




# Transfer of Learning from Mathematics to Physics

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## Acknowledgements




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## What is Transfer?

Ability to use what you have learned in one situation in a different situation.

E.g. McKeough, Lupart & Marini (1995)

## Some Early Views of Transfer

- Identical elements must exist between situations.
- Knowledge must be encoded in a coherent model.
- Researchers/educators pre-decide what must transfer.
- Static one-shot assessment e.g. tests and exams.
- Focus mainly on students' internal knowledge.

Transfer is rare.

E.g. Gick & Holyoak (1980), Reed & Ernst (1974), Thorndike (1906)

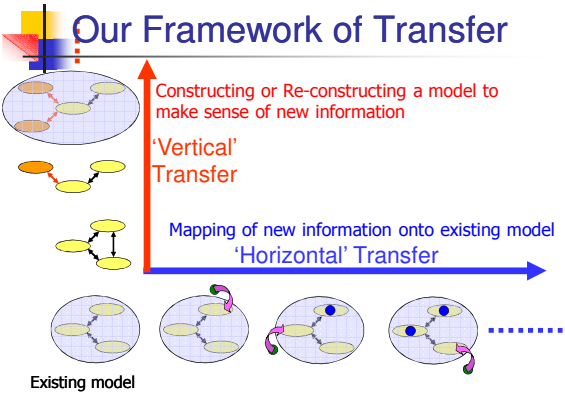
## Some Emerging Views of Transfer

- (Re) construct knowledge in new context.
- Knowledge can transfer in pieces.
- We must examine anything that transfers.
- Dynamic, real-time assessment e.g. interviews.
- Focus also on mediating factors e.g. motivation.

Transfer is ubiquitous.

Hammer *et al* (2005), diSessa & Wagner (2005);  
Bransford *et al* (1999), Lobato (2003, 1996), Greeno *et al* (1993)

## Our Framework of Transfer



Constructing or Re-constructing a model to make sense of new information

'Vertical' Transfer

Mapping of new information onto existing model

'Horizontal' Transfer

Existing model

## Some Caveats

'Horizontal' & 'Vertical' Transfer...

- are not mutually exclusive.
  - A given thinking process might involve elements of both 'horizontal' and 'vertical' transfer.
- cannot be universally labeled.
  - What is perceived as 'vertical' transfer by a novice may be perceived as 'horizontal' transfer by an expert.

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## Possible Research Questions

- How do students engage in 'horizontal' and 'vertical' transfer?
- Under what conditions do they engage in each?
- Is there a preferred sequence for these processes?

⋮


and several others....

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## 'Calculus to Physics' Study

### Research Question

To what extent do students transfer their calculus knowledge while problem solving in introductory calculus-based physics?



Lili Cui

Cui et. al. (2005)

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## 'Calculus to Physics' Study

### Research Participants

- **Students** (N = 28)
  - Enrolled in 2<sup>nd</sup> semester, calculus-based physics
  - After covering relevant topics in electricity and magnetism
- **Teachers:** Faculty, Instructors and TAs
  - Physics (N = 6)
  - Mathematics (N = 4)

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## 'Calculus to Physics' Study

### Research Plan

#### Semi-structured Interviews

- 'Horizontal' Transfer
  - Textbook-like Problems
- 'Vertical' Transfer
  - 'Contrasting Cases'
  - 'Jeopardy' Problems

<sup>1</sup> Schwartz, Bransford & Sears (2005)     <sup>2</sup> Van Heuvelen & Maloney (1999)

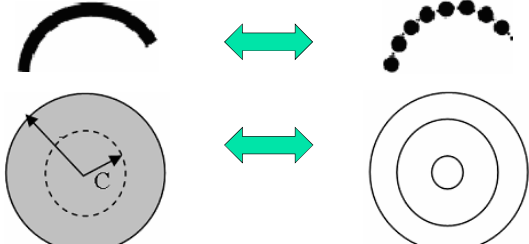
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## 'Calculus to Physics' Study

### 'Contrasting Cases'

#### Continuous vs. Discrete

When do you use integration in a problem?



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### 'Calculus to Physics' Study 'Jeopardy' Questions

Construct a physical situation that is described by the following expression

$$2 \times \int_0^{\frac{\pi}{6}} (8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2) (2 \times 10^{-10} \text{ C/m}) (5 \times 10^{-2} \text{ m}) \cos \theta d\theta$$

Our goal is not to find out whether they get these problems right, rather the process they use to attempt the problems

$2\pi R$

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### 'Calculus to Physics' Study Student Interview Results

- Textbook-like Problems
  - no difficulty in recalling
  - difficulty setting up the
- 'Contrasting Cases':
  - used similarity of textbook
  - had difficulty determining
- 'Jeopardy' Problems:
  - used pattern matching to
  - used units to find physical quantity represented by expression.
  - About half used variable of integration to figure out geometry.

"I know how to integrate it, but it is just figuring out what to integrate, that is the hard part... which is really, really, really hard."

"I look for pieces of terms that I recognize,.... they will tell what kind of problem they are, I just tend to recognize forms, like derivative... I do not know why those formula work, I just use them"

"...I do not have a good sense of what the objective of what I am actually looking for... as soon as I set it up, there is no problem"

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### 'Calculus to Physics' Study Teacher Interview Results

- Mathematics teachers..
  - focus on techniques of calculus.
  - realize value of applications, but cannot address them.
  - seldom use 'word' problems.
- Physics teachers
  - to attend to
  - math teachers
    - Do more 'word' problems, so as far as on exams I mean I was trying to put some on them, but I do not want to make the exam too hard"

"I would be happier if the mathematicians put more emphasis on the theoretical basis of calculations."

"...I do not have a good sense of what the objective of what I am actually looking for... as soon as I set it up, there is no problem"

"I do not have a good sense of what the objective of what I am actually looking for... as soon as I set it up, there is no problem"

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### 'Calculus to Physics' Study SUMMARY

- 'Horizontal' Transfer : Students have
  - no difficulty recalling model to solve calculus problems.
  - difficulty mapping physics problem variables into model.
- 'Vertical' Transfer :
  - How do we address these issues? Could some of our what we have learned elsewhere give us some clues?
- Teachers' Perspective
  - Math: Focus on techniques, not concepts or applications.
  - Physics: Would like math teachers do what they do not!

From students' perspective perhaps this was 'vertical' transfer ??

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### What Transfer do We Want?

Horizontal (Efficiency) AND Vertical (Innovation)  
Striking a Balance: 'Optimal Adaptability Corridor'<sup>1</sup>

<sup>1</sup> Schwartz, Bransford & Sears (2005)    <sup>2</sup> Murray & Arroyo (2002)

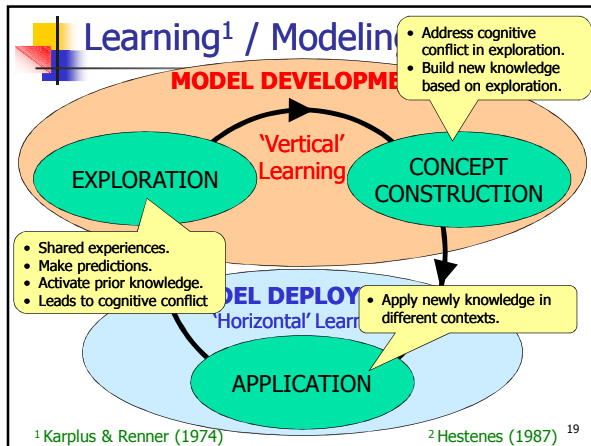
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### Characteristics of Instructional Strategies

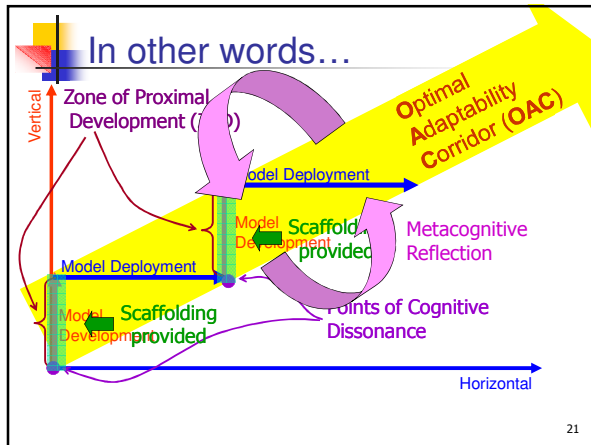
- Balance 'horizontal' and 'vertical' transfer
  - Follow an 'Optimal Adaptability Corridor'<sup>1</sup>
- Adapt proven pedagogical strategies e.g.
  - Small steps of Model Development (Vertical) followed by Model Deployment (Horizontal).<sup>2</sup>
- Emphasize multiple models
  - Sensitivity to activate appropriate model

<sup>1</sup> Schwartz, Bransford & Sears (2005)  
<sup>2</sup> Hestenes (1987)

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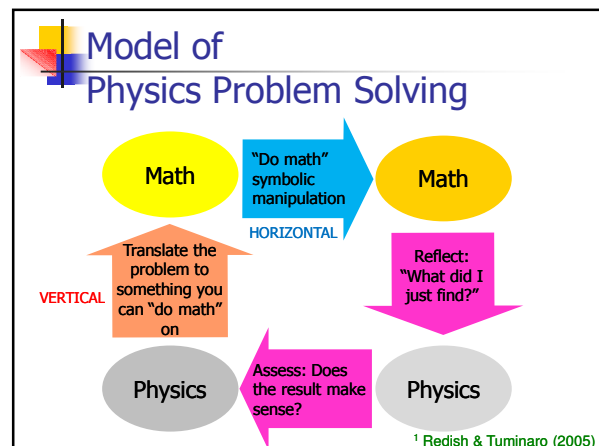


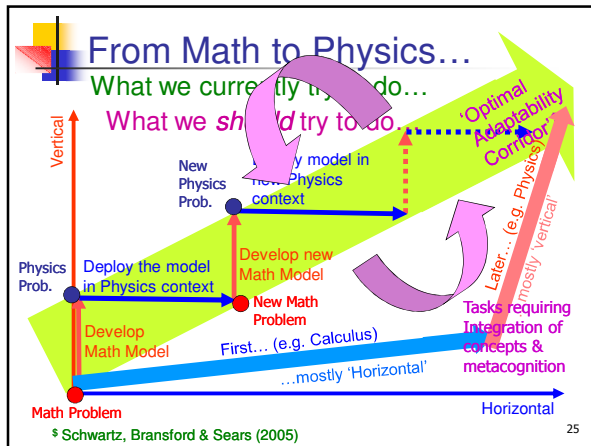
- ### Implications for Instruction
- Balance **horizontal** and **vertical** transfer
    - Follow an 'Optimal Adaptability Corridor'
  - Adapt the Modeling Cycle
    - First **Model Development**
    - Then **Model Deployment**
  - Employ strategies that ...
    - Use **cognitive conflict** to promote model development
    - Scaffold learning within **Zone of Proximal Development**
    - Use **metacognitive reflection** to create adaptive learners
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- ### We can also apply this to...
- **Learning how to Learn:**
    - Students **deploy** strategies to succeed in science/math, based on their model of what it takes to succeed in this course.
    - If they fail, they reach a **point of dissonance** – model does not work.
    - We can then facilitate a process by which they reflect and **develop** a revised model of how to learn science/math.
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- ### We can also apply this to...
- **Learning how to Teach:**
    - As teachers we **deploy** our model of how students learn and how we should teach.
    - If students fail our assessments, we reach a **point of dissonance** – our model of learning and teaching does not work.
    - We then **develop** a revised model of how they learn, and think about how we can teach more effectively.
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Is there any evidence that this will work in helping students transfer math knowledge to physics?

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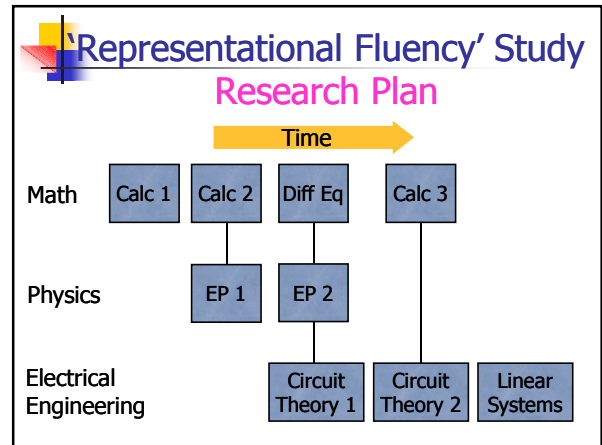
### 'Representational Fluency' Study

- How do students develop representational fluency?
- What kinds of difficulties do students encounter when solving problems in multiple representations?
- What kinds of scaffolding are useful in helping students overcome those difficulties?



Dong-Hai Nguyen

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### 'Representational Fluency' Study Data Collection (Spring & Fall 2009)

Spring 2009 Individual Interviews

n=20 EP1 students  
EE/CSE majors

1 2 3 4

Fall 2009 Individual Interviews

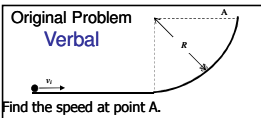
n=15 EP2 students  
EE/CSE majors

1 2 3 4

### 'Representational Fluency' Study Example Interview Problems

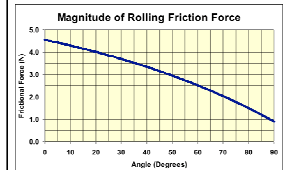
Graphical

Equational



Original Problem Verbal

Find the speed at point A.



Magnitude of Rolling Friction Force

Equational

$$F_{\text{roll}}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

### 'Representational Fluency' Study

#### Individual Interviews General Results (Spring & Fall 2009)

- All students able to solve problems with hints.
- Initially had trouble invoking **integral = area under the curve**.
- Had difficulty **coordinating geometric and algebraic** modes of thinking.
- Little evidence that students can interpret **integration as accumulation**.
- Fewer difficulties when graph problem before equation problem, than vice-versa.

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### 'Representational Fluency' Study

#### Data Collection (Spring 2010)

Spring 2010 Focus Group Interviews  
n=26 EP1 students Engineering majors

Control Treatment

### 'Representational Fluency' Study

#### Focus Group Interviews (Spring 2010)

Control Group Treatment Group

Pre-Test : Prob. 2 & 3 from Fall 2009 Interview

<ol style="list-style-type: none"> <li>1. Physics Problem (Graph)</li> <li>2. Physics Problem (Graph)</li> <li>3. Physics Problem (Equation)</li> <li>4. Physics Problem (Equation)</li> <li>5. Physics Problem (Similar to Fall 2009 Prob. 1, Verbal)</li> </ol>	<ol style="list-style-type: none"> <li>1. Math Problem (Graph)</li> <li>2. Physics Problem (Graph)</li> <li>3. Math Problem (Equation)</li> <li>4. Physics Problem (Equation)</li> <li>5. Debate Problem (Similar to Fall 2009 Prob. 1, Verbal)</li> <li>6. Problem Posing (Combine previous problem w/Graph, Equation)</li> </ol>
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Post-Test : Similar to Pre-Test, different numbers

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### 'Representational Fluency' Study

#### Focus Group Interviews (Spring 2010)

Looking at Treatment Group through Horizontal & Vertical Transfer Lens

<ol style="list-style-type: none"> <li>1. Math Problem (Graph)</li> <li>2. Physics Problem (Graph)</li> <li>3. Math Problem (Equation)</li> <li>4. Physics Problem (Equation)</li> <li>5. Debate Problem (Similar to Interview Prob. 1, Verbal)</li> <li>6. Problem Posing (Combine Prob.5 with Graph, Eqn.)</li> </ol>
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### 'Representational Fluency' Study

#### Conclusions

Interview 3  
Mann Whitney  
U = 74.0, p-value = 0.0448

Interview 4  
Mann Whitney  
U = 94.0, p-value = 0.0178


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### 'Representational Fluency' Study

#### SUMMARY

- Students have difficulty...
  - solving problems that present information in graphical and equational representations.
  - recognizing how to appropriately apply the concept of integration in physics problems
- Promising interventions...
  - involve the use of vertical and horizontal transfer.
  - use a sequence of math and physics problems.
  - use debate problems and problem posing tasks to facilitate metacognition.


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## OVERALL SUMMARY

- Transfer of learning is a complex process and must be considered from different perspectives.
- Students instinctively engage in 'horizontal' transfer and attempt 'vertical' transfer only if 'horizontal' transfer has not worked for them.
- Most of instruction focuses on 'horizontal' transfer and does not prepare students for 'vertical' transfer.
- To create adaptive learners, we must balance both; we have some evidence that this can perhaps be done through carefully designed sequences of small steps of both 'vertical' and 'horizontal' transfer.

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## Thank You

For further information  
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