Representational Competence in Learning and Problem Solving in Physics

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Dong-Hai Nguyen

Graduate Students
What is Representational Competence?

“The ability to comprehend the equivalence of different modes of representation” (Sigel & Cocking, 1977)

“Comprehend Equivalence”
⇒
- Read out same info. from different representations
- Transfer learning from one representation to another
- Transform info from one representation to another
- Others...

“Modes of Representation”
⇒
- Physical vs. Virtual
- Verbal vs. Mathematical
- Graphical vs. Equational
- Macroscopic vs. Microscopic
- Others...
Ainsworth (2006) : In interacting with an MR a learner must understand...
- the form of each representation
- the relation between representation & domain
- how to construct new representations
- the connections between different MRs
  - difficult for learners and may inhibit learning from multiple representations

Hagevik, Beilfuss & Dickerson (2006) : Mastery of MRs leads to a deep understanding
- Experts can easily shift between MRs to solve problems, but novices have difficulties
- Qualitative reps can act as a “bridge” between abstract mathematical and linguistic reps.
- Computer-based MRs provide complementary information, constrain interpretations, and promote deep understanding (Ainsworth, 1999).
Incomplete Review of MRs in Phys. Ed.

- McDermott et al (1987)
  - Students difficulties with connecting graphs to physical concepts and the real world.

- Van Heuvelen and Zou (2001)
  - Qualitative representations (sketches, diagrams, bar charts) help learning of energy concepts.

- DeLeone & Gire (2005)
  - Non-equational representations were necessary but not sufficient for problem-solving success.

- Meltzer (2005)
  - Students’ perform differently on isomorphic physics problems in different representations

- Kohl & Finkelstein (2008)
  - Students performance depends upon the representation that they were provided or chose to use

Our Research Goals

Investigate how learners...

- construct knowledge as they interact with different representations?
  - **Study 1**: Learning using Physical vs. Virtual Representations

- transfer their learning from one representation to another?
  - **Study 2**: Solving Problems in Numerical vs. Graphical vs. Equational Representations
Study 1: Background

- Previous studies -- mixed results
  - Virtual outperform analogous physical experiments
    - Zacharia, Olympiou, & Papaevripidou, 2008
    - Finkelstein, et al., 2005
  - No difference in learning: physical vs. virtual
    - Klahr, Triona, & Williams, 2007
    - Zacharia & Constantinou, 2008

- Zacharia & Constantinou (2008)
  - More research is needed to describe how physical and virtual manipulatives should be integrated in a curriculum.

Study 1: Research Questions

- RQ1.1: Is there a difference in learning from physical and virtual representation as measured by students’... 
  - performance on a multiple choice test?
  - reasoning provided as they interact with the representation?

- RQ1.2: When students use both physical & virtual representation...
  - How does their learning from the two representations compare?
  - How does the temporal order of using the physical and virtual representations affect students’ learning?
  - What views do students express about data collected from physical and virtual representations?
Study 1: Research Context

- **CoMPASS Curriculum** (Puntambekar et al, 2003)
  - Concept Mapped Project-based Activity Scaffolding System
  - Originally designed for middle school students
  - Based on “Learning by Design” (Hmelo, 2000; Kolodner et al 2003)
  - Integrates: Hypertext + Activities (Physical/Virtual)

- **Pulley Unit**
  - Two-hour Laboratory

Study 1: CoMPASS Hypertext Concept Maps

- Concept in context
- Dynamic “fish eye” concept maps
- Links in body of text
Study 1: Physical & Virtual Representations

**Pulley Simulation**

**Study 1: Research Design**

PV Sequence (N=61)

Physical-Virtual Sequence

VP Sequence (N=71)

Virtual-Physical Sequence

Pre-Test

Brainstorming & CoMPASS

Physical Experiment

Virtual Experiment

Mid-Test

Virtual Experiment

Physical Experiment

Post-Test
Study 1: Sources of Data

- Pre/Mid/Post Tests
  - 13 Conceptual multiple choice questions
  - Cronbach’s $\alpha$ Reliability 0.7

- Worksheet Questions
  - Open-ended
  - Completed while doing activity
  - Asked description of trends in observed experimental data & explain why

Study 1: Overall Test Performance

<table>
<thead>
<tr>
<th>Effect</th>
<th>Comparison</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Score</td>
<td>Pre – Mid</td>
<td>&lt; .001</td>
</tr>
<tr>
<td></td>
<td>Mid – Post</td>
<td>&lt; .001</td>
</tr>
<tr>
<td>Score * Treatment</td>
<td>Pre – Mid</td>
<td>.19</td>
</tr>
<tr>
<td></td>
<td>Mid – Post</td>
<td>.001</td>
</tr>
</tbody>
</table>

Graph showing percentage mean score over time with pre, mid, and post data points. The graph compares scores between the physical and virtual conditions with a line for each condition.
Study 1: ‘Force’ Questions on Test

<table>
<thead>
<tr>
<th>Effect</th>
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<th>p-value</th>
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<tbody>
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<td>Pre – Mid</td>
<td>&lt;.001</td>
<td></td>
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<tr>
<td>Mid – Post</td>
<td>.19</td>
<td></td>
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</tbody>
</table>

Study 1: ‘Work’ Questions on Test

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<tr>
<th>Effect</th>
<th>Comparison</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre – Mid</td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>Mid – Post</td>
<td>&lt;.001</td>
<td></td>
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Study 1: ‘Force’ Questions on Test

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<th>Comparison</th>
<th>p-value</th>
</tr>
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<tbody>
<tr>
<td>Pre – Mid</td>
<td>&lt;.001</td>
<td></td>
</tr>
<tr>
<td>Mid – Post</td>
<td>.98</td>
<td></td>
</tr>
</tbody>
</table>

Study 1: ‘Work’ Questions on Test

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<tr>
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<th>Comparison</th>
<th>p-value</th>
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<td>&lt;.001</td>
<td></td>
</tr>
<tr>
<td>Mid – Post</td>
<td>&lt;.001</td>
<td></td>
</tr>
</tbody>
</table>
**Study 1: Why these Results?**

Two possible effects:

- Differential cue salience?  
  (Denton & Kruschke, 2006)

- Anomalous Data?  
  (Chinn & Brewer, 1993)

**Study 1: What Causes Differential Cue Salience?**

- Superiority / Noticing effect?  
  (Lindgren & Schwartz, 2009)
Study 1: Implication of Differential Cue Salience

Blocking? (Heckler, et al 2006)

<table>
<thead>
<tr>
<th>'Force' Cues</th>
<th>'Work' Cues</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Physical P</strong></td>
<td>High</td>
</tr>
<tr>
<td><strong>Virtual V</strong></td>
<td>High</td>
</tr>
<tr>
<td><strong>Virtual V</strong></td>
<td>High</td>
</tr>
<tr>
<td><strong>Physical P</strong></td>
<td>High</td>
</tr>
</tbody>
</table>

Salience is high in both: Learning from whichever cue is presented first: Primacy effect
Decreasing Salience: Blocking

Study 1: Anomalous Data

Anomalous data (Chinn & Brewer, 1993)

Students’ responses to anomalous data:
- Ignore
- Reject
- Exclude from the domain
- Hold in abeyance
- Reinterpret and retain
- Reinterpret and make peripheral changes
- Accept and change theory

In the right circumstances, can facilitate conceptual change
Study 1: What causes Anomalous data?

Anomalous data (Chinn & Brewer, 1993) due to...

- Prior knowledge
- Processing Strategy
- Characteristics of Data
  - Credibility
  - Ambiguity

Possible reasons...
- Measurement error
- Friction not being accounted for
- Others?

Study 1: An Example of Ambiguous & Unambiguous Data

<table>
<thead>
<tr>
<th>Type of Pulley System</th>
<th>Work value determined in PHYSICAL experiment</th>
<th>Work value measured in VIRTUAL experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Fixed</td>
<td>.49 J</td>
<td>.50 J</td>
</tr>
<tr>
<td>Single Movable</td>
<td>.52 J</td>
<td>.50 J</td>
</tr>
<tr>
<td>Single Compound</td>
<td>.48 J</td>
<td>.50 J</td>
</tr>
<tr>
<td>Double Compound</td>
<td>.54 J</td>
<td>.50 J</td>
</tr>
</tbody>
</table>
Study 1: Worksheet Q – Equivalence of ‘Work’ across Pulley Systems

Based on your data, when you changed the pulley setup, how did it affect the work required?

<table>
<thead>
<tr>
<th></th>
<th>Physical Stayed Same</th>
<th>Physical Changed Slightly</th>
<th>Virtual Changed Slightly</th>
<th>Virtual Stayed Same</th>
<th>Physical</th>
<th>Virtual</th>
<th>Virtual</th>
<th>Physical</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV Sequence</td>
<td>Physical: Higher Ambiguity</td>
<td>Virtual: Lower Ambiguity</td>
<td>VP Sequence</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

0% 20% 40% 60% 80% 100%

Study 1: Test Q – Equivalence of ‘Work’ across Pulley Systems

Alice is using pulley setup A, Brenda is using B, and Carl is using C. What can you tell about the work needed to lift the same load by each of them through the same height, if friction is not a factor?

A) Alice does most work.  B) Brenda does most work.  C) Carl does most work.  D) Work done by all three is the same.
Study 1: Test Q – Equivalence of ‘Work’ across Pulley Systems

How does the work done in different pulley systems to lift the same load the same height compare, if friction is not a factor?

**PRE to MID**

- PV Sequence
  - After completing Physical
  - PRE: Same
  - MID: Diff

- VP Sequence
  - After completing Virtual
  - PRE: Same
  - MID: Diff

**MID to POST**

- PV Sequence
  - After completing Virtual
  - MID: Same
  - POST: Diff

- VP Sequence
  - After completing Physical
  - MID: Diff
  - POST: Same
Study 1: What are the implications of differential ambiguity?

Differential ambiguity in data

⇒ Different response to data by learner

<table>
<thead>
<tr>
<th>Physical Data for ‘Work’</th>
<th>Virtual Data for ‘Work’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Higher ambiguity</td>
<td>Lower ambiguity</td>
</tr>
<tr>
<td>Less likely to change students’ conceptions</td>
<td>More likely to change students’ conceptions</td>
</tr>
</tbody>
</table>

Study 1*: What would students’ prefer?

We asked students about data collected from physical and virtual representations

- Which set of data is more useful in different hypothetical situations?
  - Different contexts (exam, rental store, missed lab)
  - Different concepts (force, work)
  - Different pulley systems (fixed, movable, compound)
- How are the data collected physical and virtual similar and different?

*101 future elementary teachers in a conceptual physics class did activities in PV and VP sequences in Activities Center
Study 1*: Sample Question

Q1) On an exam, your professor has asked you some questions about several pulley setups.

A) On the first question on the test, you have to decide whether a fixed or movable pulley requires the least effort force to lift the load.

Which experience in the Activity Center would better help you answer this question? (Check one)

- Experiment with real pulleys
- Computer simulation of pulleys
- Both are equally helpful

- Explain what led you to make the choice above.

Study 1*: What would students’ prefer?

Which set of manipulatives would you use to decide:

- On a test, whether a fixed or movable pulley requires less force?
- In a rental store, whether a fixed or movable pulley will better help you lift a bed?
Study 1*: Why?

Reasoning given by students

- Physical allows you see/feel
- Physical better fit
- Both offer helpful info
- Computer more accurate
- Computer easier

<table>
<thead>
<tr>
<th>Reasoning</th>
<th>Rental</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical allows you see/feel</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Physical better fit</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Both offer helpful info</td>
<td>10</td>
<td>30</td>
</tr>
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<td>Computer more accurate</td>
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<td>20</td>
</tr>
<tr>
<td>Computer easier</td>
<td>10</td>
<td>20</td>
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</table>

Number of Students

Students' preference depends most on context

- 22-38% changed answer when concept changed
- 20-30% changed answer when pulley setup changed
- 53-68% changed answer when context changed

Other factors remaining same, students most chose...

- Virtual for “Test” context
- Physical and Virtual for “Rental Store” context

Students’ reason that...

- Simulation data is free from certain types of errors
- Physical provides more kinesthetic experience
- Physical may be a better fit to a real life situation
**Study 1: Conclusions**

- **RQ1.1:** Is there a difference in learning from physical and virtual representations as measured by...
  - Some concepts (e.g. force) are better learned using physical, others (e.g. work) better using virtual representations.
  - Differential ambiguity, cue salience in representations facilitates learning of different concepts, so both representations needed.

- **RQ1.2:** When students use both physical & virtual representations...
  - Overall, if physical is used first, students continue to learn when virtual is used afterward, but if they use virtual first they do not appear to continue learning with the physical.
  - Students appear to prefer the virtual in “test” contexts, but both equally when in “real world” situations.

**Study 2: Background**

Questions addressed in recent research on MRs

- Does using MR help students learn concepts and learn to better solve problems?
- What instructional innovations help students use MRs while solving problems?
  - Do those innovations help them solve problems?
- How does the representation in which the problem is posed affect student performance and their decision to use another type of representation when solving the problem?
Multiple Representations (MRs) useful in solving physics problems

- Several studies addressing the benefits of using MRs in solving physics problems.
- Not as many studies on how students transfer their problem solving skills in physics across different MRs.

**Study 2: Research Questions**

**RQ2.1**: What kinds of difficulties do students encounter when solving problems in multiple representations?
- What scaffolding is helpful to facilitate learners to overcome these difficulties

**RQ2.2**: How do those difficulties change which the sequence in which these representations are presented?
Study 2: Theoretical Perspective

Zone of Proximal Development

(Vygotsky, 1978)

- Problem presented within students’ ZPD
- Hints provided to scaffold process of problem solving within ZPD.

Study 2: Research Context

- N=20 participants
- Engineering majors
- Enrolled in 1st semester calc-based physics
- Topics: Kinematics, Work-Energy
Study 2: Research Methodology

Teaching/Learning Interviews (Steffe et al., 2003)

- Four sessions: One after each class exam
- Each session: 60 minutes, video/audio taped
- Three problems per session
- Hints provided when students expressed difficulties

Study 2: Research Design

<table>
<thead>
<tr>
<th>EG Sequence (N=10)</th>
<th>GE Sequence (N=10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation-Graph Sequence</td>
<td>Graph – Equation Sequence</td>
</tr>
</tbody>
</table>

Original (Verbal) Problem

- Equational Problem
- Graphical Problem

Equational Problem
- Graphical Problem
- Equational Problem
Study 2: Data Analysis

Phenomenographic Approach (Marton, 1986)

- Categorized students’ difficulties
- Categorized hints provided by interviewer
- Inter-rater reliability ~ 0.8

Example: Original Problem (Verbal)

A hoop radius \( r = 1 \text{ cm} \) and mass \( m = 2 \text{ kg} \) is rolling at an initial speed \( v_i \) of 10 m/s along a track as shown. It hits a curved section (radius \( R = 2.0 \text{ m} \)) and is launched vertically at point A.

What is the launch speed of the hoop as it leaves the curve at point A?
Example: Graphical Problem

A sphere radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed $v_i$ of 5 m/s along a track as shown. It hits a curved section (radius $R = 1.0$ m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle $\theta$ as per the graph shown.

What is the launch speed of the hoop as it leaves the curve at point A?

Example: Equational Problem

A sphere radius $r = 1$ cm and mass $m = 2$ kg is rolling at an initial speed $v_i$ of 5 m/s along a track as shown. It hits a curved section (radius $R = 1.0$ m) and is launched vertically at point A. The rolling friction on the straight section is negligible. The magnitude of the rolling friction force acting on the sphere varies as angle $\theta$ (radians) as per the equation shown.

$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

What is the launch speed of the hoop as it leaves the curve at point A?
**Study 2: Results -- Difficulties**

- GRAPH: difficulty processing information from graph
- FUNCTION: inappropriate interpretation of equation
- PRINCIPLE: inappropriate use of physical principles
- QUANTITY: incorrect use of physical quantities
- FORMULA: incorrect interpretation of formulae
- VALUE: Using incorrect value of physical quantities
- MATH: Inability to manipulate math processes
- CALCULATION: simple calculation errors

**Study 2: Results -- Hints**

- **Implicit: Questions**
  - Asking students to reflect on what physics knowledge and math processes are applicable
  - Facilitating students integrate math knowledge and processes to apply that knowledge to physics problems

- **Explicit: Statements**
  - Cuing students to refer to some information, prior knowledge
  - Enabling students to recall or apply physics or math knowledge
Study 2: Results -- Common Themes

- Case Reuse (Jonassen, 2006)
  - Tried to mimic the previous problems
    - Example: Finding potential energy for a spring by trying to find the spring constant.

- Graphical Interpretation
  - Instinctively tried to calculate the slope of graph
  - Several hints to recognize integral is area under graph

- Physical Interpretation of Math Procedures
  - Adequate knowledge of math procedures
  - Inability to apply these procedures in physics problems
  - Hints on reflecting on units of physical quantities effective

Study 2: Results - Sequencing Effect

- Equation-Graph sequence may cause more difficulties to students than the Graph-Equation sequence*

* Not statistically significant
**Study 2: Toy Model of Difficulty Contributions**

\[ D_{\text{Total}} = D_{\Delta R} + D_{\Delta C} + D_{\Delta O} \]

- Total # of Difficulties
- Difficulties due to Change in Representation
- Difficulties due to Change in Context
- Difficulties due to all Other Changes

**Study 2: Results - Sequencing Effect**

- Most Difficulties are due to change in Representation \( (D_{\Delta R}) \)
- Decline in \( D_{\Delta R} \) in going from 2\(^{nd}\) problem to 3\(^{rd}\) problem regardless of sequence
- \( D_{\Delta R} [\text{Verbal} \rightarrow \text{Equation}] > D_{\Delta R} [\text{Verbal} \rightarrow \text{Graph}] \)

* Not statistically significant
**Study 2: Results - Sequencing Effect**

**Difficulties Due to Change in Context**

<table>
<thead>
<tr>
<th>Problem Encountered</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average # Difficulties</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**Difficulties Due to Other Changes**

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**Study 2: Conclusions**

**RQ2.1:** What kinds of difficulties do students encounter when solving problems in multiple representations?

- Students had difficulty interpreting physical meaning of mathematical processes.
  - Thus had difficulties solving problems in graphical and functional representations.

- When the context of the problem changed, could not relate the new problem to original problem.
  - Thus had difficulties identifying the principle and physical quantities needed to solve the new problem.
Study 2: Conclusions

RQ2.2: How do those difficulties change which the sequence in which these representations are presented?

- Verbal -> Graphical -> Equation sequence has fewer overall difficulties
  - Most of the observed difficulties are related to change in representation, rather than change in context.
  - Difficulties due to change in representation are fewer in the G-E sequence compared to E-G sequence.

SUMMARY

- Research on multiple representations in physics is a useful endeavor: much done, but much more can be done.
- Different representations offer different salient cues, and different levels of ambiguity to facilitate, block learning of different concepts.
- The sequence in which representations are presented can have important implications for learning and problem solving in physics.
Thank You
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