Tutorials to Facilitate Students' Learning of Integration in Physics

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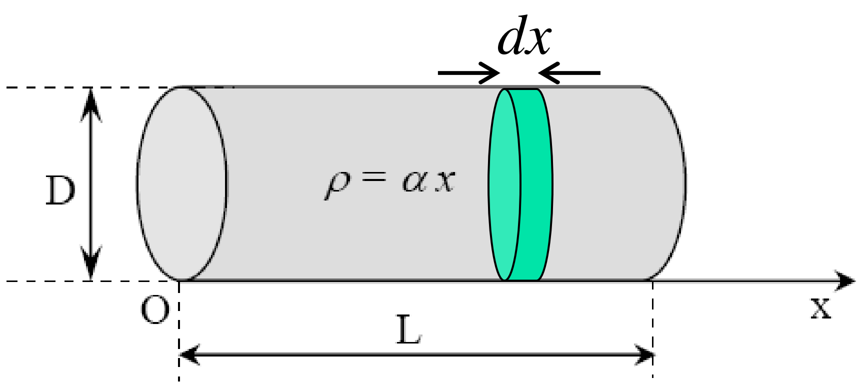
Review of Previous Study

- Investigate students' difficulties with integration in Electricity and Magnetism
- Major difficulties
  - Set up the expression for the infinitesimal quantity
  - Accumulate the infinitesimal quantities

Students Difficulties with Integration

- Set up the expression for the infinitesimal term
  - Students usually wrote $\int f(x)$ instead of $\int f(x)\,dx$
  - Example: Find the resistance of a conductor whose resistivity is given as per the equation $\rho(x) = \alpha x$



Correct solution:

$$dR = \frac{\rho(x)\,dx}{A} = \frac{4\alpha x\,dx}{\pi D^2}$$

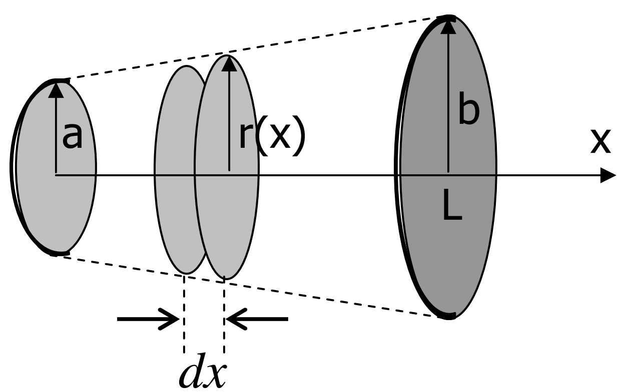
$$\Rightarrow R = \int dR = \int_0^L \frac{4\alpha x\,dx}{\pi D^2} = \frac{2\alpha L^2}{\pi D^2}$$

Common error:

$$dR = \frac{\rho(x)L}{A} = \frac{4\alpha x\,L}{\pi D^2} \quad \Rightarrow \quad R = \int dR = \int_0^L \frac{4\alpha x\,L}{\pi D^2}dx = \frac{2\alpha L^3}{\pi D^2}$$

Students Difficulties with Integration

- Incorrect accumulation of infinitesimal quantities
  - Students did not attend to how the infinitesimal quantities should be added up
  - Example: Find the capacitance of the capacitor



$$dC = \varepsilon_0 \frac{A}{dx} = \varepsilon_0 \frac{\pi\left[r(x)\right]^2}{dx}$$

Correct: $\frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^L \frac{dx}{\varepsilon_0 \pi\left[r(x)\right]^2}$

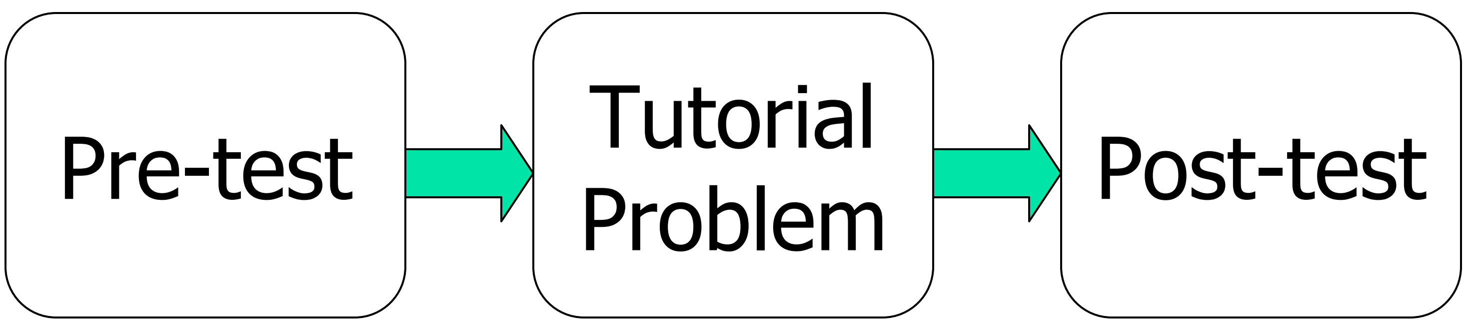
Incorrect: $C_{eq} = \int dC = \int_o^L \varepsilon_0 \frac{\pi\left[r(x)\right]^2}{dx}$

The tutorials

- This study: Tutorial to facilitate students' problem solving with integration in E&M
- The tutorials were designed to help students learn to
  - set up correct expression for infinitesimal quantity
    - understand the term "dx" in the integral
  - attend to how the infinitesimal quantities are added up

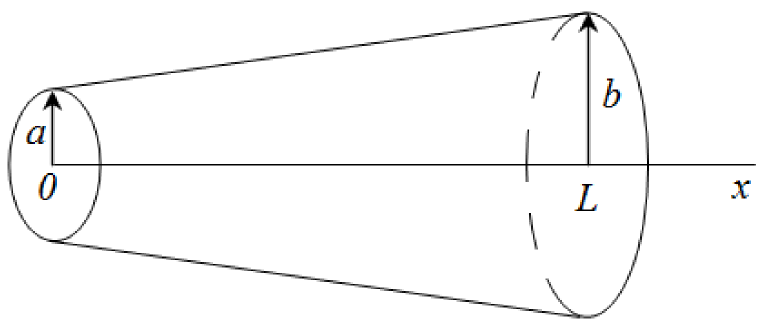
Experimental Design

- Tutorial done during problem solving session of the class (~60 mins)
- 200+ students divided into 6 sessions
- 3 sessions served as control group, others as treatment group



The Pre- and Post-test Problems

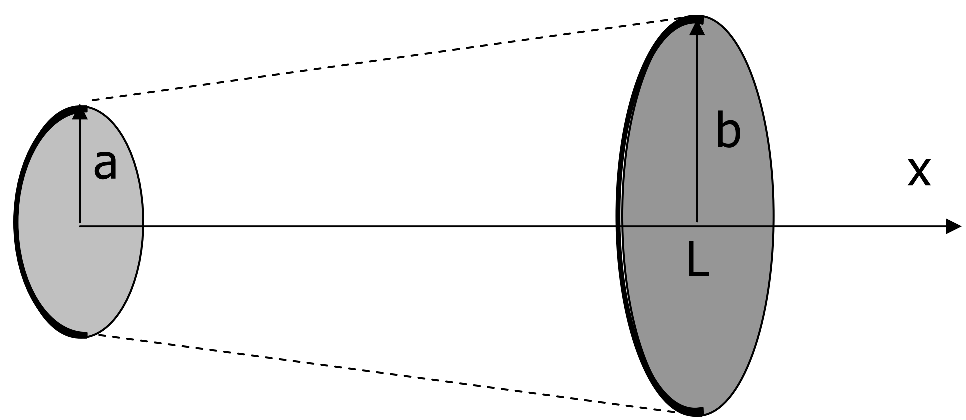
Consider a wire of length L in the shape of a truncated cone. The radius of the wire varies with distance x from the narrow end according to $r = a + \frac{b-a}{L}x$, where $0 < x < L$. Derive an expression for the resistance of this wire in terms of its length L, radius a, radius b, and resistivity ρ.



The Tutorial Problem

Consider a capacitor of material, permittivity ε. The capacitor consists of two circular plates of radii *a* and *b* placed at a distance *L* apart.

Derive an expression for the capacitance of this capacitor in terms of its length *L*, radius *a*, radius *b*, and permittivity ε.



The Solution to the Tutorial Problem

The control group received the written solution to the tutorial problem

Imagine that there are several fictitious plates which are a distance dx apart in the region between the two plates of the capacitor. The given capacitor is now a ***series*** combination of several capacitors made by the fictitious plates. If dx is small enough, then the radii of two adjacent plates are almost equal, so the capacitance of the capacitor made by these two adjacent plates is:

$$dC = \varepsilon_0 \frac{A}{dx} = \varepsilon_0 \frac{\pi\left[r(x)\right]^2}{dx}$$

At $x = 0$, $r = a$, and at $x = L$, $r = b$, so the expression of $r(x)$ is: $r(x) = a + \frac{b-a}{L}x$

Then $dC = \varepsilon_0 \frac{\pi\left[a + \frac{b-a}{L}x\right]^2}{dx}$

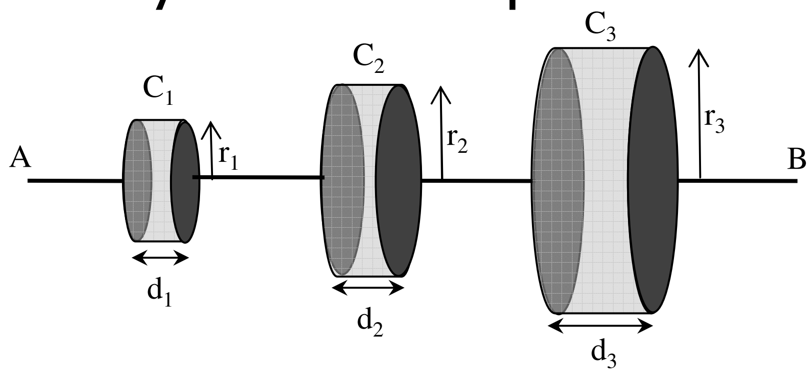
The equivalent capacitance of all fictitious capacitors from $x = 0$ to $x = L$ is

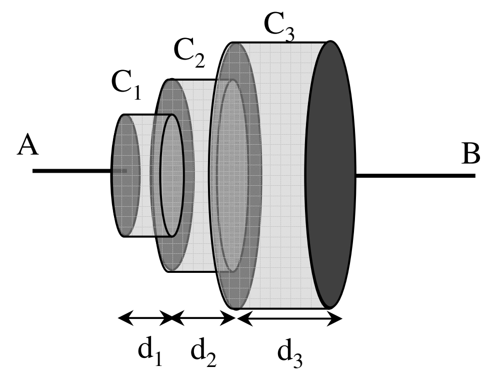
$$\frac{1}{C} = \int \frac{1}{dC} = \int_0^L \frac{dx}{\varepsilon_0 \pi\left[a + \frac{b-a}{L}x\right]^2} = \frac{1}{\varepsilon_0 \pi} \frac{L}{b-a}\left(-\frac{1}{a + \frac{b-a}{L}x}\right)\Bigg|_0^L = \frac{L}{\varepsilon_0 \pi ab}$$

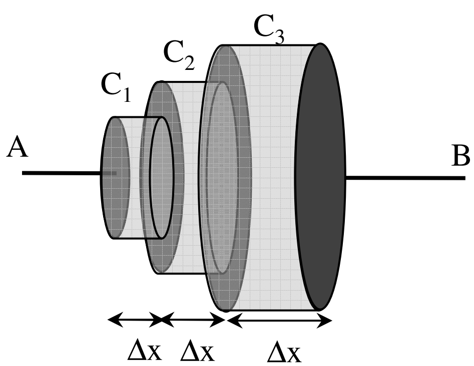
So the capacitance of the given capacitor is: $\boxed{C = \frac{\varepsilon_0 \pi ab}{L}}$

The Treatment Problems

Find the equivalent capacitance of the following systems of capacitors







N capacitors, plates are Δx apart, radius of the i-th plate: $r_i = a + bx_i$

Infinite number of capacitors, plates are dx apart, radius of the plate at position x is $r(x) = a + bx$

Results of Fisher's Exact Test

Pre-test

	Correct	Incorrect	**Total**
Control	8	97	**105**
Treatment	8	98	**106**
Total	**16**	**195**	**211**

p = 1.00

No significant difference between the two groups

Post-test

	Correct	Incorrect	**Total**
Control	33	72	**105**
Treatment	49	57	**106**
Total	**82**	**129**	**211**

p = 0.03

The treatment group outperformed the control group

Conclusion

- Our treatment problems helped students learn to set up an integral better than the written solution did.
- Scaffolding problems are better than examples and written solutions in facilitating students' learning about integration in physics problems.

Thank you

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