

## Energy Diagrams II

### Goal

- To gain a correct understanding of potential energy diagrams.
- To create links between a potential energy setting and appropriate physical systems.

### Introduction

We will continue to explore potential energy diagrams for mechanical systems. In this tutorial, we will investigate situations using a more quantitative approach. We shall ignore friction and gravity, and assume that energy is conserved.

### A. One-Dimension Systems

Two hollow shells are made with porous metal sheets. The shell on the left is grounded and the one on the right is set at 5 volts as shown in Figure 1. A small bead (mass = 1g) is positively charged with  $+e$ , where  $-e$  is the charge on an electron. In region I, the bead is moving towards the right with a kinetic energy  $K_0 = 6.0$  eV. The holes of the metal sheet are large enough so that the bead can freely pass through.

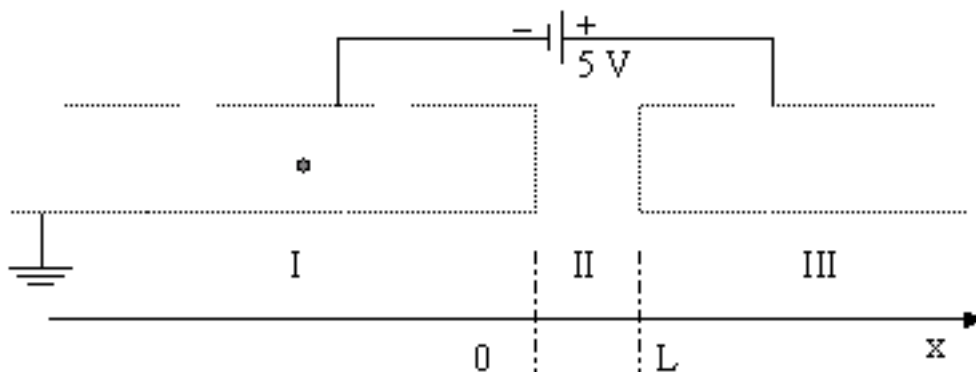
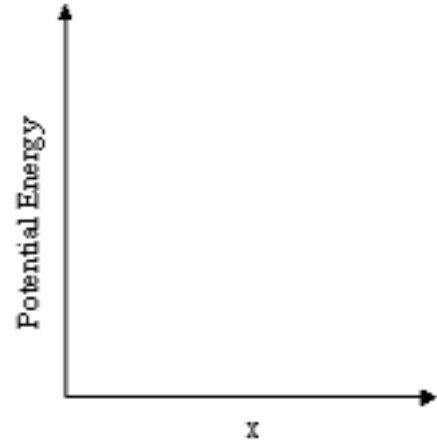
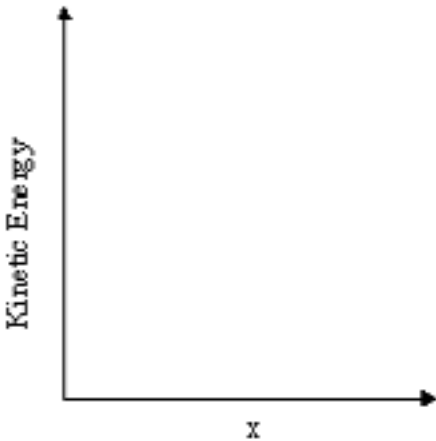


Figure 1: Diagram of the setup for a one-dimensional system with two hollow shells and a charged bead.

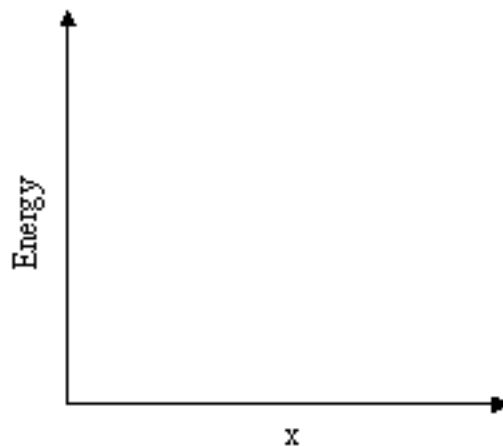
A-1. What is the total energy of the bead at  $x = 0$  and at  $x = L$ ? Why?

A-2. On the axes provided, sketch a graph of the kinetic energy and a graph of the potential energy of the bead as a function of  $x$ .



A-3. On your graph of the potential energy indicate the total energy of the bead. Explain your reasoning.

A-4. Suppose the initial kinetic energy of the bead is 3.0 eV instead of 6.0 eV. Sketch a new graph of the potential energy and the total energy of the bead in all three regions. Explain your reasoning.



Now suppose the system is changed to the structure shown in Figure 2 below. The bead is initially moving left with  $K = 6.0 \text{ eV}$ .

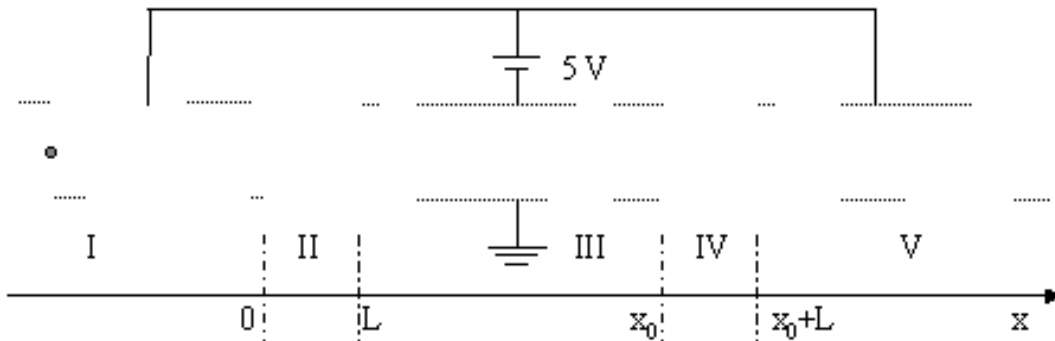
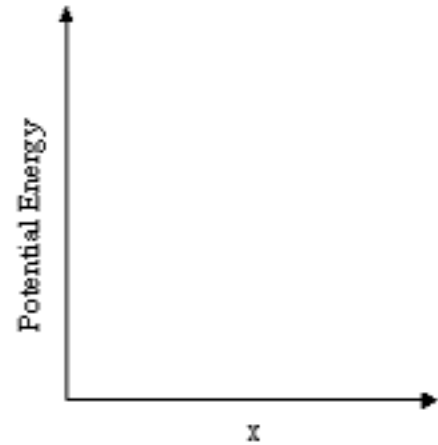
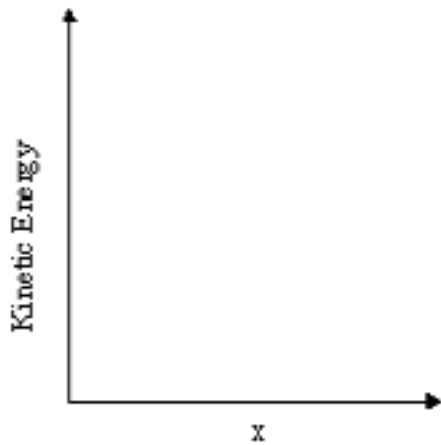


Figure 2: Diagram of the setup for a one-dimensional system with three hollow shells and a charged bead.

- A-5. For this new system, sketch the kinetic energy and the potential energy of the bead as a function of  $x$ . Include the total energy on your potential energy diagram and explain your reasoning.



- A-6. For region II, find an algebraic expression to represent the kinetic energy in terms of  $x$ . Explain how you arrived at your answer.

## B. One-dimensional Bound System

Now consider the oscillator shown in Figure 3. The system is available for viewing in the lab. The largest possible displacement of the glider is  $A$ . The glider has a mass  $m_0$  and a spring constant  $k_s$ .

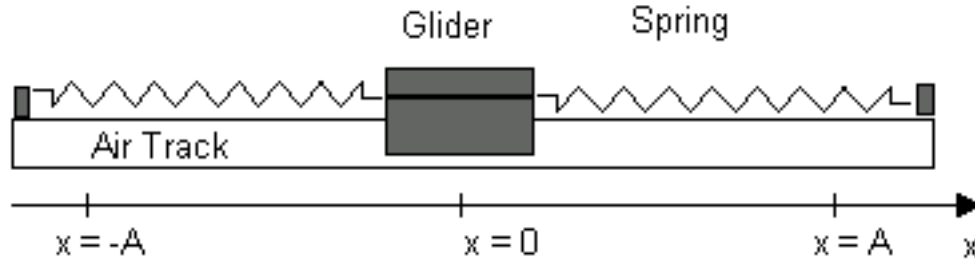


Figure 3: Diagram of the glider and springs set up.

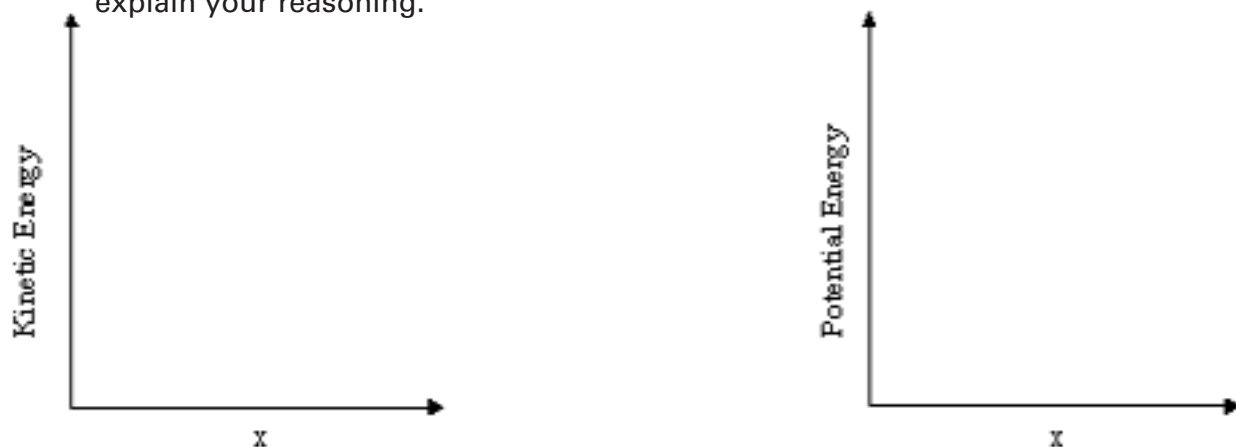
B-1. Assume the glider is oscillating. Describe, in words, the change in kinetic and potential energies as it moves through one full cycle.

B-2. Make a qualitative sketch of the kinetic and potential energies versus  $x$ .



B-3. Recall that the force of a spring is  $F = -kx$  where  $x$  is the displacement measured from the equilibrium position and that the potential energy is  $U = \int_{x_1}^{x_2} \mathbf{F}(x) \cdot d\mathbf{x}$ . What are the analytic forms for kinetic and potential energy?

- B-4. Using these results, sketch the kinetic energy and the potential energy of the bead as a function of  $x$ . Include the total energy on your potential energy diagram and explain your reasoning.



- B-5. Find an expression to represent the kinetic energy in terms of  $x$ . Explain how you arrived at your answer and show that your result is consistent with the potential energy diagram you plotted.

Suppose we remove the spring and attach two springy bumpers at both ends of the track to produce elastic collisions. The glider has a mass  $m_0$  and the spring constant of the springy bumpers is  $k_b$ . Initially, the glider is moving to the right with  $K = K_0$ .

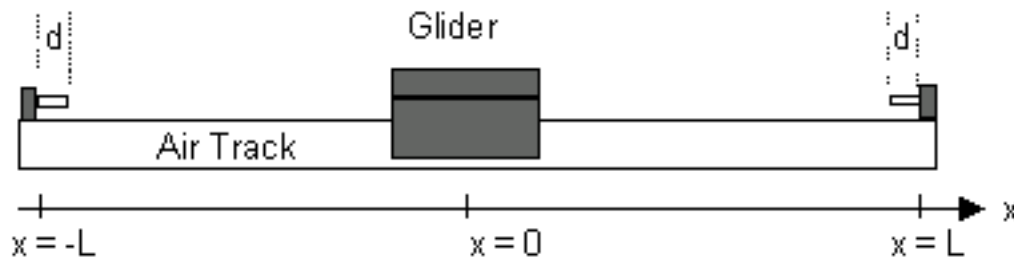
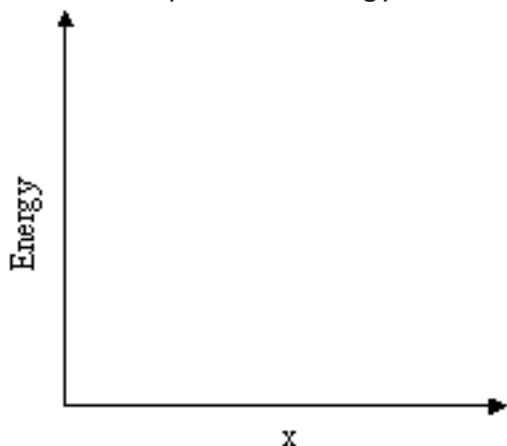


Figure 4: Diagram of the glider with springy bumpers instead of springs.

- B-6. Sketch the forces exerted on the glider as a function of  $x$ .

B-7. In the space below, sketch the potential energy and the total energy of the glider.



B-8. For the above case, find a general expression to represent the relationship between potential energy, force and position (hint: use an integral). Show that your result is consistent with the potential energy diagram you just sketched.

### C. Measurements with One-Dimension Systems

Now you will complete some measurements with the one-dimensional system that you considered in part B. The data will be collected using a force probe, a motion detector, and a computer data acquisition program. The motion detector uses a form of sonar to measure the distance from it to the nearest object in front of it. For this reason, a reflector or flag on the cart is often needed to increase the size of the detected object. The force probe generates a voltage signal proportional to the pull or push exerted on it. Both signals are received and processed by ScienceWorkshop.

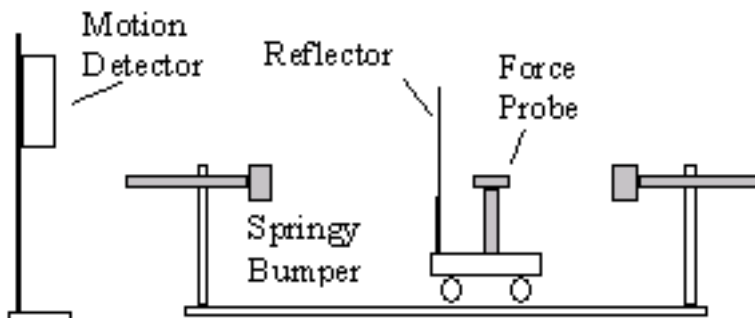
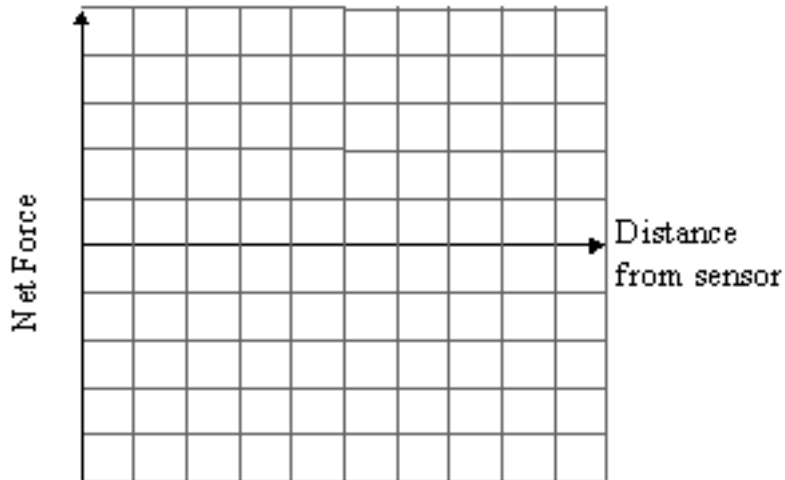


Figure 5: Diagram of the cart and springy bumpers ready for data collection.

C-1. First, predict and sketch how a graph of the net force felt by the cart will look as a function of the cart's position.

Now open the *PED-2* file on the computer. Click on the *Record* button and observe how the graphs on the screen change as you move the cart up and down the track. If your graphs look exceedingly spiky, it is possible the equipment is out of line. Check that the motion detector and the reflector are aligned, and keep the motion detector at least 40 cm away from the nearest end of the track.

C-2. You are now ready to measure the net force the cart feels as it bounces back and forth between the two bumpers. Press the *Record* button and give the cart an initial push so that it hits each bumper once. Sketch the resulting force-distance graph on the grid below.



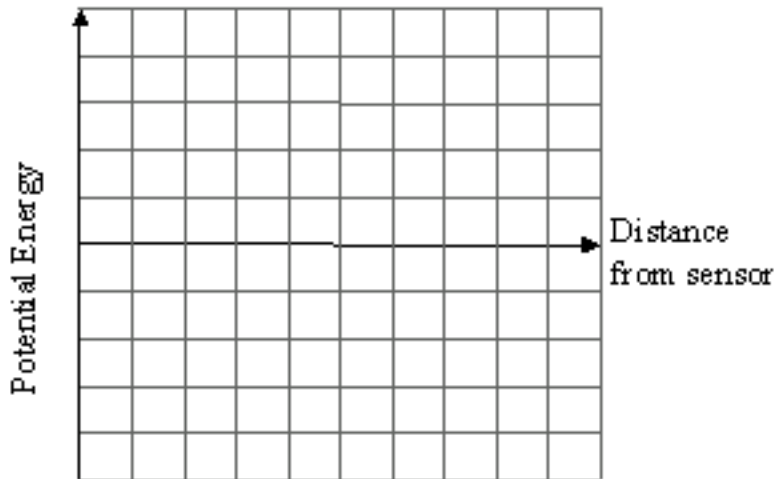
C-3. Compare the results with your prediction and resolve any discrepancies.

The potential energy,  $U(x)$ , is given by an integral from a reference point (taken here to be  $x = 0$  at the sensor)

$$U(\mathbf{x}) = -\int_0^{\mathbf{x}} \vec{F} \cdot d\vec{r} = -\int_0^{\mathbf{x}} F(x') dx'$$

where the integral on the right is for the one-dimensional case.

C-4. Using your  $F$  vs.  $x$  graph as a reference, sketch the qualitative shape of the potential energy vs.  $x$  graph for the cart on the grid below. Explain how you arrived at your answer.



In the following experiments, you will use the computer and sensors to record the motion of the cart, measuring simultaneously its position and the force it experiences when connected to two springs.

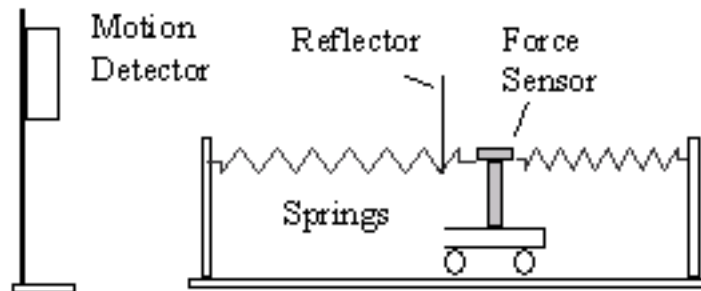
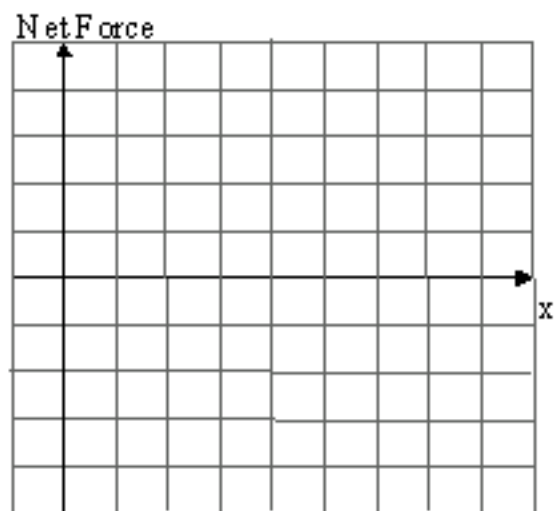


Figure 6: Diagram of the cart and springs ready for data collection.



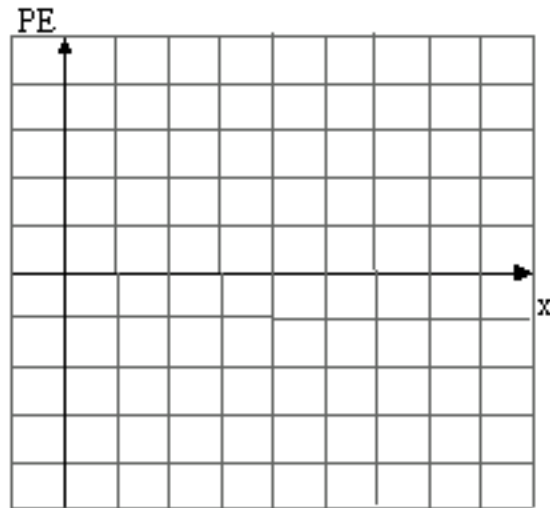
C-5. First predict and sketch a graph of the net force on the cart at different positions.

C-6. Now measure the force on the cart and sketch the resulting force-position graph from the computer on the grid below.



C-7. Does the force-position data match your predictions? Resolve any discrepancies.

- C-8. Using your  $F$  vs.  $x$  graph as a reference, sketch the qualitative shape of the potential energy vs.  $x$  graph for the cart on the grid below. How did you determine the shape of this graph?



#### D. Summary

- D-1. Write a summary of the way you can determine motion when given a potential energy diagram. Include in the summary, how to determine
- changes in speed as a function of potential energy,
  - the location at which a classical object turns around, and
  - regions in which classical objects cannot go.

- D-2. Write a paragraph describing the relationship between forces and kinetic and potential energy diagrams.

## E. Questions

- E-1. An object is moving in a classical system represented by the potential energy diagram shown in Figure 7. Ignore all friction and assume total energy is conserved. Sketch a graph of the speed of the object with respect to position.

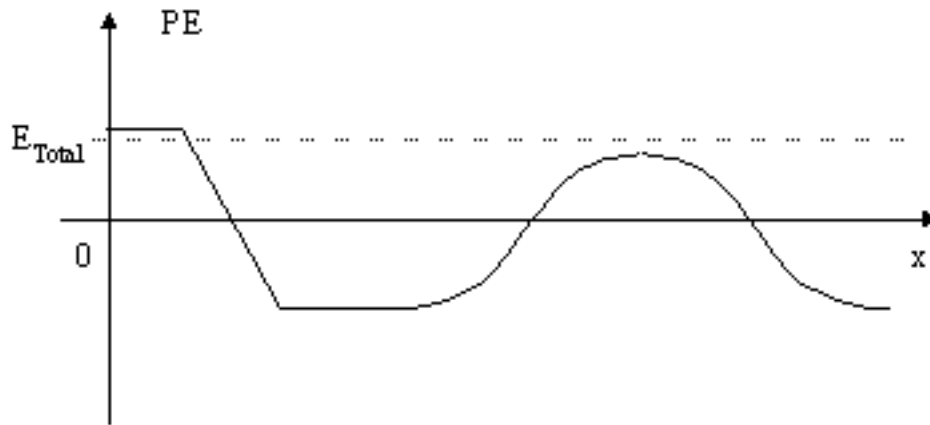


Figure 7: Potential Energy Diagram for question E-1.

- E-2. An object moving in a system has the potential energy diagram shown in Figure 8. Describe what physical system can give this energy diagram.

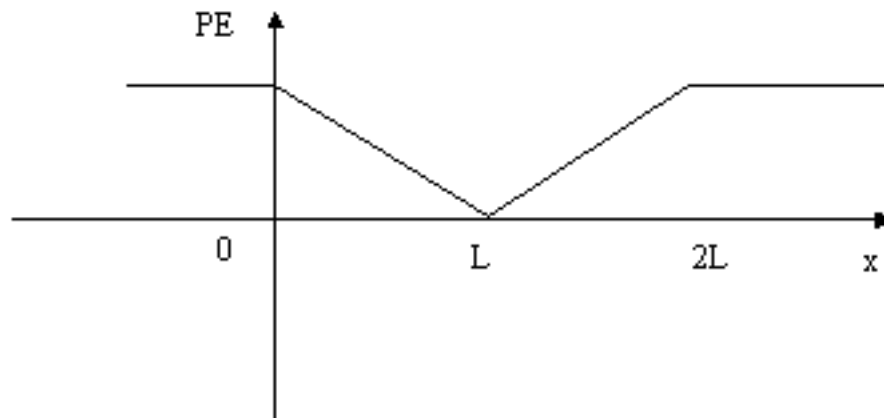


Figure 8: Potential Energy Diagram for question E-2.