# Representational Competence in Learning and Problem Solving in Physics

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## What is Representational Competence?

"The ability to comprehend the equivalence of different modes of representation" (Sigel & Cocking, 1977)

"Comprehend Equivalence" ⇒

- Read out same info. from different representations
- Transfer learning from one representation to another
- Transform info from one representation to another.
- Others…

"Modes of Representation" ⇒

- Physical vs. Virtual
- Verbal vs. Mathematical
- Graphical vs. Equational
- Macroscopic vs. Microscopic
- Others…

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## Incomplete Review of Previous Research on Multiple Representations (MRs)

Ainsworth (2006) : In interacting with an MR a learner must understand…

- the form of each representation
- the relation between representation & domain
- how to construct new representations
- the connections between different MRs
  - difficult for learners and may inhibit learning from multiple representations

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## Incomplete Review of Previous Research on Multiple Representations (MRs)

Hagevik, Beilfuss & Dickerson (2006) : Mastery of MRs leads to a deep understanding

- Experts can easily shift between MRs to solve problems, but novices have difficulties
- Qualitative reps can act as a "bridge" between abstract mathematical and linguistic reps.
- Computer-based MRs provide complementary information, constrain interpretations, and promote deep understanding (Ainsworth, 1999).

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## Incomplete Review of MRs in Phys. Ed.

- McDermott et al (1987)
  - Students difficulties with connecting graphs to physical concepts and the real world.
- Van Heuvelen and Zou (2001)
  - Qualitative representations (sketches, diagrams, bar charts) help learning of energy concepts.
- DeLeone & Gire (2005)
  - Non-equational representations were necessary but not sufficient for problem-solving success.
- Meltzer (2005)
  - Students' perform differently on isomorphic physics problems in different representations
- Kohl & Finkelstein (2008)
  - Students performance depends upon the representation that they were provided or chose to use

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## Our Research Goals

Investigate how learners…

- construct knowledge as they interact with different representations?
  - Study 1: Learning using Physical vs. Virtual Representations
- transfer their learning from one representation to another?
  - Study 2: Solving Problems in Numerical vs. Graphical vs. Equational Representations

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## Study 1: Background

- Previous studies -- mixed results
  - Virtual outperform analogous physical experiments
    - Zacharia, Olympiou, & Papaevripidou, 2008
    - Finkelstein, et al., 2005
  - No difference in learning : physical vs. virtual
    - Klahr, Triona, & Williams, 2007
    - Zacharia & Constantinou, 2008
- Zacharia & Constantinou (2008)
  - More research is needed to describe how physical and virtual manipulatives should be integrated in a curriculum.

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## Study 1: Research Questions

- RQ1.1: Is there a difference in learning from physical and virtual representation as measured by students'…
  - performance on a multiple choice test?
  - reasoning provided as they interact with the representation?
- RQ1.2: When students use both physical & virtual representation…
  - How does their learning from the two representations compare?
  - How does the temporal order of using the physical and virtual representations affect students' learning?
  - What views do students express about data collected from physical and virtual representations?

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# Study 1: Research Context

- CoMPASS Curriculum (Puntambekar et al, 2003)
  - Concept Mapped Project-based Activity Scaffolding System
  - Originally designed for middle school students
  - Based on "Learning by Design" (Hmelo, 2000; Kolodner et al 2003)
  - Integrates: Hypertext + Activities (Physical/Virtual)
- Pulley Unit
  - Targeted concepts: Force, Force-Distance tradeoff, Work done, Equivalence of Work, Equality of Work & PE, MA.
  - Two-hour Laboratory

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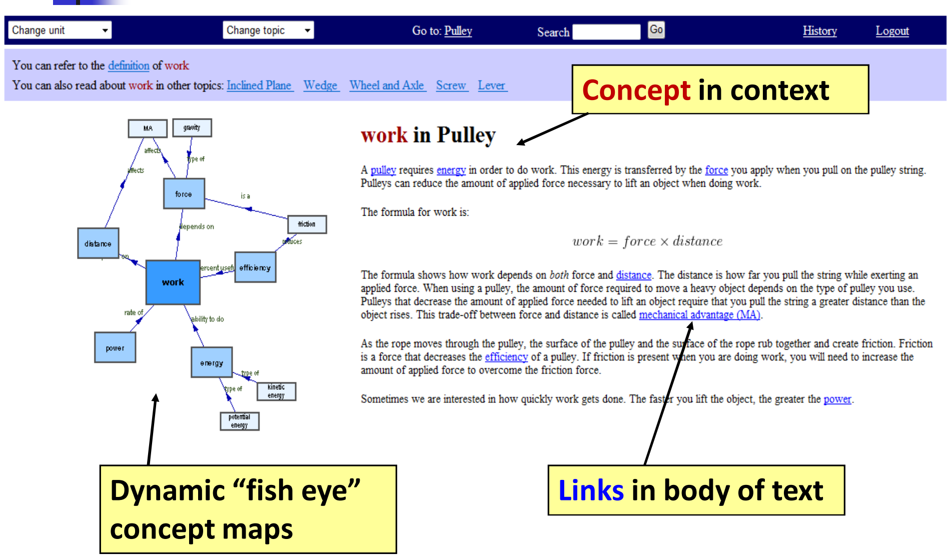
# Study 1: CoMPASS Hypertext Concept Maps

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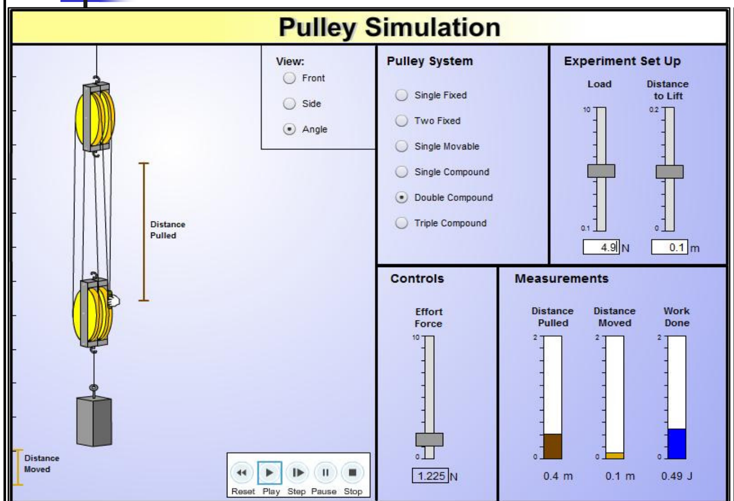
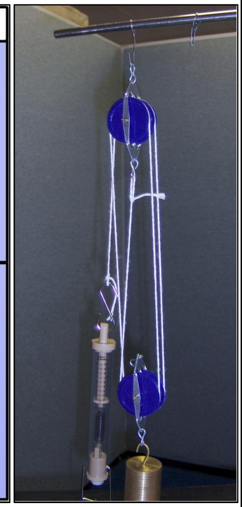
## Study 1: Physical & Virtual Representations



Virtual Physical

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## Study 1: Research Design

**PV Sequence (N=61)** | **VP Sequence (N=71)**

**Physical-Virtual Sequence** | **Virtual-Physical Sequence**

Pre-Test
↓
Brainstorming & CoMPASS
↓
Physical Experiment | Virtual Experiment
↓
Mid-Test
↓
Virtual Experiment | Physical Experiment
↓
Post-Test

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# Study 1: Sources of Data

- Pre/Mid/Post Tests
  - 13 Conceptual multiple choice questions
  - Topics covered: Force, Work, Force-distance tradeoff, Work-PE equivalence, MA
  - Cronbach's $\alpha$ Reliability 0.7
- Worksheet Questions
  - Open-ended
  - Completed while doing activity
  - Asked description of trends in observed experimental data & explain why

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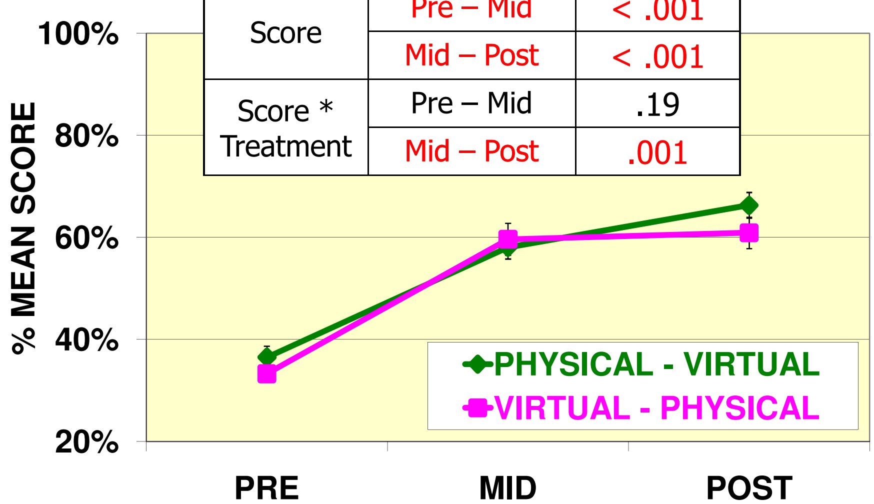
# Study 1: Overall Test Performance

| Effect | Comparison | p-value |
|---|---|---|
| Score | Pre – Mid | < .001 |
| | Mid – Post | < .001 |
| Score * Treatment | Pre – Mid | .19 |
| | Mid – Post | .001 |

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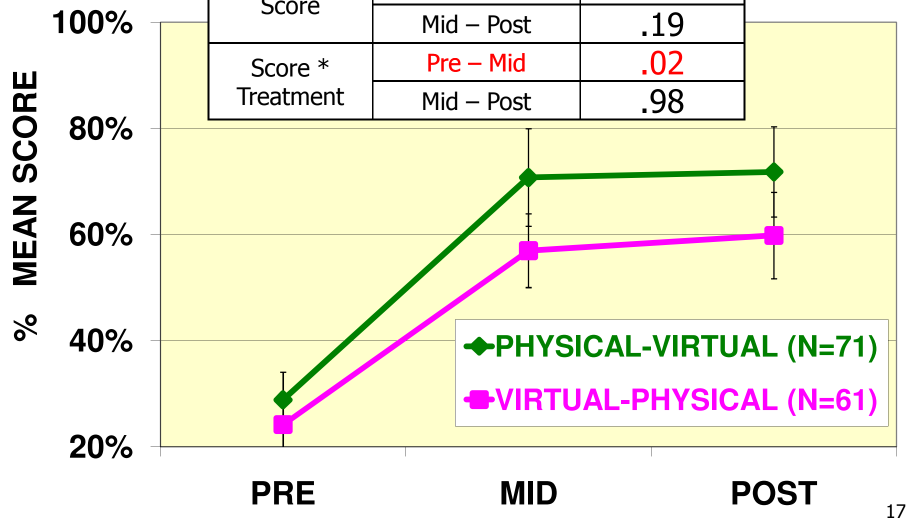
## Study 1: 'Force' Questions on Test

| Effect | Comparison | p-value |
|---|---|---|
| Score | Pre – Mid | <.001 |
| | Mid – Post | .19 |
| Score * Treatment | Pre – Mid | .02 |
| | Mid – Post | .98 |

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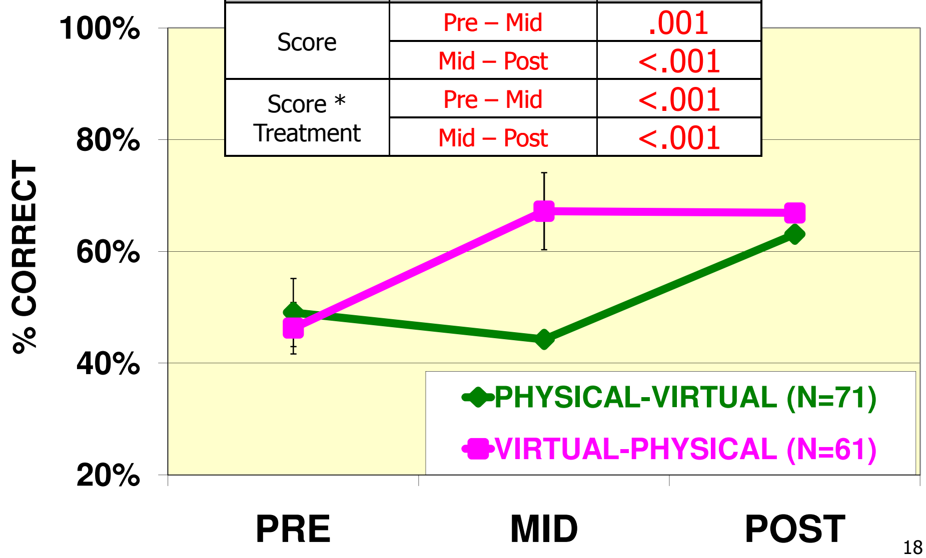
## Study 1: 'Work' Questions on Test

| Effect | Comparison | p-value |
|---|---|---|
| Score | Pre – Mid | .001 |
| | Mid – Post | <.001 |
| Score * Treatment | Pre – Mid | <.001 |
| | Mid – Post | <.001 |

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## Study 1: Why these Results?

Two possible effects:

- Differential cue salience?
  (Denton & Kruschke, 2006)
- Anomalous Data?
  (Chinn & Brewer, 1993)

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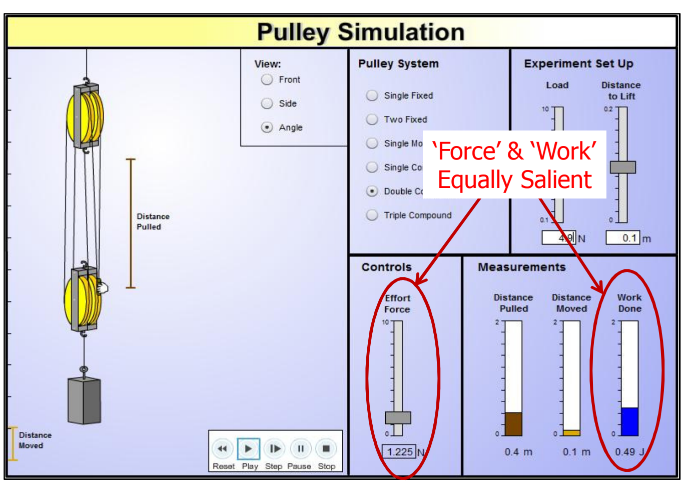
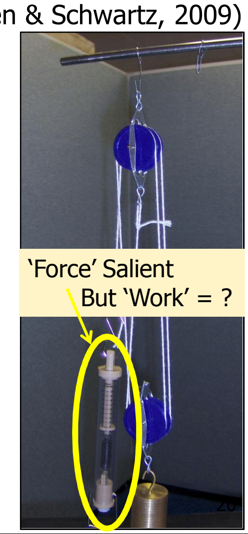
## Study 1: What Causes Differential Cue Salience?

- Superiority / Noticing effect? (Lindgren & Schwartz, 2009)



## Study 1: Implication of Differential Cue Salience

- Blocking? (Heckler, et al 2006)

| | Physical P | Virtual V |
|---|---|---|
| 'Force' Cues | High | High |
| 'Work' Cues | Low | High |

| | Virtual V | Physical P |
|---|---|---|
| 'Force' Cues | High | High |
| 'Work' Cues | High | Low |

Increasing Salience: Learning

Salience is high in both: Learning from whichever cue is presented first: Primacy effect

Decreasing Salience: Blocking

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## Study 1: Anomalous Data

Anomalous data (Chinn & Brewer, 1993)

Students' responses to anomalous data:

- Ignore
- Reject
- Exclude from the domain
- Hold in abeyance
- Reinterpret and retain
- Reinterpret and make peripheral changes
- Accept and change theory

In the right circumstances, can facilitate conceptual change

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## Study 1: What causes Anomalous data?

Anomalous data (Chinn & Brewer, 1993) due to…

- Prior knowledge
- Processing Strategy
- Characteristics of Data
  - Credibility
  - Ambiguity

Possible reasons..
- Measurement error
- Friction not being accounted for
- Others?

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## Study 1: An Example of Ambiguous & Unambiguous Data

| Type of Pulley System | Work value determined in PHYSICAL experiment | Work value measured in VIRTUAL experiment |
|---|---|---|
| **Single Fixed** | .49 J | .50 J |
| **Single Movable** | .52 J | .50 J |
| **Single Compound** | .48 J | .50 J |
| **Double Compound** | .54 J | .50 J |

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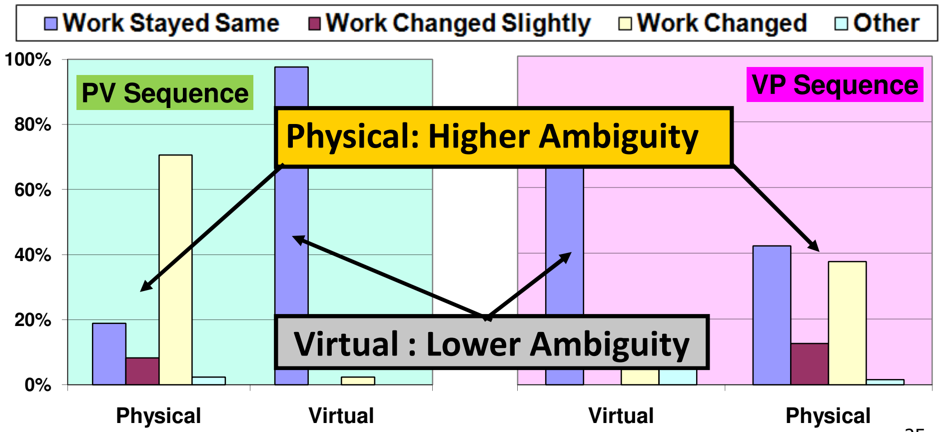
## Study 1: Worksheet Q – Equivalence of 'Work' across Pulley Systems

Based on your data, when you changed the pulley setup, how did it affect the work required?

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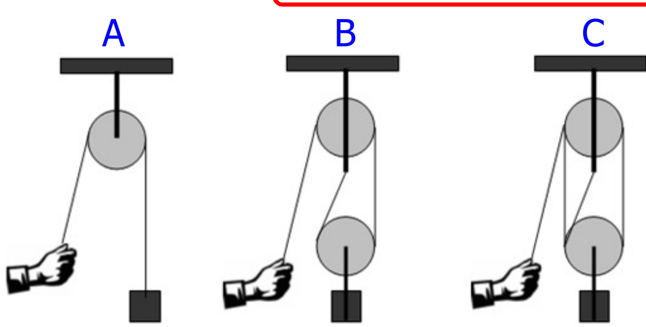
## Study 1: Test Q – Equivalence of 'Work' across Pulley Systems

Alice is using pulley setup A, Brenda is using B, and Carl is using C. What can you tell about the *work needed* to lift the same load by each of them through the same height, if friction is not a factor?

A) Alice does most work.
B) Brenda does most work.
C) Carl does most work.
D) Work done by all three is the same.

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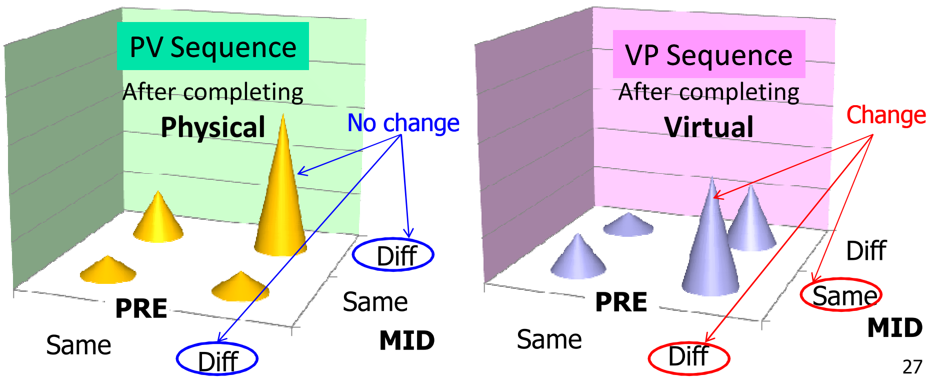
## Study 1: Test Q – Equivalence of 'Work' across Pulley Systems

How does the work done in different pulley systems to lift the same load the same height compare, if friction is not a factor?

### PRE to MID

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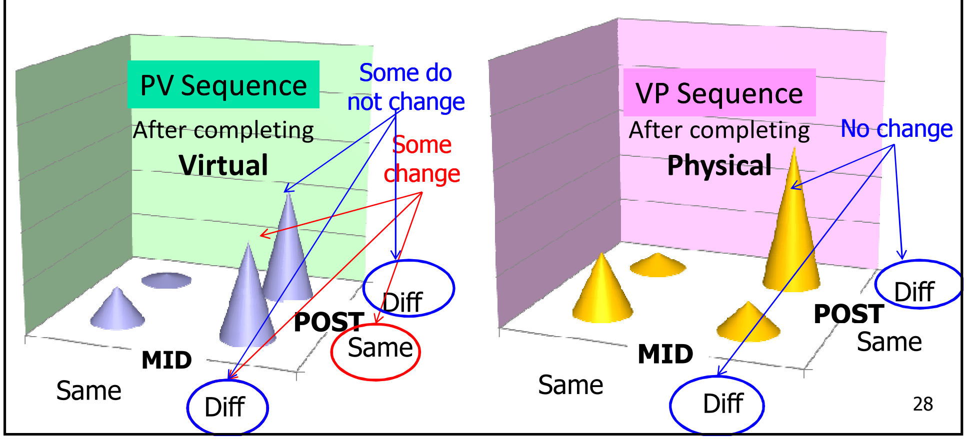
## Study 1: Test Q – Equivalence of 'Work' across Pulley Systems

How does the work done in different pulley systems to lift the same load the same height compare, if friction is not a factor?

### MID to POST

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## Study 1: What are the implications of differential ambiguity?

Differential ambiguity in data

⇒ Different response to data by learner

| Physical Data for 'Work' | Virtual Data for 'Work' |
| --- | --- |
| ▪ Higher ambiguity<br>▪ Less likely to change students' conceptions | ▪ Lower ambiguity<br>▪ More likely to change students' conceptions |

## Study 1*: What would students' prefer?

We asked students about data collected from physical and virtual representations

- Which set of data is more useful in different hypothetical situations?
  - Different contexts (exam, rental store, missed lab)
  - Different concepts (force, work)
  - Different pulley systems (fixed, movable, compound)
- How are the data collected physical and virtual similar and different?

*101 future elementary teachers in a conceptual physics class did activities in PV and VP sequences in Activities Center

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## Study 1*: Sample Question

Context — Pulley type

Q1) On an exam, your professor has asked you some questions about several pulley setups.

A) On the first question on the test, you have to decide whether a **fixed** or **movable** pulley requires the *least effort force* to lift the load.
Which experience in the Activity Center would better help you answer this question? (Check **one**)

- ☐ Experiment with real pulleys
- ☐ Computer simulation of pulleys
- ☐ Both are equally helpful

Concept

- Explain what led you to make the choice above.

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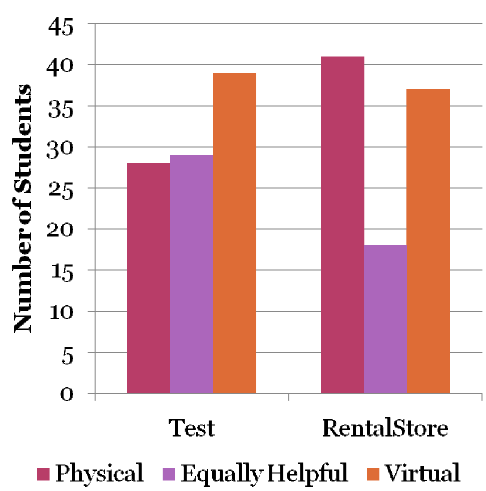
## Study 1*: What would students' prefer?

Which set of manipulatives would you use to decide:

- On a **test**, whether a fixed or movable pulley requires less force?
- In a **rental store**, whether a fixed or movable pulley will better help you lift a bed?

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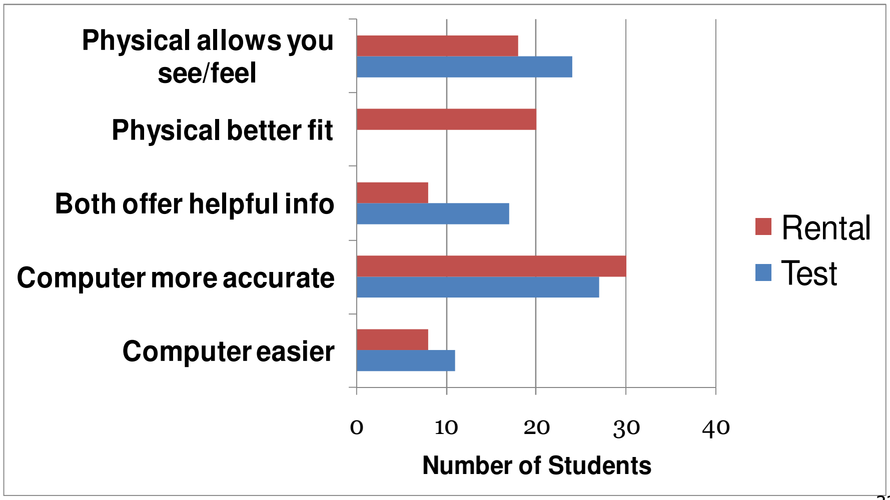
## Study 1*: Why?

### Reasoning given by students

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## Study 1*: Students Preferences

- Students preference depends most on **context**
  - 22-38% changed answer when **concept** changed
  - 20-30% changed answer when **pulley setup** changed
  - 53-68% changed answer when **context** changed
- Other factors remaining same, students most chose…
  - **Virtual** for "Test" context
  - **Physical** and **Virtual** for "Rental Store" context
- Students' reason that…
  - Simulation data is free from certain types of errors
  - Physical provides more kinesthetic experience
  - Physical may be a better fit to a real life situation

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## Study 1: Conclusions

- RQ1.1: Is there a difference in learning from physical and virtual representations as measured by…
  - Some concepts (e.g. force) are better learned using physical, others (e.g. work) better using virtual representations.
  - Differential ambiguity, cue salience in representations facilitates learning of different concepts, so both representations needed.
- RQ1.2: When students use both physical & virtual representations…
  - Overall, if physical is used first, students continue to learn when virtual is used afterward, but if they use virtual first they do not appear to continue learning with the physical.
  - Students appear to prefer the virtual in "test" contexts, but both equally when in "real world" situations.

## Study 2: Background

Questions addressed in recent research on MRs

- Does using MR help students learn concepts and learn to better solve problems?
- What instructional innovations help students use MRs while solving problems?
  - Do those innovations help them solve problems?
- How does the representation in which the problem is posed affect student performance and their decision to use another type of representation when solving the problem?

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# Study 2: Motivation

Multiple Representations (MRs) useful in solving physics problems

- Several studies addressing the benefits of using MRs in solving physics problems.
- Not as many studies on how students transfer their problem solving skills in physics across different MRs.



# Study 2: Research Questions

RQ2.1: What kinds of difficulties do students encounter when solving problems in multiple representations?

- What scaffolding is helpful to facilitate learners to overcome these difficulties

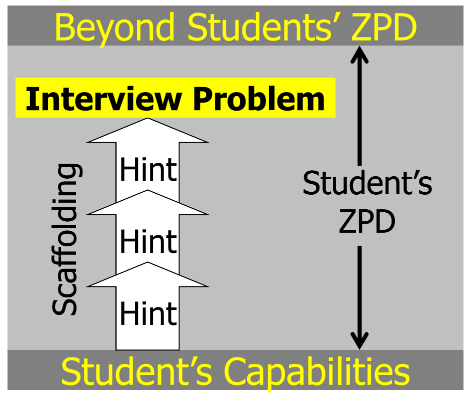
RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

# Study 2: Theoretical Perspective

Zone of Proximal Development

(Vygotsky, 1978)

- Problem presented within students' ZPD
- Hints provided to scaffold process of problem solving within ZPD.

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# Study 2: Research Context

- N=20 participants
- Engineering majors
- Enrolled in 1st semester calc-based physics
- Topics: Kinematics, Work-Energy

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## Study 2: Research Methodology

**Teaching/Learning Interviews** (Steffe et al , 2003)

- Four sessions: One after each class exam
- Each session: 60 minutes, video/audio taped
- Three problems per session
- Hints provided when students expressed difficulties

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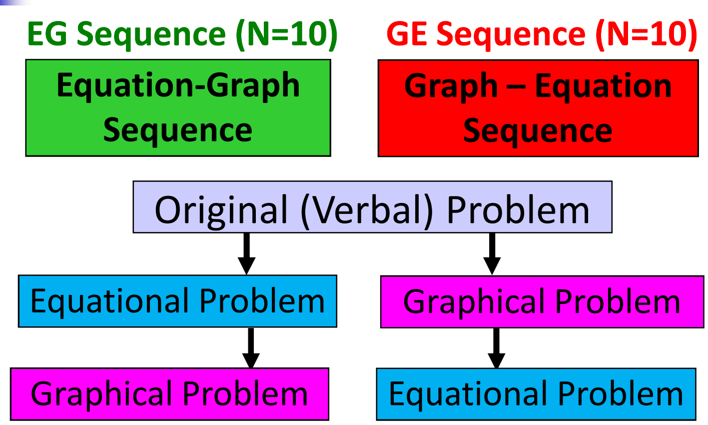
## Study 2: Research Design

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## Study 2: Data Analysis

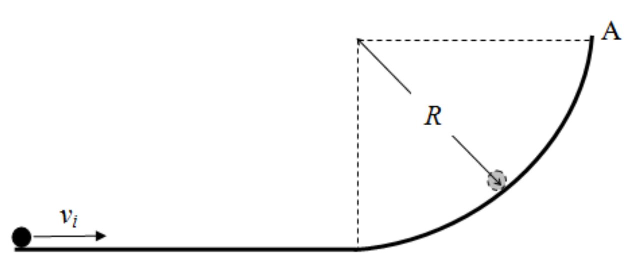
Phenomenographic Approach (Marton, 1986)

- Categorized students' difficulties
- Categorized hints provided by interviewer
- Inter-rater reliability ~ 0.8



## Example: Original Problem (Verbal)

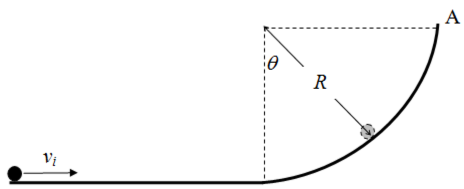
A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.

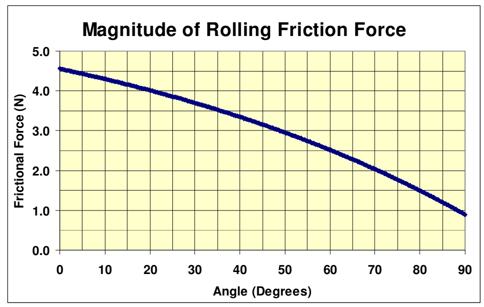



What is the launch speed of the hoop as it leaves the curve at point A?

# Example: Graphical Problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown.




What is the launch speed of the hoop as it leaves the curve at point A?

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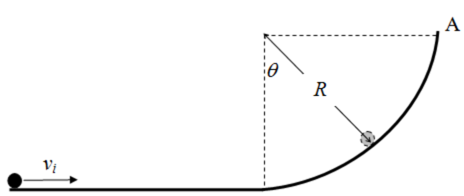
# Example: Equational Problem

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed vi of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle θ (radians) as per the equation shown.




$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

What is the launch speed of the hoop as it leaves the curve at point A?

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## Study 2: Results -- Difficulties

- GRAPH: difficulty processing information from graph
- FUNCTION: inappropriate interpretation of equation
- PRINCIPLE: inappropriate use of physical principles
- QUANTITY: incorrect use of physical quantities
- FORMULA: incorrect interpretation of formulae
- VALUE: Using incorrect value of physical quantities
- MATH: Inability to manipulate math processes
- CALCULATION: simple calculation errors

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## Study 2: Results -- Hints

- Implicit: Questions
  - Asking students to reflect on what physics knowledge and math processes are applicable
  - Facilitating students integrate math knowledge and processes to apply that knowledge to physics problems
- Explicit: Statements
  - Cuing students to refer to some information, prior knowledge
  - Enabling students to recall or apply physics or math knowledge

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## Study 2: Results -- Common Themes

- Case Reuse (Jonassen, 2006)
  - Tried to mimic the previous problems
    - Example: Finding potential energy for a spring by trying to find the spring constant.
- Graphical Interpretation
  - Instinctively tried to calculate the slope of graph
  - Several hints to recognize integral is area under graph
- Physical Interpretation of Math Procedures
  - Adequate knowledge of math procedures
  - Inability to apply these procedures in physics problems
  - Hints on reflecting on units of physical quantities effective

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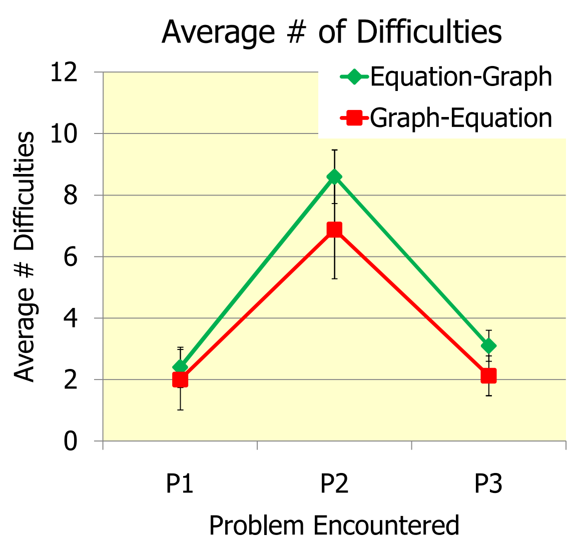
## Study 2: Results - Sequencing Effect

- Equation-Graph sequence may cause more difficulties to students than the Graph-Equation sequence*




* Not statistically significant

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## Study 2: Toy Model of Difficulty Contributions

$$D_{Total} = D_{\Delta R} + D_{\Delta C} + D_{\Delta O}$$

- $D_{Total}$ → Total # of Difficulties
- $D_{\Delta R}$ → Difficulties due to Change in Representation
- $D_{\Delta C}$ → Difficulties due to Change in Context
- $D_{\Delta O}$ → Difficulties due to all Other Changes

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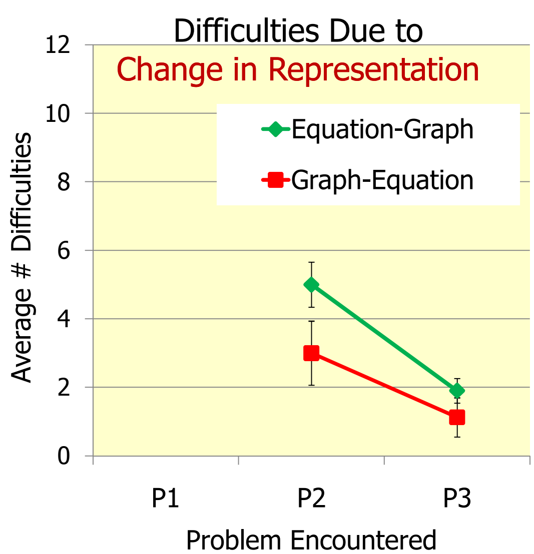
## Study 2: Results - Sequencing Effect

- Most Difficulties are due to change in Representation ($D_{\Delta R}$)
- Decline in $D_{\Delta R}$ in going from 2nd problem to 3rd problem regardless of sequence
- $D_{\Delta R}$ [Verbal → Equation] > $D_{\Delta R}$ [Verbal → Graph]*




* Not statistically significant

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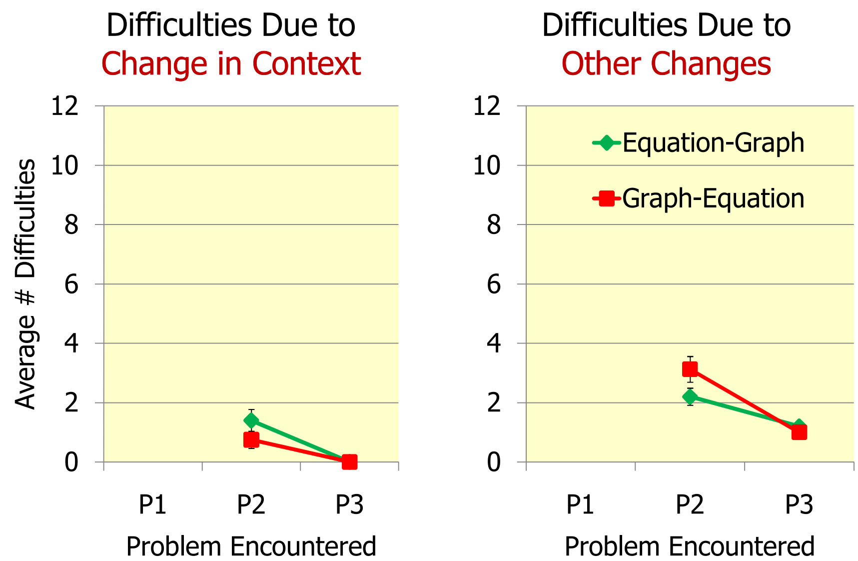
# Study 2: Results - Sequencing Effect

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# Study 2: Conclusions

RQ2.1: What kinds of difficulties do students encounter when solving problems in multiple representations?

- Students had difficulty interpreting physical meaning of mathematical processes.
  - Thus had difficulties solving problems in graphical and functional representations.

- When the context of the problem changed, could not relate the new problem to original problem.
  - Thus had difficulties identifying the principle and physical quantities needed to solve the new problem

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## Study 2: Conclusions

RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

- Verbal -> Graphical -> Equation sequence has fewer overall difficulties
  - Most of the observed difficulties are related to change in representation, rather than change in context.
  - Difficulties due to change in representation are fewer in the G-E sequence compared to E-G sequence.

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## SUMMARY

- Research on multiple representations in physics is a useful endeavor: much done, but much more can be done.
- Different representations offer different salient cues, and different levels of ambiguity to facilitate, block learning of different concepts.
- The sequence in which representations are presented can have important implications for learning and problem solving in physics.

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# Thank You

## For further information


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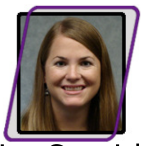

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