ACTIVITY 12

It Was Probably Heisenberg

Goal
We look more carefully at the uncertainties in position and momentum. We will learn how the uncertainty in one variable is related to the other. Finally, we will speculate about the value of the Heisenberg Compensator in the StarTrekTransporter.

In Activity 11 we saw that the uncertainty in momentum at one time affects our uncertainty in position at a later time. Now, we wish to investigate the relation between the two variables at any one time.

To begin let's use the *Wave Packet Explorer* to create wave functions with different uncertainties in momentum. Your instructor will select a different wave function from Figure 12-1 for each group to create.

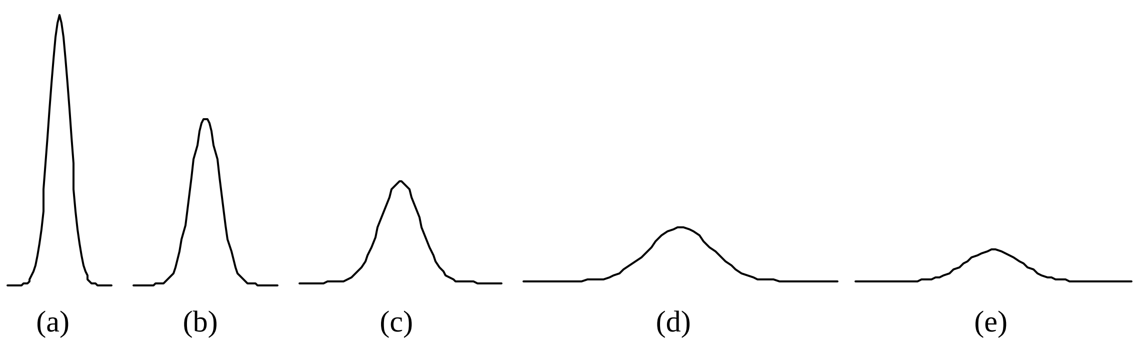

Figure 12-1: Wave functions with different uncertainties in position.

? Compare the result of all groups. Which one needed to include the largest number of momenta?

? the smallest number of momenta?

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? How is the uncertainty in position related to the uncertainty in momentum?

This exercise indicates that as one of the uncertainties increases the other decreases. This conclusion is built into the wave nature of matter. It does not depend on our measurement instruments. (We have not discussed measurement here — only creating wave functions.)

Heisenberg's Uncertainty Principle

The mutual dependence of the uncertainty in position and the uncertainty in momentum was first stated by Werner Heisenberg. His statement is known as *Heisenberg's Uncertainty Principle*. It says that the uncertainty in position and the uncertainty in momentum are closely related. If one decreases, the other increases by the same factor. Mathematically, the Uncertainty Principle says:

(Uncertainty in position) x (Uncertainty in momentum) = constant

Or, if you like symbols for uncertainty in position (Δx) and uncertainty in momentum (Δp).

$$\Delta x \cdot \Delta p = \text{constant}$$

The constant is Planck's constant (*h*) divided by 2π:

This principle states that we can never know both the exact position and the exact momentum of an electron at the same time. The best we could do is that the uncertainties are related by this equation. We would get these results only with perfect measuring instruments. We can always do worse. For this reason, Heisenberg's Uncertainty Principle is usually stated as an inequality:

(Uncertainty in position) x (Uncertainty in momentum) is at least $\frac{h}{2p}$.

$$\Delta x \cdot \Delta p \geq \frac{h}{2p}$$

? For example, suppose we establish the position of an electron precisely to within a tenth of a nanometer (0.0000000001 m). Using D$x = 10^{-10}$ m, what would be the <u>minimum</u> uncertainty in the electron's momentum (Dp)? (The value of Planck's constant is h = 6.63´ 10^{-34} J×s)

? With this uncertainty in momentum, what would be the corresponding uncertainty in the *speed* of the electron? (The electron's *mass* is 9.11×10^{-31} kg.) Recall that momentum = mass x velocity.

? A reasonable speed for an electron might be around 10^6 m/s. Is the uncertainty in speed that you calculated significant when compared to this speed? Explain.

The Heisenberg Uncertainty Principle is applied to all types of matter. One of the "controversies" in the early Star Trek series involved these uncertainties. Some Trekkies claimed that the Uncertainty Principle means that transporters would not be feasible. The uncertainties meant that the transporter would never be able to measure the location of the matter in people well enough. So they could not be beamed elsewhere.

Maybe these uncertainties are so small that they are unimportant in the transporter. Suppose a gnat is flying around in the transporter room. When it flies over the transporter pad, we measure its position and momentum. Then try to transport it to a location near somebody's compost pile. We measure the gnat's position only fairly accurately. The uncertainty of its position is within one millimeter (10^{-3} m). According to Heisenberg's Uncertainty Principle, what would be the *minimum* uncertainty in the gnat's momentum (Δp)?

So, the gnat's uncertainty position and its momentum are rather small. We could conclude that the Uncertainty Principle is not a problem. But, the transporter does not "work" by beaming whole objects. Instead it measures the locations of all atoms (maybe even all protons, neutrons, and electrons). Then, it disassembles the object and reassembles it elsewhere. Look at the uncertainty calculation for an electron. How likely is it that the gnat can even be reassembled intact? Explain.

? How would the Uncertainty Principle affect our ability to transport objects even larger than gnats? (They will have many more atoms.)

The Star Trek writers worked around this problem with a component they called a "Heisenberg compensator." This device somehow allows precise measurement of both position and momentum. They admit that they don't know how such a device would work. However, they recognize that it is needed to overcome these limitations of quantum physics. Its failure also makes for interesting stories. It should be noted that the Star Trek writers have also been insightful and creative enough to invent "subspace." This spatial continuum is hidden within our own, familiar three-dimensional space. It allows the transporter to "beam" people and objects instantaneously from one site to another since, in subspace, travel is not restricted to speeds below the speed of light. Fortunately, fiction is not restricted by the concepts of physics.

Today we lack the technology to construct "Heisenberg compensators." But, we cannot rule out the possibility that something similar may one day be developed. Talk of transporters may sound fantastic, but some current research indicates that "quantum teleportation" (as it is known to scientists) may actually be possible with individual atoms. Researchers at IBM have shown that teleportation of one atom is theoretically possible, but only if the original is destroyed. One atom is far from a person (or even a gnat) and many problems are involved in scaling up. But, it is fun to think about it.

Perhaps the most important aspect of the Uncertainty Principle is its philosophical implications. It states that humans cannot know everything about an object. Even if we try to imagine the "perfect" measuring instruments, we cannot determine the exact position and exact momentum simultaneously. Such a device can never be built. Thus, the Uncertainty Principle places limitations on humankind's knowledge of everything.

This limitation on our knowledge is inherent in nature. We observe the spectra of atoms. To explain them we conclude that matter behaves as waves. If this description is effective, it must describe individual electrons. This description leads to the Heisenberg Uncertainty Principle. Then, we find we cannot know everything precisely, even if we had perfect measuring instruments.

This lack of ability to know is a major departure from the physics of Newton. In 1787 Pierre Simon LaPlace, a mathematician, considered Newton's Laws carefully. If he knew the initial position and velocity, and the forces on an object, he could measure these variables for every object in the universe. Then suppose he could calculate really fast. He could know all of the future because he knows Newton's Laws. The only thing stopping him is good measurements and slow computers.

Quantum mechanics (particularly the Heisenberg Uncertainty Principle) says, “not true.” Even with perfect measuring instruments we cannot know enough. The nature of matter, not of measurement limits our knowledge.

Homework

12-1. Suppose we determine the position of an electron to within 10 nanometers (10×10^{-9} m). What will be the best possible measurement of its momentum?

12-2a. Suppose you are able to determine the speed of a 100 kg pole vaulter to within .04 m/s. Within what accuracy would you be able to determine her position?

12-2b. Based on your answer above, do you think that we need to worry about the Uncertainty Principle in measurements that we make in our everyday lives? Explain your answer.

Nanotechnology is the science of building tiny machines by constructing them atom-by-atom. This construction concept is rapidly generating interest among scientists and engineers. Claims and promises are being made that this technology will revolutionize our lives. However, we need to think about what happens when the “machines” become extremely small. Perhaps the Uncertainty Principle will make nanotechnology difficult or even impossible to use. Explain why nanotechnologists may need to worry about the limits imposed by the Uncertainty Principle.