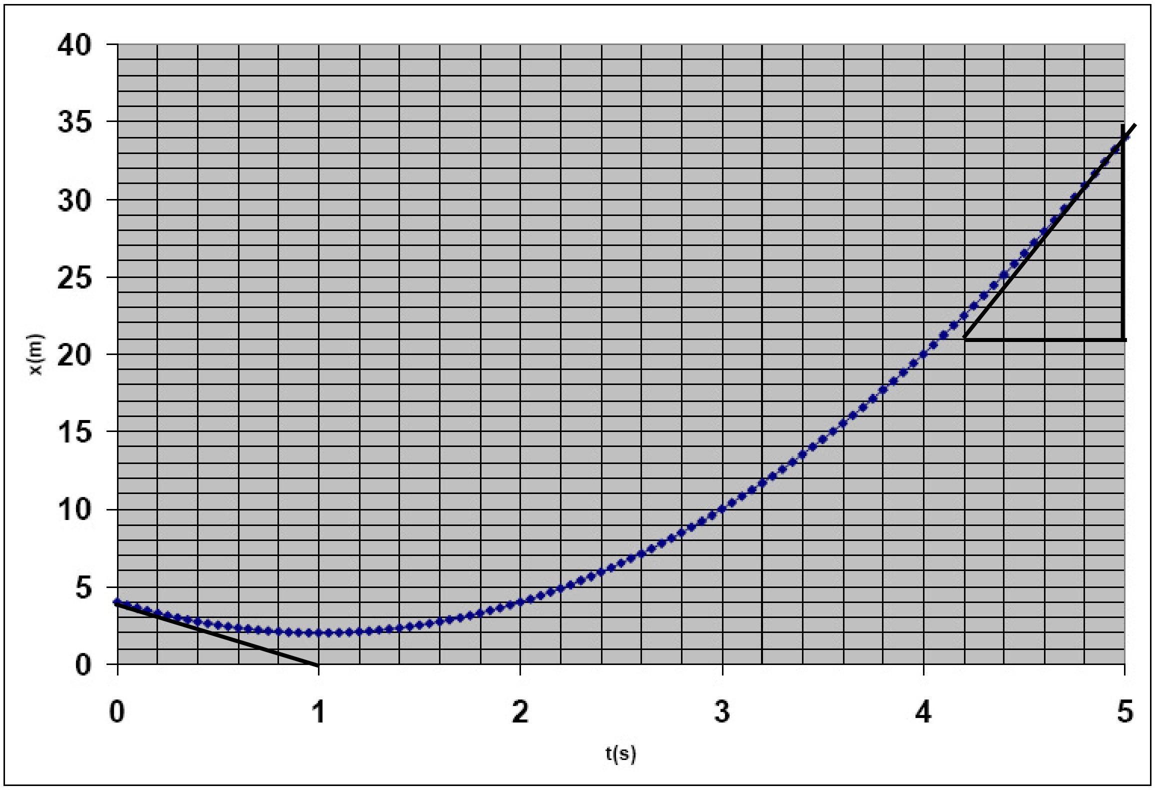
$a_{avg} = (v - v0)/\ \Delta t = (79 - (-2))/\ 3 = 27$ m/s^2

c) Acceleration is derivative of velocity : $a(t) = dv/dt = 18t$

At $t = 3$: $a(t) = 54$ m/s^2.

Problem 10

The position of an object moving along an x axis versus time is given by the graph below, where x is in meters and t in seconds.



a) Find at least one time when the velocity is zero.

b) What is the average acceleration between 0 and 5 seconds?

c) What is the acceleration at t = 3 seconds?

SOLUTION

a) Velocity is the slope of the x-t graph. On the graph, the slope is zero at t = 1.0 second, and that's when velocity is zero too.

b) At t = 0 s: v = slope = (0 m – 4 m)/(1 s – 0 s) = - 4 m/s

   At t = 5 s: v = slope = (34 m – 21 m)/(5.0 s – 4.2 s) = 16.25 m/s

   $a_{avg} = \Delta v/\Delta t$ = (16.25 m/s – (-4 m/s))/(5 s – 0 s) = 4.05 m/s^2.

c) The graph is parabolic so the position function is quadratic, so the acceleration function is constant. So the acceleration at 3 seconds also has the value of average acceleration: a (t = 3s) = 4.05 m/s^2.

Name:____ Date:____________________________

FOCUS GROUP INTERVIEW 2

Please discuss these problems with your partner as you work through them.

Please alert the instructor if you are unable to proceed. You will be provided with a solution.

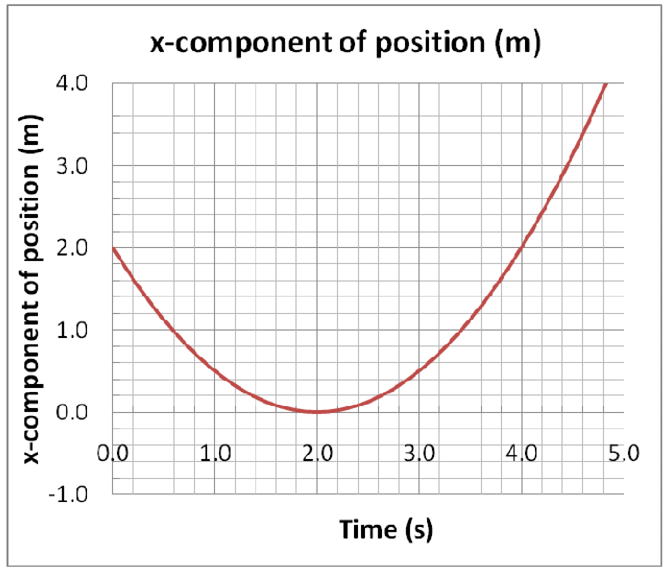
Problem 1

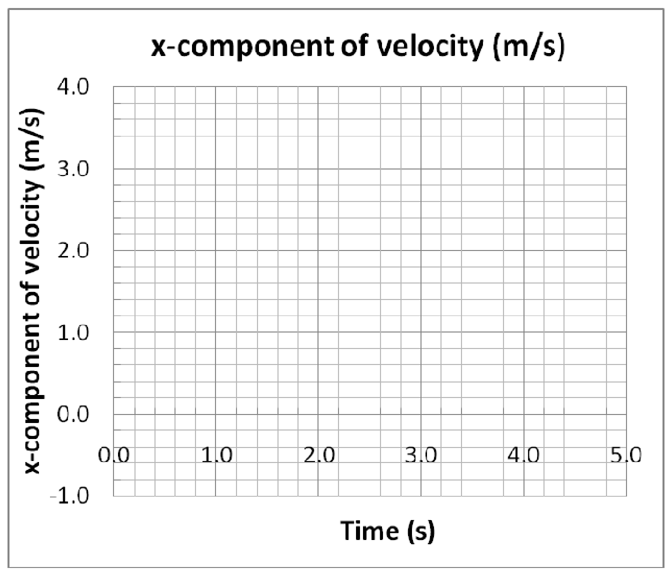
Partner:____

A graph of position versus time of an object under the influence of a *constant* force is shown.

In the empty graph provided below, sketch the graph of the velocity vs. time. Be as accurate as you can with the scales and numbers on your graph.

Show all of your work and computations in the space below. Explain any assumptions and approximations you make.





STOP

Please ask the instructor for the solution to this problem and check *your* solution with it.

Problem 2
The velocity as a function of an object under the influence of a *constant* force is given by

$$v(t) = -7t + 9$$

What is the acceleration of the object?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Problem 3

A 5 kg block sliding across a horizontal frictionless floor moves such that its acceleration in the x- and y-directions respectively is given by

$$a_x(t) = 3m/s^2 \qquad a_y(t) = -2m/s^2$$

What is the magnitude of the Force acting on the block and its direction relative to the positive x axis?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Problem 4

A 7 kg block sliding across a horizontal floor under the influence of two forces

$$F_x = 49N \qquad F_y = -63N$$

If the coefficient of friction between the box and floor is 0.2, what is magnitude and direction of acceleration?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name:____ Date:___________________________

FOCUS GROUP INTERVIEW 2

Please discuss these problems with your partner as you work through them.

Please alert the instructor if you are unable to proceed.

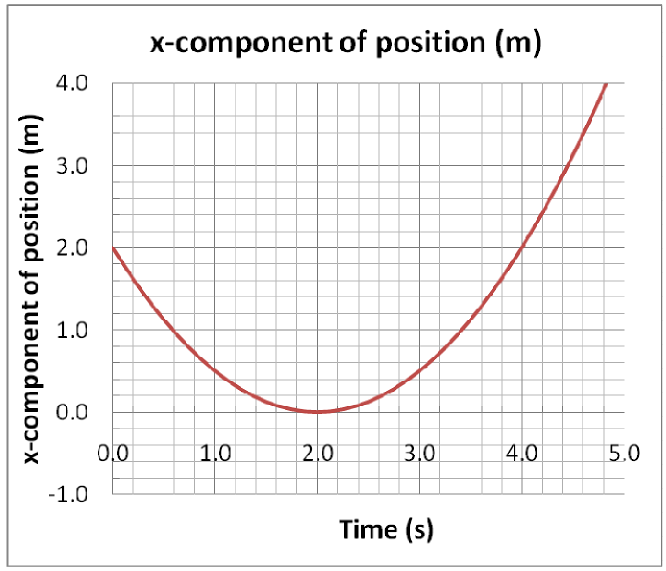
Problem 1

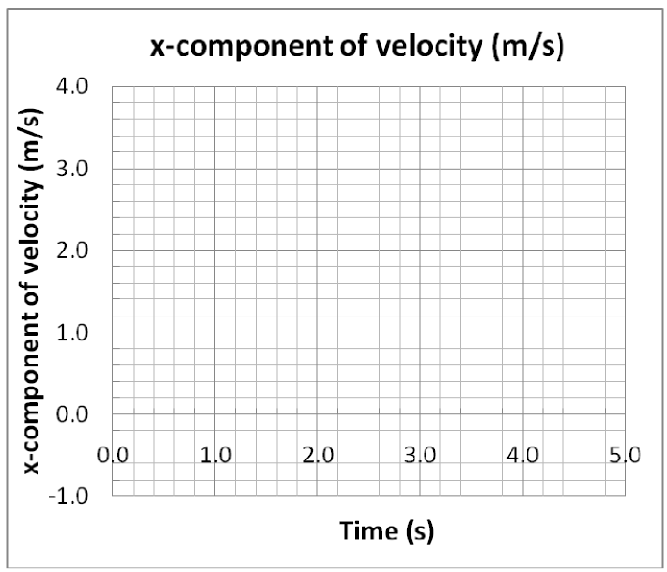
Partner:____

A graph of position versus time of an object under the influence of a *constant* force is shown.

(a) In the empty graph provided below, sketch the graph of the velocity vs. time. Be as accurate as you can with the scales and numbers on your graph.

(b) Show all of your work and computations in the space below. Explain any assumptions and approximations you make.





(c) Now imagine that you are explaining the solution of this problem to a classmate. Write down step-by-step instructions that you would give to your classmates to show them how you solve the problem.

Please alert the instructor that you have completed this problem before proceeding to the next problem.

Problem 2

The velocity of an object moving under the influence of a *constant* force is given as the following function of time:

$$v(t) = -7t + 9$$

(a) What is the acceleration of the object as a function of time?

(b) Now imagine that you are explaining the solution of this problem to a classmate. Write down step-by-step instructions that you would give to your classmates to show them how you solve the problem.

Please alert the instructor that you have completed this problem before proceeding to the next problem.

Problem 3

Three students are discussing their strategies for solving the problem below.
Determine whether each strategy is correct or incorrect and write your comments on each solution in the space provided. (Note that it is possible that more than one student has the correct approach.)

A 5 kg block starts from rest and moves under the influence of an applied force F_A. It slides across a horizontal floor with an x-y coordinate system drawn on the floor. The acceleration of the block in the x- and y-directions is given as:

$$a_x(t) = 3m/s^2 \qquad\qquad a_y(t) = -2m/s^2$$

If the coefficient of kinetic friction is 0.2, what is the magnitude of the applied force F_A and what is its direction relative to the positive x axis?

Student	Solutions	Comments
Alice	First, we must find the net acceleration using the Pythagorean formula using the values of ax and ay. Then we multiply this acceleration with the mass. To that we add the frictional force μmg, to find the applied force.	
Bob	No, you have to first find the components of the net force from the acceleration components. Then find the components of the friction force and add them to the net force to find applied force.	
Cary	No, that will not work either. We find the applied force by taking the acceleration in the x-direction, multiplied by the mass and then add the friction force. We don't have to worry about the y-component because it is vertical.	

Please alert the instructor that you have completed this problem before proceeding to the next problem.

Problem 4

(a) Create a *solvable* problem of your own which can be created by putting together the elements of the previous three problems.

(b) Now imagine that you are explaining the solution of this problem to a classmate. Write down step-by-step instructions that you would give to your classmates to show them how you solve the problem.

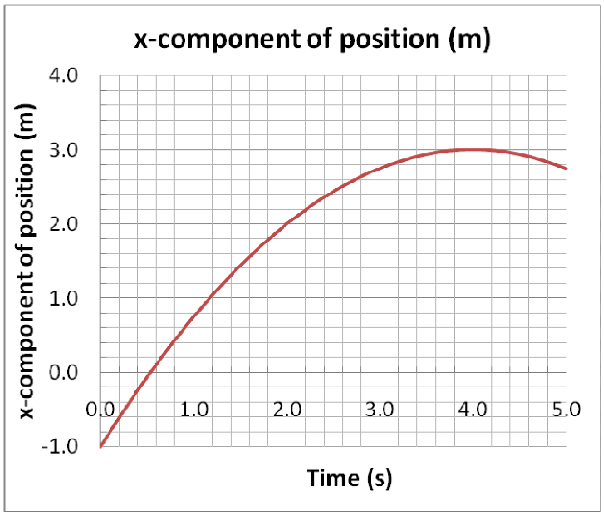
Please alert the instructor that you have completed this problem.

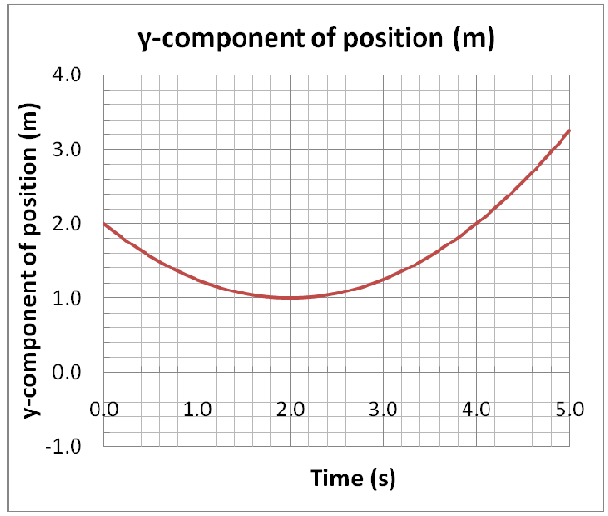
Name:____ Date:_____________________________

FOCUS GROUP INTERVIEW 2
Please work on these problems individually

Problem C

A *constant* force $\mathbf{F_C}$ is applied on a 3.00 kg block moving across a horizontal floor on which an x-y, coordinate system is drawn. The coefficient of kinetic friction between the block and floor is 0.20. The graphs below give the block's x and y components of the position vs. time t.





Find the magnitude of force $\mathbf{F_C}$ and the angle it makes relative to the positive x-axis.

Problem D

A *constant* force $\mathbf{F_D}$ is applied on a 2.00 kg block moving across a horizontal floor on which an x-y, coordinate system is drawn. The coefficient of kinetic friction between the block and floor is 0.1. The block's x and y positions (in meters) as a function of time t (in seconds) is given by

$$x = -0.6t^2 + 7t + 5 \qquad\qquad y = 0.8t^2 + 4t - 1$$

Find the magnitude of force $\mathbf{F_D}$ and the angle it makes relative to the positive x-axis.

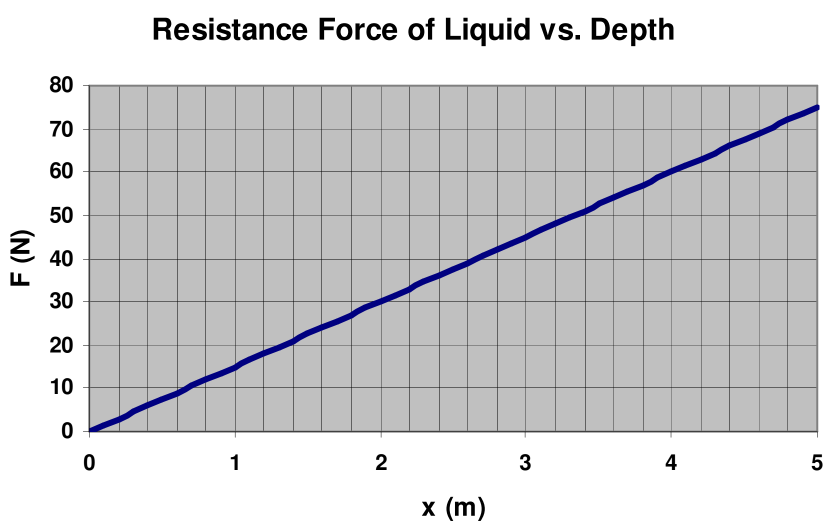
Name: ____________________________ Date: ________________________

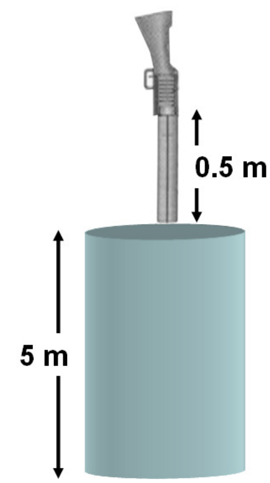
FOCUS GROUP INTERVIEW 3
Please work on these problems individually

<u>Problem A</u>

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.



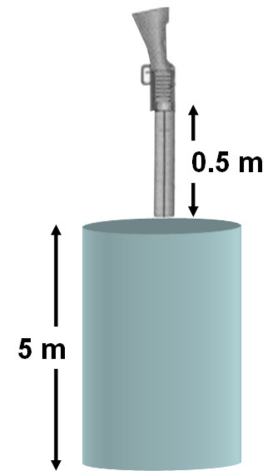


What is the spring compression *x*?

Name: ___________________________ Date: ______________________

Problem B

A 0.05 kg bullet is loaded into a gun compressing a spring which has spring constant k = 5000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.



The barrel of the gun is frictionless. The magnitude of the resistance force F (in Newtons) provided by the liquid changes with depth x (in meters) as per the following function:

$$F(x) = 8x + 0.5x^2$$

The bullet comes to rest at the bottom of the drum

What is the spring compression x?

Name: ______________________________ Date: __________________

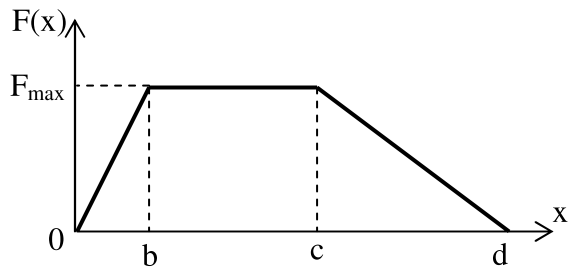
FOCUS GROUP INTERVIEW 3

Please discuss these problems with your partner as you work through them.
Please alert the instructor if you are unable to proceed. You will be provided with a solution.

Partner: ______________________________

Problem 1

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newtons and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.

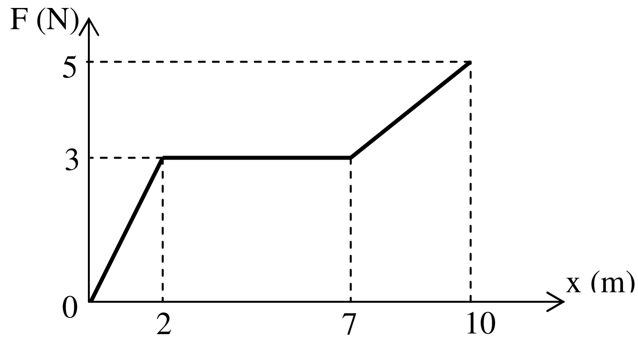


Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 2

The graph below shows the magnitude of a force F (in Newtons) acting on an object with respect to the displacement x (in meters) of the object. Find the work done by force F on the object over the displacement from 0 m to 10 m.



Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 3

A block is pulled on a horizontal frictionless floor by a force whose magnitude F (in Newtons) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (a, b, c are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 4

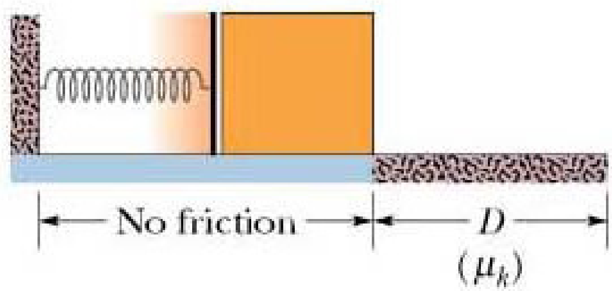
A block is pulled on a horizontal frictionless floor by a force F whose magnitude depends on the displacement of the block as per the function: $F(x) = 2x^3 - 3x + 2$ (x is in meters, F is in Newtons). Find the work done by force F when the block has been moved from 0 m to 2 m.

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 5

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression *x*?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ____________________________ Date: _____________________

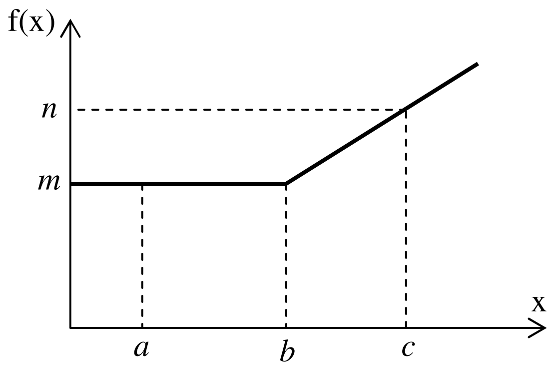
FOCUS GROUP INTERVIEW 3
Please discuss these problems with your partner as you work through them.
Please alert the instructor if you are unable to proceed.

Partner: ______________________________

Problem 1

The graph of a function f(x) is given below.

Find the value of the integral $\int_a^c f(x)\,dx$ in terms of the constants a, b, c, m, n.

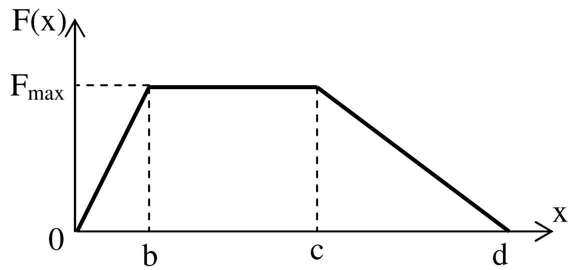


Please notify the instructor before proceeding to the next problem.

Name: ____________________________ Date: _____________________

Problem 2

The graph below shows the magnitude of a force F(x) acting on an object with respect to the displacement x of the object (F is in Newtons and x is in meters). Find the work done by force F on the object over the distance d that the force is acting.



Please notify the instructor before proceeding to the next problem.

Name: ____________________________ Date: _____________________

Problem 3

Find the area of the region surrounded by the graphs of the following functions:
$f(x) = x^3 + 2x + 1,\ f(x) = 0,\ x = x_1,\ x = x_2.$

Please notify the instructor before proceeding to the next problem.

Name: ___________________________ Date: ____________________

Problem 4

A block is pulled on a horizontal frictionless floor by a force F whose magnitude (in Newtons) depends on the displacement x of the block (in meters) as per the function: $F(x) = ax^2 + bx + c$ (*a*, *b*, *c* are constants). Find the work done by force F when the block has been moved from x_1 to x_2.

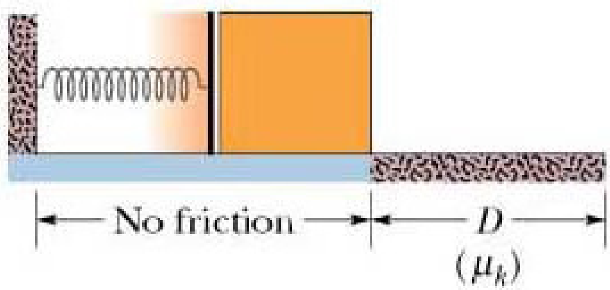
Please notify the instructor before proceeding to the next problem.

Name: ___________________________ Date: ____________________

Problem 5

Five students are discussing their strategies to solve the following problems.

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance D = 7.8 m.



What was the spring compression *x*?

Below are parts of the students' strategies. Comment on each student's ideas. Explain who you agree with most and why. For the students who make statements you disagree with, try to identify what went wrong in the student's reasoning.

Student	Strategy	Comments
David	Energy is conserved so all the changes in energy add to zero. The block starts from rest and then comes to a stop, so there is no change in kinetic energy. The only energy that changes is the spring's potential energy and that's good because that involves the compression of the spring. You can calculate the change in potential energy and solve for the compression.	
Mary	Friction is involved so you need to use $\Delta K + \Delta U = W$, where $W = -\mu_k mgD$ is the work done by friction. ΔK is zero because initial and final speeds are zero. The initial U is that of the spring and final U is zero. Then put everything into the equation and solve for x.	
Eric	Isn't the work $+\mu_k mgD$, because W in that equation is the amount of work done and therefore it must be positive?	

Name: ____________________________ Date: _____________________

Susan	But the spring does work on the block too and you have to take that into account. Work is force times distance, and since the force of the spring is -kx and the spring pushes the block a distance x, the work done by the spring is $-kx^2$. That's the formula you should use to find the compression.	
Mike	All you have to do to calculate the work done by the spring is to plug in the total distance the spring pushes the block into the force -kx. So, if the initial compression is L, the work done by the spring is -kL.	

Please notify the instructor before proceeding to the next problem.

Name: ___________________________ Date: ____________________

Problem 6

a. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 2 to create a new solvable problem of your own. Write your instructions to solve that new problem.

Name: ___________________________ Date: ____________________

b. Start with the physics problem in problem 5, modify it by including in it the physics ideas in problem 4 to create a new solvable problem of your own. Write your instructions to solve that new problem.

Please notify the instructor before proceeding to the next problem.

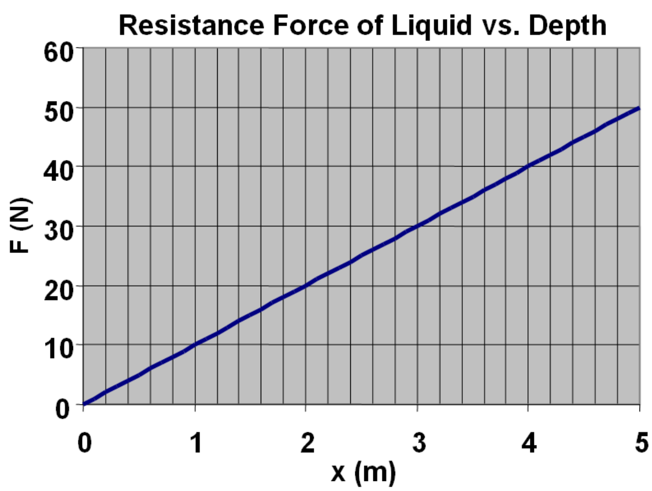
Name: ______________________ Date: ____________________

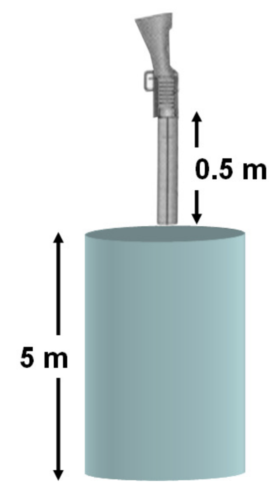
FOCUS GROUP INTERVIEW 3
Please work on these problems individually

Problem C

A 0.1 kg bullet is loaded into a gun compressing a spring which has spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

The barrel of the gun is frictionless. The magnitude of the resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.



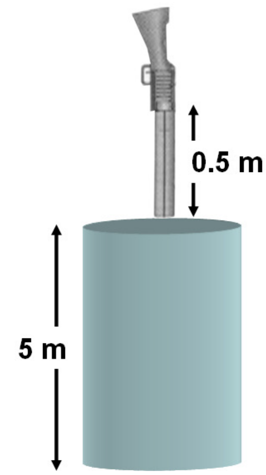


What is the spring compression *x*?

Name: ___________________________ Date: _______________________

Problem D

A 0.1 kg bullet is loaded into a gun compressing a spring which has spring constant k = 6000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.



The barrel of the gun is frictionless. The magnitude of the resistance force F (in Newtons) provided by the liquid changes with depth x (in meters) as per the following function:

$$F = 10x + 0.6x^2$$

The bullet comes to rest at the bottom of the drum

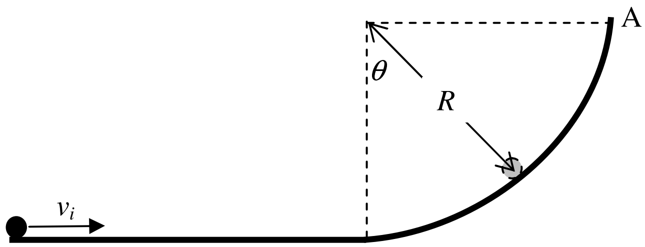
What is the spring compression x?

Name: ___________________________ Date: ______________________

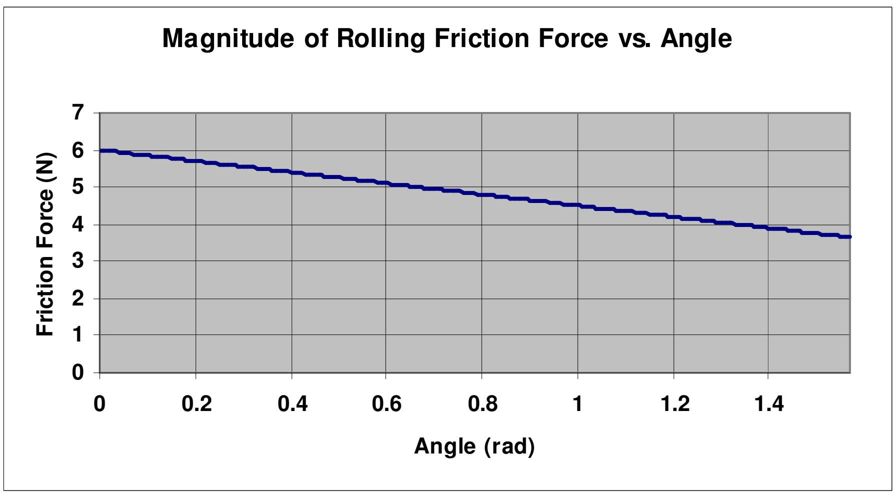
FOCUS GROUP INTERVIEW 4
Please work on these problems individually

Problem A

A sphere radius r = 2 cm and mass m = 1.0 kg is rolling at an initial speed v_i = 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown below

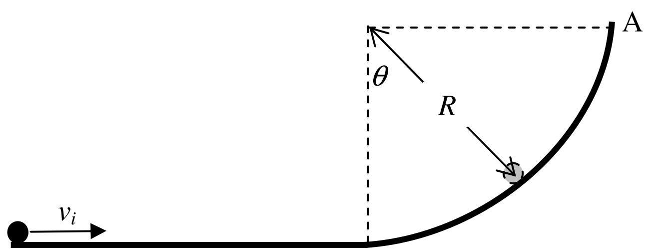


What is the launch speed of the sphere as it leaves the curve at point A?

Name: ________________________ Date: ____________________

Problem B

A sphere radius r = 2 cm and mass m = 1.0 kg is rolling at an initial speed v_i = 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force F (in Newtons) acting on the sphere varies with angle θ (radians) as per the following function

$$F(\theta) = 5.0 - 1.5\theta$$

What is the launch speed of the sphere as it leaves the curve at point A?

Name: ______________________________ Date: __________________

FOCUS GROUP INTERVIEW 4

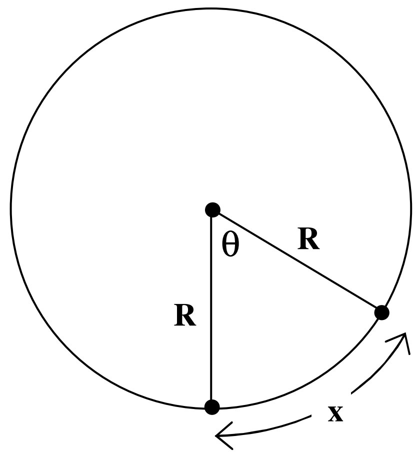
Please discuss these problems with your partner as you work through them.

Please alert the instructor if you are unable to proceed. You will be provided with a solution.

Partner: ______________________________

Problem 1

A bug sits on the edge of the turn table of radius R = 2.0 m which is rotating around its center. What is the distance 'x' that the bug has traveled after the turn table has rotated by an angle $\theta = \pi/4$?

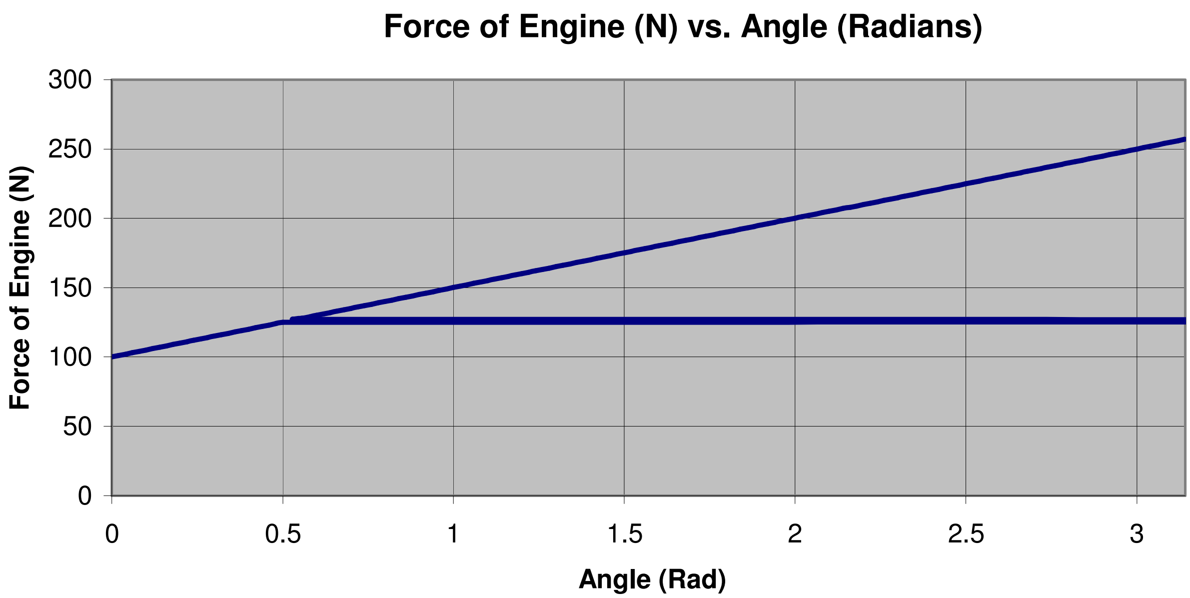


Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 2

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R = 3.0 m. The graph below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians).
Find the work done by the engine when the plane travels from its initial point to the final point.



Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 3

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R = 3.0 m. The equation below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians)

$$F(\theta) = 50 + 2\theta$$

where, F is in Newtons and θ is in radians.

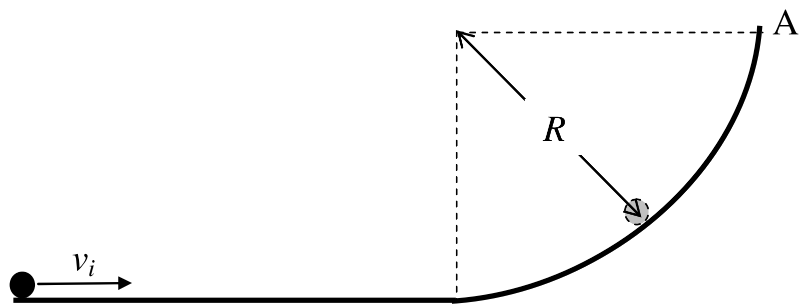
Find the work done by the engine when the plane travels from its initial point to the final point.

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: ___________________

Problem 4

A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.



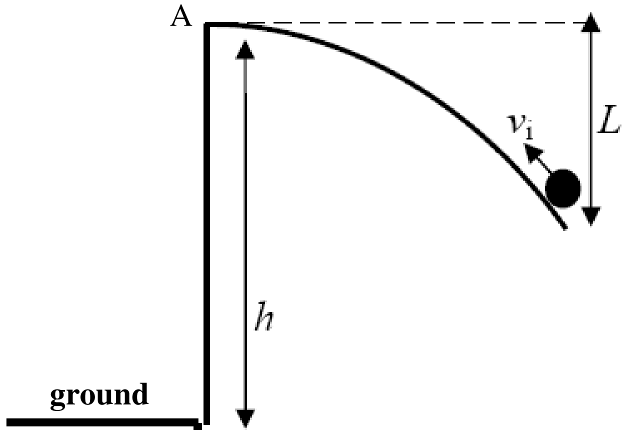
What is the launch speed of the hoop as it leaves the slope at point A?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ______________________________ Date: __________________

Problem 5

A hoop of mass 0.5 kg starts with an speed v_i= 12 m/s and rolls without slipping up a slope of height L = 6.0 m and is launched horizontally at point A The point of launch is at a height h = 12 m above the ground.



What is the launch speed of the hoop as it leaves the slope at point A?

Please ask the instructor for the solution to this problem and check *your* solution with it.

Name: ____________________________ Date: _____________________

FOCUS GROUP INTERVIEW 4

Please discuss these problems with your partner as you work through them.

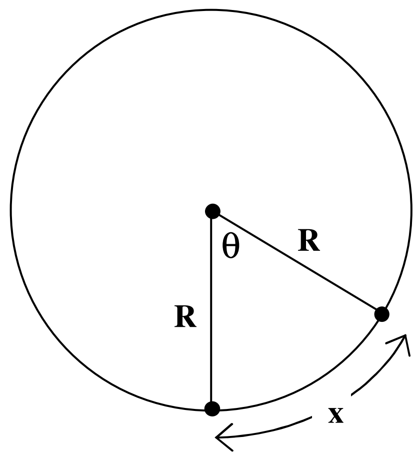
Please alert the instructor if you are unable to proceed.

Partner: ______________________________

Problem 1

a. What is the length of the arc 'x' along a circle in terms of radius R and angle θ (in radians)?

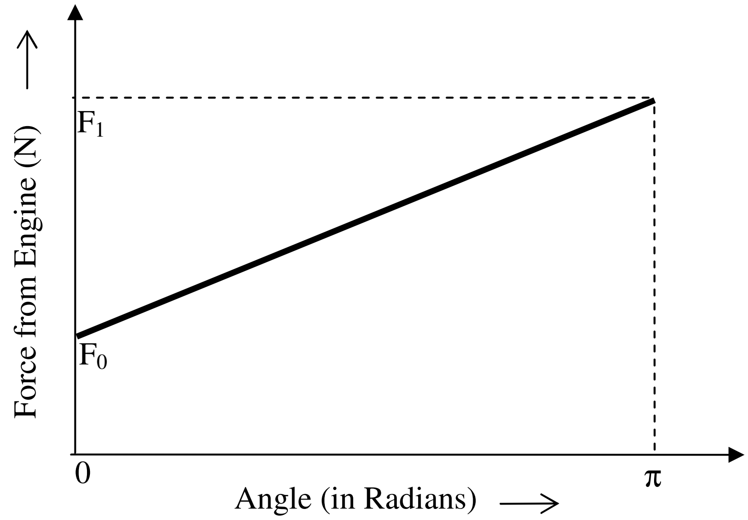
b. A bug sits on the edge of the turn table of radius R = 2.0 m which is rotating around its center. What is the distance 'x' that the bug has traveled after the turn table has rotated by an angle $\theta = \pi/4$?



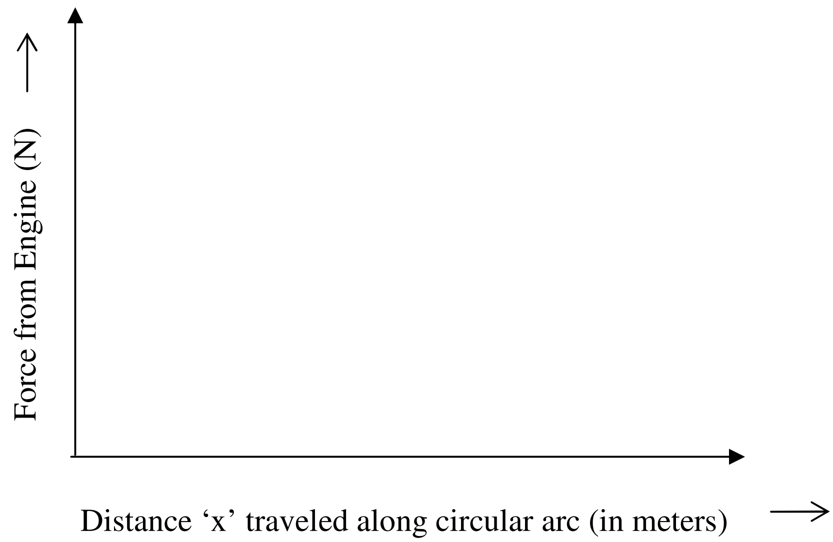
Please notify the instructor before proceeding to the next problem.

Name: ____________________________ Date: ____________________

Problem 2
A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R (in meters). The graph below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians).



a. Plot the graph of force of the engine (in Newtons) with respect to the distance ‘x’ (in meters) that the plane travels along the circular arc from its initial to its final point.



b. Find the work done by engine when the plane travels from its initial point to the final point.

Please notify the instructor before proceeding to the next problem.

Name: ____________________________ Date: ____________________

Problem 3

A toy plane is attached to a pole by a string and flies around it in a circular arc of radius R = 3.0 m. The equation below shows the force exerted by the engine of the plane as it starts from rest from its initial position ($\theta = 0$ radian) to the final position ($\theta = \pi$ radians)

$$F(\theta) = \mathrm{a}\theta + \mathrm{b}$$

where, a, b are constants; F is in Newtons, and θ is in radians.

a. Write down the equation of force of the engine as a function of the distance ‘x’ the plane travels along the circular arc from its initial to its final point.

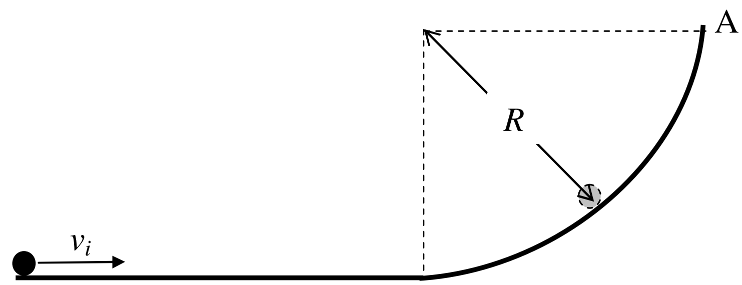
b. Find the work done by the engine when the plane travels from its initial point to the final point in terms of a and b.

Please notify the instructor before proceeding to the next problem.

Name: ____________________________ Date: _____________________

Problem 4
Five students are discussing their strategies to solve the following problem.

A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.



What is the launch speed of the hoop as it leaves the slope at point A?

Below are parts of the students' strategies. They may not be the complete solutions.
Comment on each student's ideas. Explain who you agree with most and why. For the students who make statements you disagree with, explain what you think is wrong in the student's reasoning.

	Strategy	Comments
David	Energy of the hoop is conserved. On the straight part of the track, the hoop's energy includes both translational and rotational kinetic energy. At point A, the hoop's energy includes potential and translational kinetic energy. When the hoop flies off the track, it does not roll any more, so it does not have rotational kinetic energy at point A.	
Mary	Yes, the hoop does not have rotational energy at point A, but it does not have translational energy on the straight part of the track either. The hoop doesn't have translational motion. It moves forward because it is rolling along the track.	
Eric	The hoop has both translational and rotational motion both on the straight part of the track and at point A. So there are two kinds of kinetic energy in both initial and final energy.	

Name: ___________________________ Date: ____________________

Susan	Both gravity and normal forces, which are acting on the sphere, do not cause any torque to the sphere so angular momentum of the sphere is conserved between initial point and point A. Angular momentum equals to moment of inertia times angular speed, so I can find angular speed at point A. This angular speed divided by the radius of the sphere is the linear speed of the sphere at point A.	
Jim	I will use kinematics equation: $v^2 = v_0^2 + 2ad$, where a is acceleration due to gravity which is acting on the sphere as it climbs up the track and d is the distance along the track. Then I can find speed of the sphere at point A.	

Please notify the instructor before proceeding to the next problem.

Name: ___________________________ Date: ____________________

Problem 5

a. Start with the physics problem in problem 4, modify it by including in it the physics ideas in problem 2 to create a new solvable problem of your own. Write your instructions to solve that new problem.

Name: ____________________________ Date: ____________________

b. Start with the physics problem in problem 4, modify it by including in it the physics ideas in problem 3 to create a new solvable problem of your own. Write your instructions to solve that new problem.

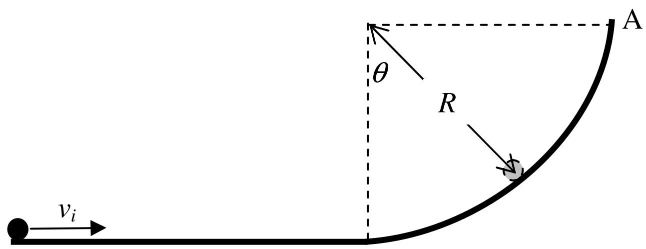
Please notify the instructor before proceeding to the next problem.

Name: ___________________________ Date: ______________________

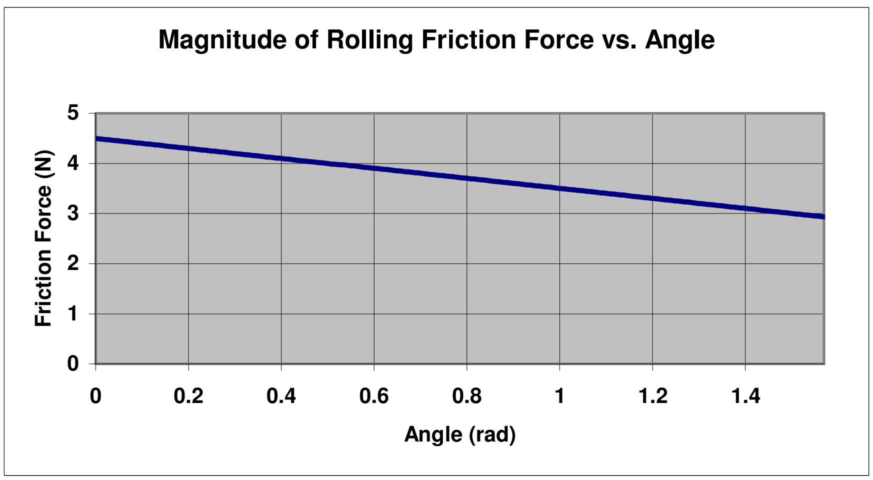
FOCUS GROUP INTERVIEW 4
Please work on these problems individually

<u>Problem C</u>

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i = 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies with angle θ as per the graph shown below

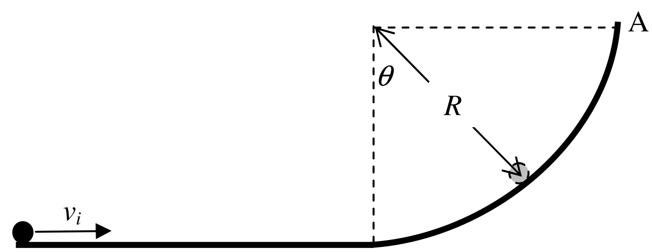


What is the launch speed of the sphere as it leaves the curve at point A?

Name: ________________________ Date: ____________________

Problem D

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i = 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies with angle θ (radians) as per the following function

$$F(\theta) = 4.5 - 1.2\theta$$

What is the launch speed of the sphere as it leaves the curve at point A?

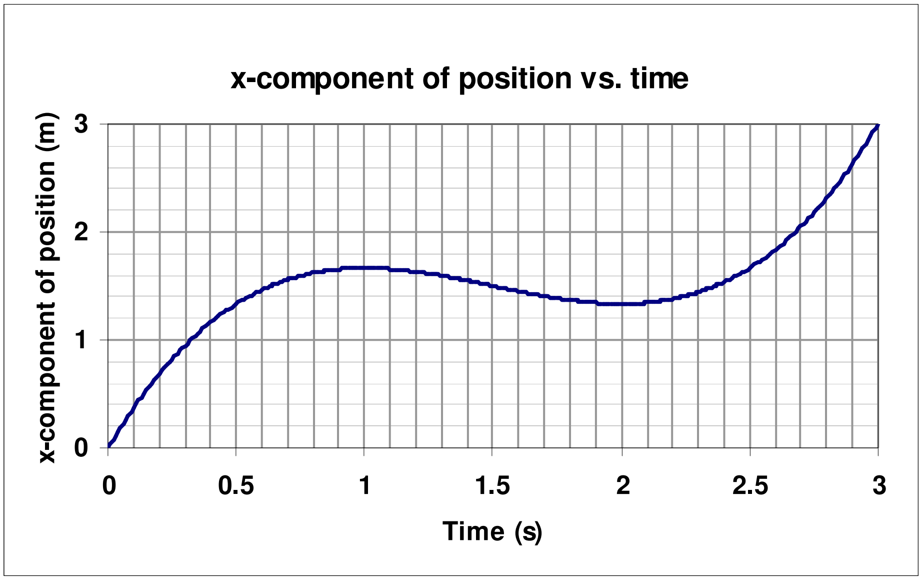
NAME (Please Print)____

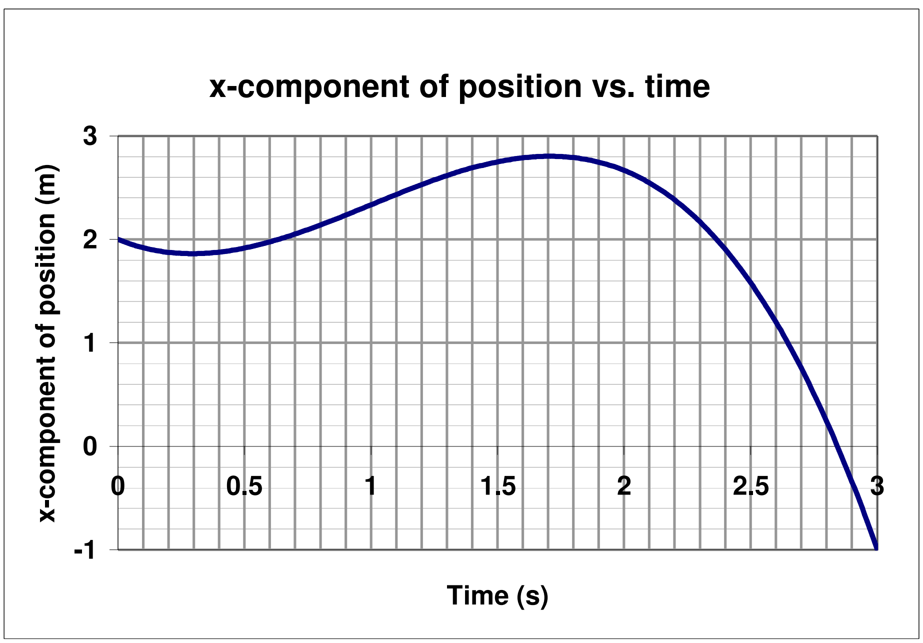
Please answer the questions below INDIVIDUALLY

FOCUS GROUP INTERVIEW 5

Problem 1

A particle is moving across a horizontal floor on which an x-y coordinate system is drawn. The following graphs give the x and y components of the position (in meters) of the particle versus time t (in seconds).
Find the change in velocity of the particle during from 0.5 seconds to 2.0 seconds.





Problem 2

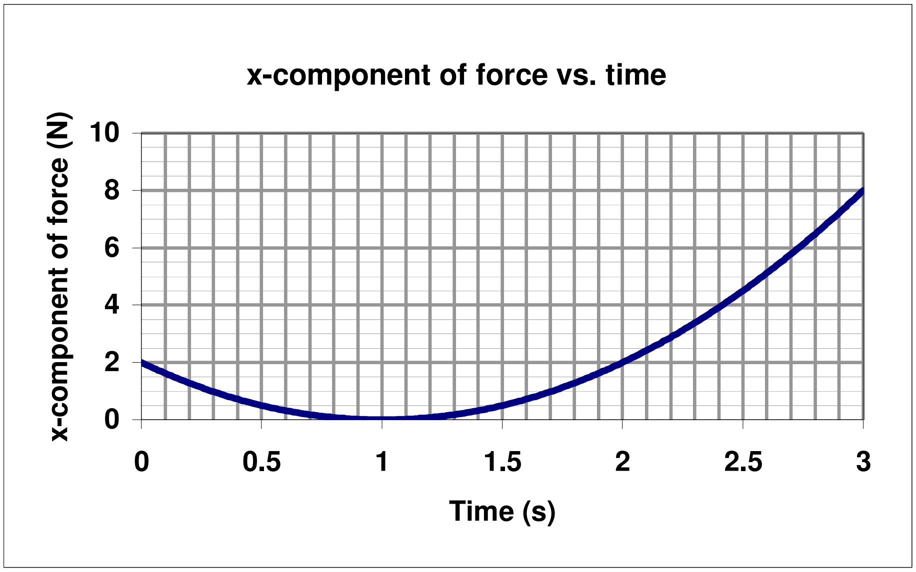
A particle is moving across a horizontal floor on which an x-y coordinate system is drawn. The x and y components of the position (in meters) of the particle are given as following functions of time t (in seconds).

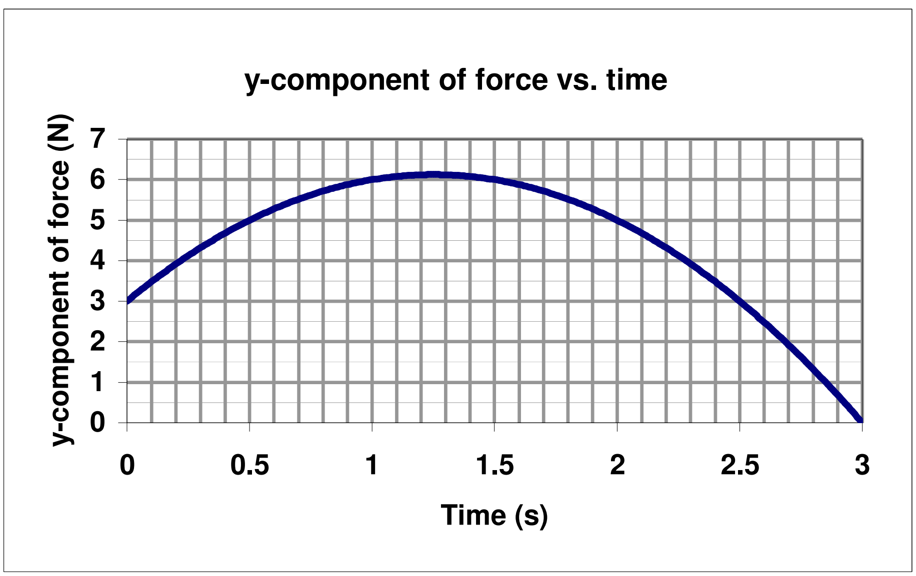
$$x(t) = t^3 - 4t^2 + 2t + 1 \qquad\qquad y(t) = -2t^3 + 3t^2 + 2t - 5$$

Find the change in velocity of the particle during the time from 1.0 second to 3.0 seconds.

Problem 3

A force **F** is applied on a 2.0 kg block moving across a horizontal floor on which an x-y coordinate system is drawn. The graphs below give the x and y components of **F** versus time t. Find the magnitude of net acceleration **a** of the block and the angle it makes with the positive x-axis at t = 2.0 seconds.





Find the magnitude of force **F** and the angle it makes relative to the positive x-axis.

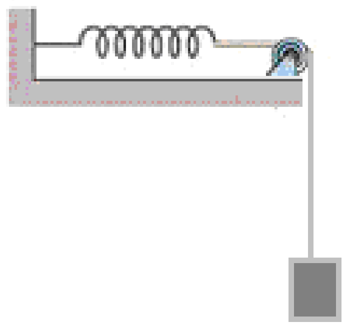
Problem 4
A force **F** is applied on a 2.5 kg block moving across a horizontal floor on which an x-y coordinate system is drawn. The x and y components of **F** (in Newtons) are given as following functions of time (in seconds):

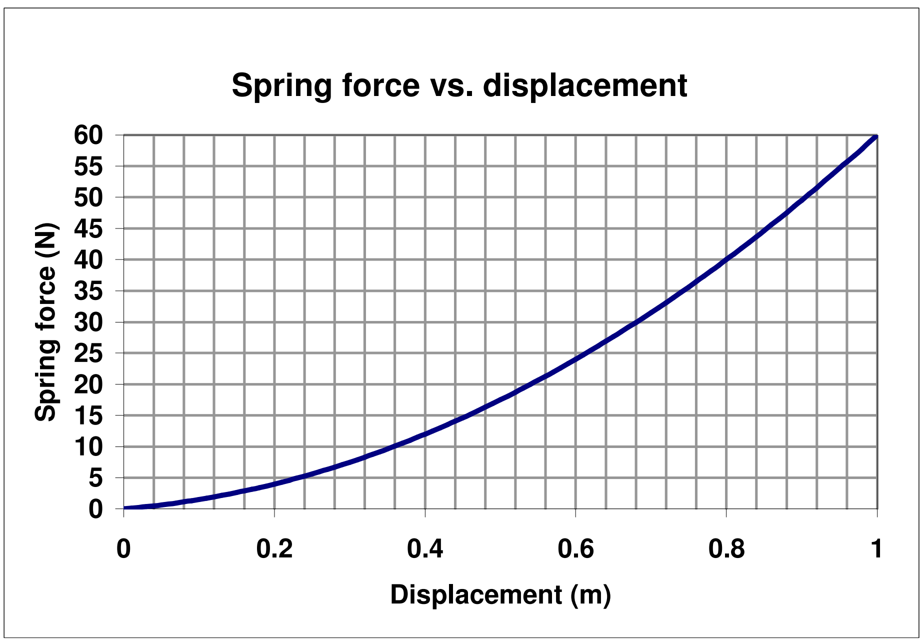
$$F_x(t) = 2t^2 - 5t + 7 \qquad\qquad F_y(t) = 3t^2 + 4t - 6$$

Find the magnitude of net acceleration **a** of the block and the angle it makes with the positive x-axis at t = 3.0 seconds.

Problem 5

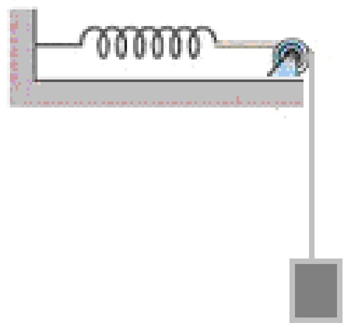
A 3.0 box is hanging by a cord that runs over a pulley connected to a spring as shown in the figure. The force (in Newtons) that the spring exerts when stretched a distance x (in meters) is plotted in the following graph. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless. What is the speed of the box when it has moved vertically down a distance of 0.8 m from its initial position?





Problem 6

A 2.0 box is hanging by a cord that runs over a pulley connected to a spring as shown in the figure. The force (in Newtons) that the spring exerts when stretched a distance x (in meters) is found to have magnitude $F(x) = 38.4x^2 + 52.8x$ in the direction opposite the stretch. The box is released from rest when the spring is unstretched. Assume that the pulley is massless and frictionless. What is the speed of the box when it has moved vertically down a distance of 0.4 m from its initial position?



ECE Fourier Series Interview Protocol for

ECE 512 – Linear Systems Students

Chen Jia and Steve Warren, Spring 2010

1. To get a sense of how students handle a basic Fourier series, let them solve one problem (~15 minutes).

   Function: $f\ (t) = t^2 - 2$ over t = [-2, 2]

- Before they begin, ask them to estimate the sign of a_0
- Q: "Can you describe to me how you solved the problem?" (Even, odd or neither; the main process; any difficulties)
- Q: "Starting with $f_{TFS}(t)$, what is the value of $f\ (t)$ at t = 0?"

2. Ask questions about 3 or 4 individual concepts. Show the student two figures for the new Fourier series: the plot of the original function, and the plot of an altered function. (~10 minutes)

- Q: "How does a_0 change?"
- Q: "How do a_n and b_n change?"
- Q: "How does w_0 changes?"

3. Mixed trigonometric Fourier series concepts (~10 minutes)

- Q: "If we flip the signal horizontally, how does the TFS representation change?"
- Q: "If we make $a_0 = a_0 + 3$ and $a_n = -a_n$, how will the figure look?"

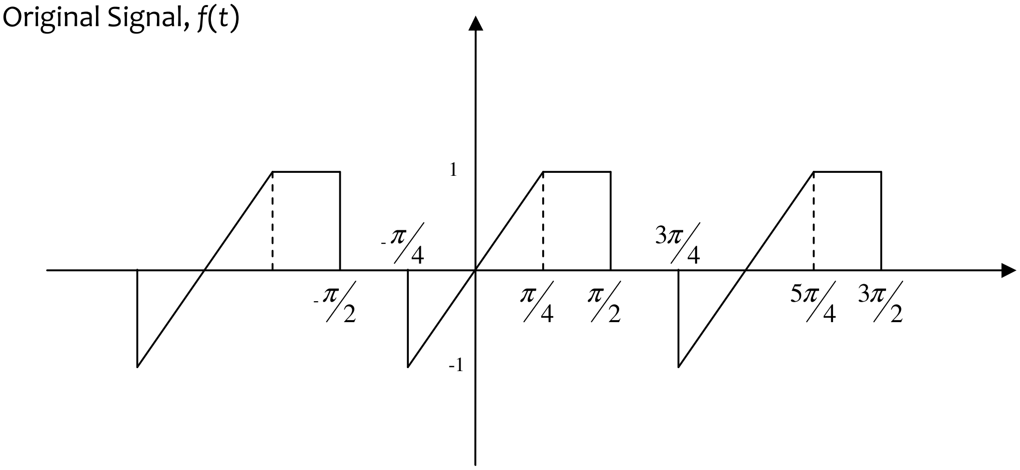
4. Mixed compact trigonometric Fourier series concepts (~10 minutes)

- Q: "If we delay the signal 1 second and double the amplitude, what will change in the CTFS and by how much?"

5. Construction of a new signal. This part depends on student success to this point and on the overall time limit (1 hour).

- Q: "If we want to construct a new signal like this, which base signals and representations (TFS or CTFS) should we choose? Do we need to change some parameters to build the new one?"

Original Signal, $f(t)$



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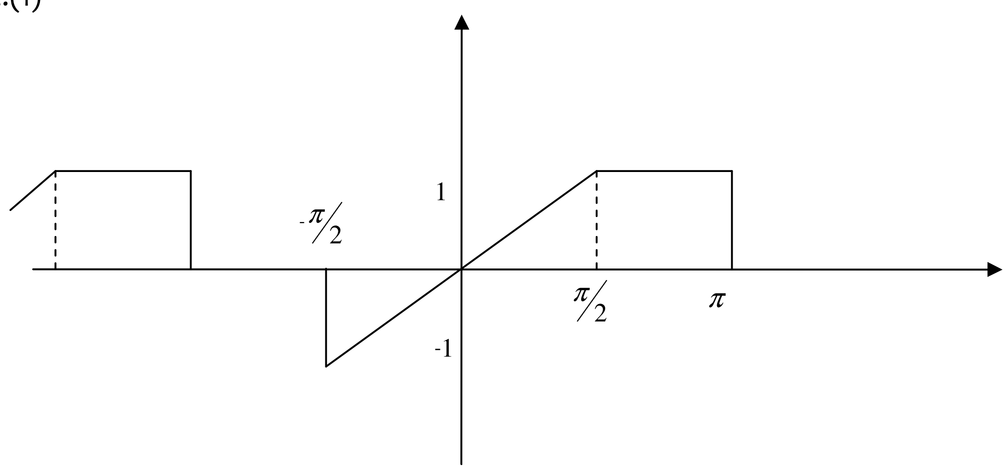

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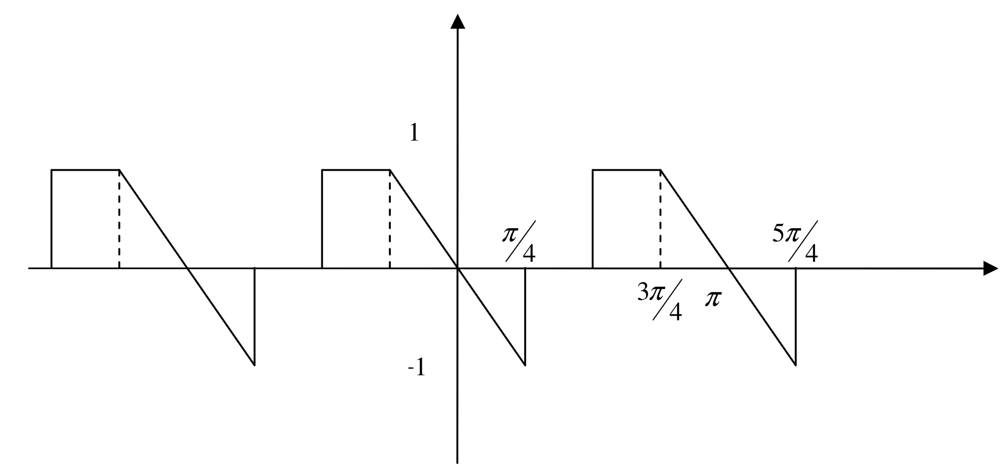

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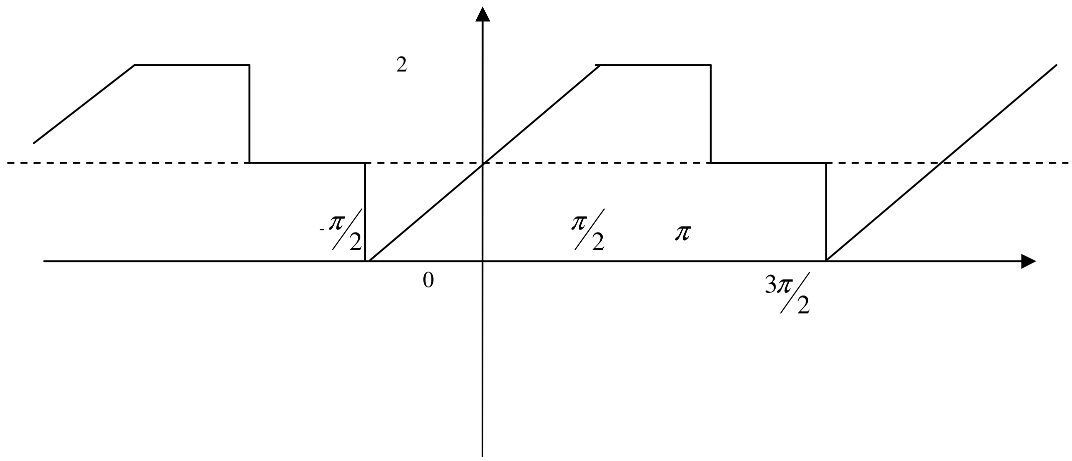

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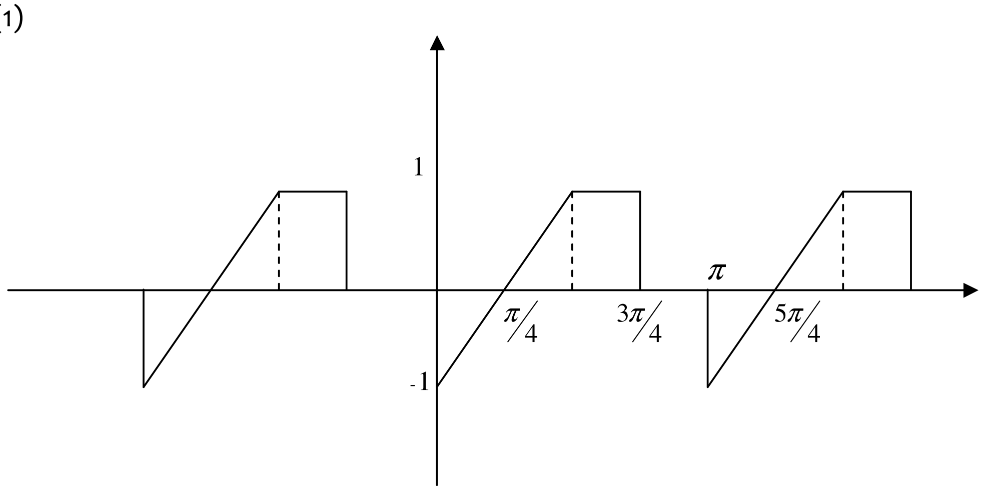

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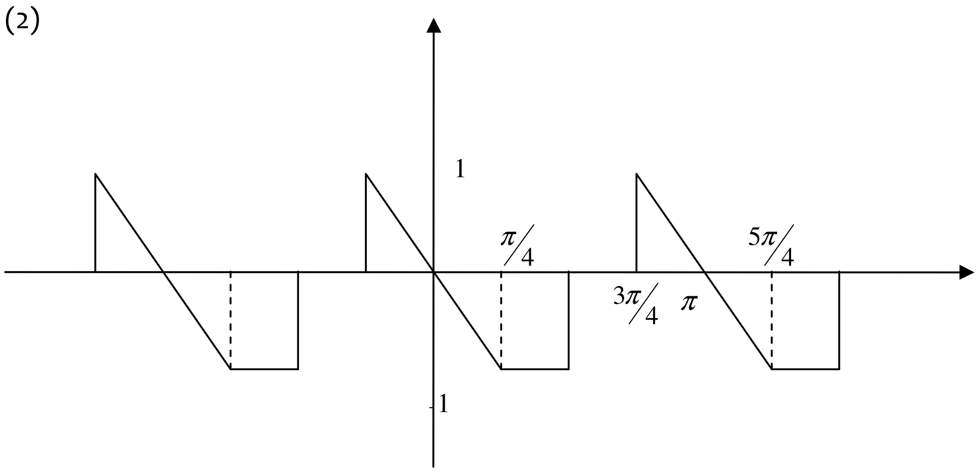

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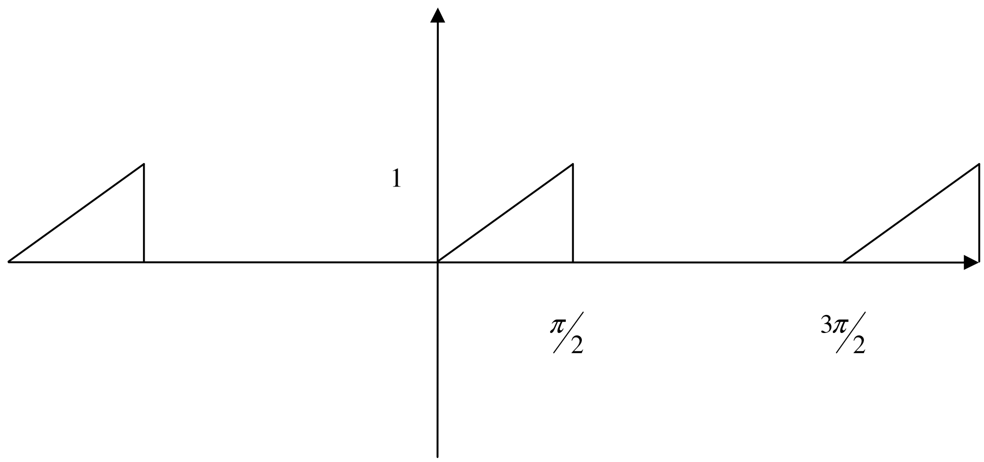
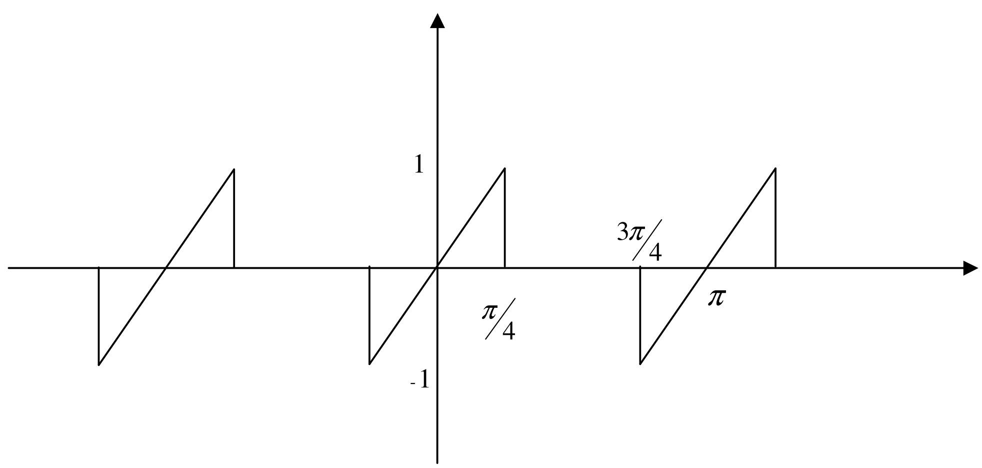
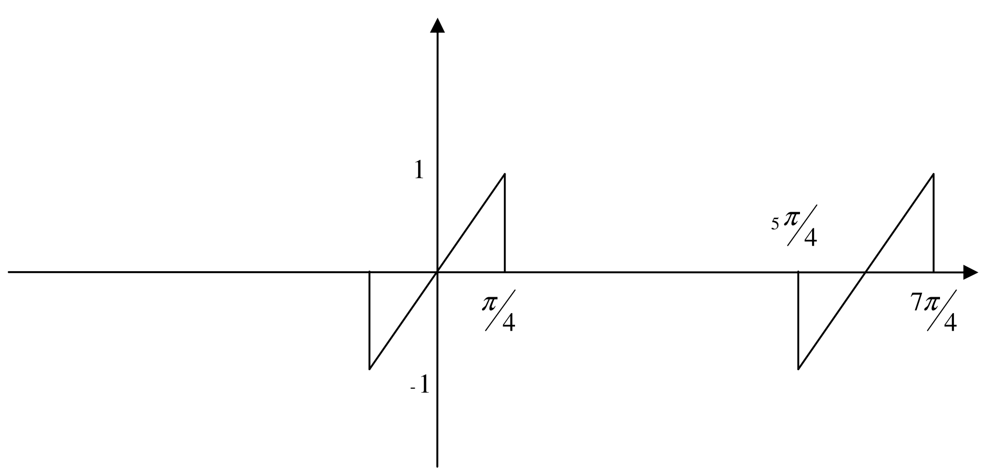
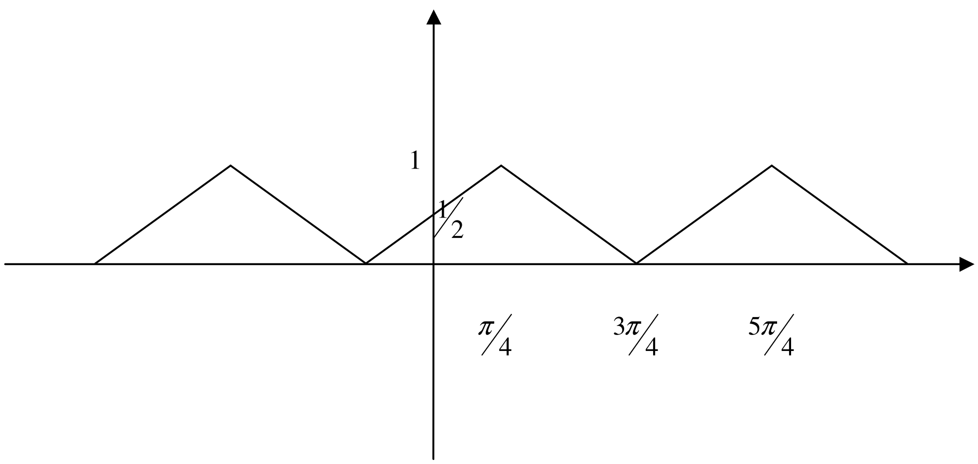






(1)



(2)