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WAVES of matter

Visual Quantum Mechanics

ACTIVITY 1 Exploring Light from Gases

Goal

We will view the colors of light which are emitted by different gases. From these patterns of light we gain information about energies in atoms.

The importance of light as a form of energy is apparent in our everyday lives. In addition to enabling us to see and plants to grow the energy of light has provided important clues in understanding the structure of matter. By looking carefully at light scientists have made surprising (at the time) discoveries about atoms and molecules. The explanations of these observations have led to a new level of understanding about all types of matter. This understanding has provided an entirely different view of atoms, molecules and, even, what we can know.

In this unit we will explore and learn about some of the important discoveries in the 20th Century. These observations and theories have helped scientists reach their present level of knowledge about very small objects such as electrons, atoms and molecules. Our investigations will begin with observations of light coming from atoms.

We begin by considering objects that emit their own light. Think about how you could get an object to emit light. What would you do to get light from objects?

If you look at your answer and all of your classmates, you will probably have several ways that light is produced. All involve providing some type of energy to the object. In everyday life providing electricity or heat is the most common way to create light. Sometimes light from electricity produces a large amount of heat. For example, a light bulb or a toaster produces both light and heat. In other cases, such as a fluorescent tube, the amount of heat is rather small. The relatively large and small amounts of heat indicate different processes for creating light. We will concentrate on processes similar to those occurring *inside* a fluorescent tube.

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Gas tubes contain atoms of one element. When supplied with electrical energy, the atoms emit light. You will use a spectroscope to break this light into its component colors. The display of colors is called a spectrum.

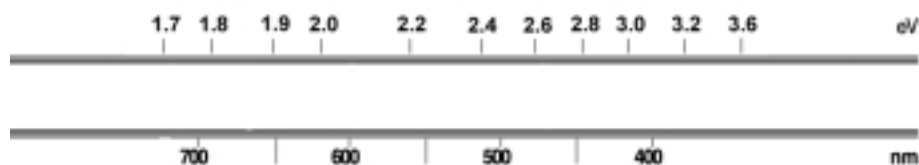
Caution: (1) Some power supplies for gas tubes have exposed metal contacts. Because the gas lamp is a high voltage light source, do not touch the metal contacts that connect the gas tube to the power supply.
 (2) Never look at the sun or a tanning lamp with a spectroscope. Eye damage may occur from brightness and from high energy ultraviolet photons.

On the following scales, draw the pattern of emitted light observed with the spectroscope for three gas lamps.^{Hint} Use colored pencils or markers to indicate the position of color(s). Add a written description to record which colors seem bright or dim.

Light Patterns Emitted by Gas Lamps

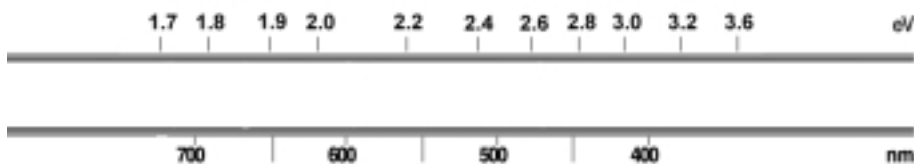
Hydrogen or _____:

Color of the light without spectroscope _____



Helium or _____:

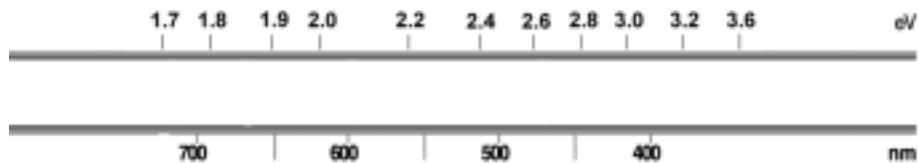
Color of the light without spectroscope _____



Hint To ensure that the light patterns are clearly visible, position the vertical slit of the spectrometer (found on the end with a screen) so that it is directly facing the light source and, if possible, hold the spectrometer less than a foot away from the light source. Dim the lights of the room so that the light patterns may be seen. The room, however, should be lighted enough for the energy scale to be seen.

Mercury or _____:

Color of the light without spectroscope _____



As you can see, different elements emit different colors. These colors can be described quantitatively in terms of wavelength which is the nanometer (nm) scale above. A nanometer is .000000001 meters, so the wavelength of light is very small. Alternately we can describe the colors in terms of energy. The electron-volt (eV) scale provides an energy associated with the light. An electron volt is also very small — 1.6×10^{-19} Joules.

In our investigations we will be particularly interested in the energy of the light emitted by the gas. Two factors — brightness and color — contribute in very different ways to the energy of a light.

When we think about the definition of energy, the brightness makes sense. A bright light has more energy in it than a dim light.

The color connection is not quite so obvious. Atoms emit light in small packets of energy. These packets are called photons. Each individual photon contains an amount of energy that is related to its color. So, if we wish to discuss the energy of one of these photons, we need to know its color.

For light that we can see the energy ranges from red at the low energy to violet at the high-energy end. Not visible but still a form of light are infrared photons with an energy lower than red and ultraviolet photons which have energies higher than violet. The order of energies for the various colors of photons is shown below.

- | | |
|------------------------------|-------------|
| | Infrared |
| Low energy visible photons: | Red |
| | Orange |
| | Yellow |
| | Green |
| | Blue |
| High energy visible photons: | Violet |
| | Ultraviolet |

Each time an atom produces light, it emits a photon. In our investigations we will be primarily interested in the energy of individual photons. As we will see, this energy will tell us something about the atoms of a material. Thus, the color of a light will be an important variable.

Each photon of visible light carries a very small amount of energy. This energy ranges from about 2.56×10^{-19} Joules for red light to 4.97×10^{-19} Joules for violet. Using these very small numbers is inconvenient, so we will use different units – the electron volt (eV). In these units, visible light energies range from about 1.6 eV (red) to 3.1 eV (violet) – much easier numbers to deal with.

The brightness of the light is related to the number of photons emitted. A dim light will emit fewer photons than a bright light. Thus, we have two measures of energy — brightness and color. Because color is related to the light from each individual atom, we will concentrate on it.

In the table below record the color of light emitted by each gas lamp that is related to the greatest and least energy per photon.

Gas	Greatest Energy	Least Energy
Hydrogen		
Helium		
Mercury		

? How can you tell which particular color of light emitted by each gas lamp results in the greatest number of photons emitted?

In the table below record the color(s) of light for which the greatest numbers of photons are emitted by each gas lamp.

Gas	Greatest Number of Photons
Hydrogen	
Helium	
Mercury	

? What are the similarities among the light patterns observed for the various gases?

? What are the differences?

Atoms have a nucleus and electrons. These components are attracted to each other. The attraction between the electrons and nucleus means that energy in the form of electrical potential energy is stored in the atom. In addition the electron's motion contributes kinetic energy. So, each electron has a total energy that is equal to its kinetic energy plus its electrical potential energy.

Electrical potential energy occurs for attraction (opposite charges) and repulsion (same charges). To distinguish these two situations we use positive and negative numbers. The positive numbers indicate potential energy associated with repulsion, while negative numbers go with attraction. Because we will work with attraction, we will be using negative potential energies.

To get the total energy we add kinetic energy (a positive number) and potential energy (a negative number). For an electron in an atom the result for the total energy will always be negative. The idea of a negative energy may seem strange at first. To get an idea of its meaning consider an electron which is *not* attached to an atom, not near any other electrical charges, and is not moving. It is interacting with nothing and not moving, so it has zero potential energy, zero kinetic energy and zero total energy.

If this electron is attached to an atom, its energy becomes negative. The *magnitude* of the energy must be added to the electron to get it back to zero energy — to get it to be no longer attached to an atom and not moving.

For example, suppose we know that an electron has an energy of -13.6 eV. From this information we know that

- the electron is attached to an atom, and
- to get the electron completely free from that atom we must give it 13.6 eV of energy.

Thus, the negative total energy can convey some valuable information about the electrons. The questions below will help you check your understanding of these ideas.

A. Which energies below indicate that the electron is attached to an atom?

-1 eV 0 eV 18 eV -8.6 eV

B. For each of the energies below indicate how much energy you must add to get the electron free from the atom.

-3.4 eV -54.4 eV -11.5 eV

C. An electron has an energy of -4.6 eV. An interaction occurs and it loses 5.1 eV of energy. What is its new energy?

D. Is it still attached to the atom? Explain your answer.

E. An electron is attached to an atom and has a total energy of -8.9 eV. An interaction adds 12.0 eV to this atom. What will be the electron's new energy?

F. Will it be moving away from the atom? Explain your answer.

A useful way to describe the energy of electrons in an atom is to use an energy diagram. The diagram plots the electron's energy on the vertical axis of a graph. We simply draw a line at the energy of the electron. As an example the diagram in Figure 1-4 represents an energy of -3.4 eV.

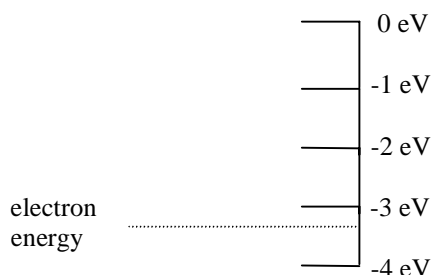


Figure 1-4: An energy diagram for an electron with -3.4 eV of energy.

In this scheme the horizontal axis has no particular meaning. We are only dealing with one variable — the electron's energy. We could just draw dots on the energy axis, but lines are easier to see.

In our studies we will always be interested in electrons that are attached to atoms. So, we place zero energy at the top of the diagram and do not include positive energies.

Changing Energies — Transitions

To emit light an electron must change its energy. This statement reflects conservation of energy.

$$\text{Electron energy before} = \text{Electron energy after} + \text{Light (photon) energy}$$

Each time an electron decreases its energy it emits one photon. Thus, by looking at the energy of photons we can learn about what is happening in an atom. From what we can see (light) we infer about what we cannot see (the atom). This process allows us to build models of the atom.

We will use energy diagrams to indicate the changes in the electron's energy. The process is shown in Figure 1.5

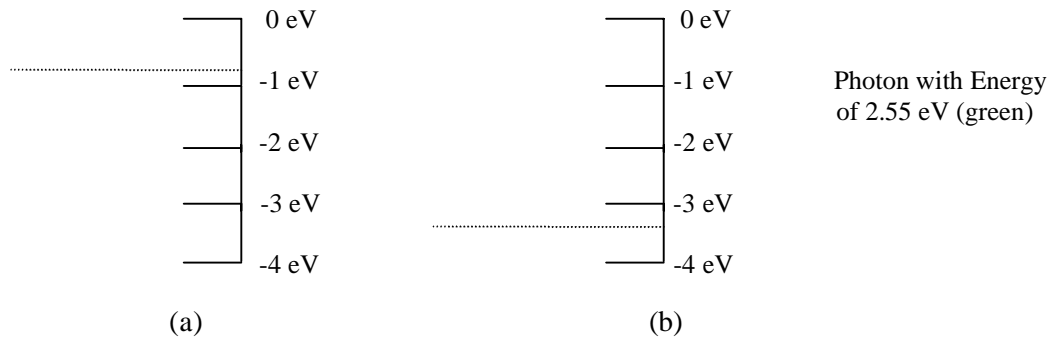


Figure 1-5 (a) Before the emission of light the electron has an energy of -0.85 eV.

(b) After the emission of light the electron has an energy of -3.40 eV and a photon of 2.55 eV has been emitted.

The diagrams show the before and after pictures for the electron's energy and indicate that a photon was emitted. To simplify our drawings we generally combine all of the information onto one graph as in Figure 1-6.

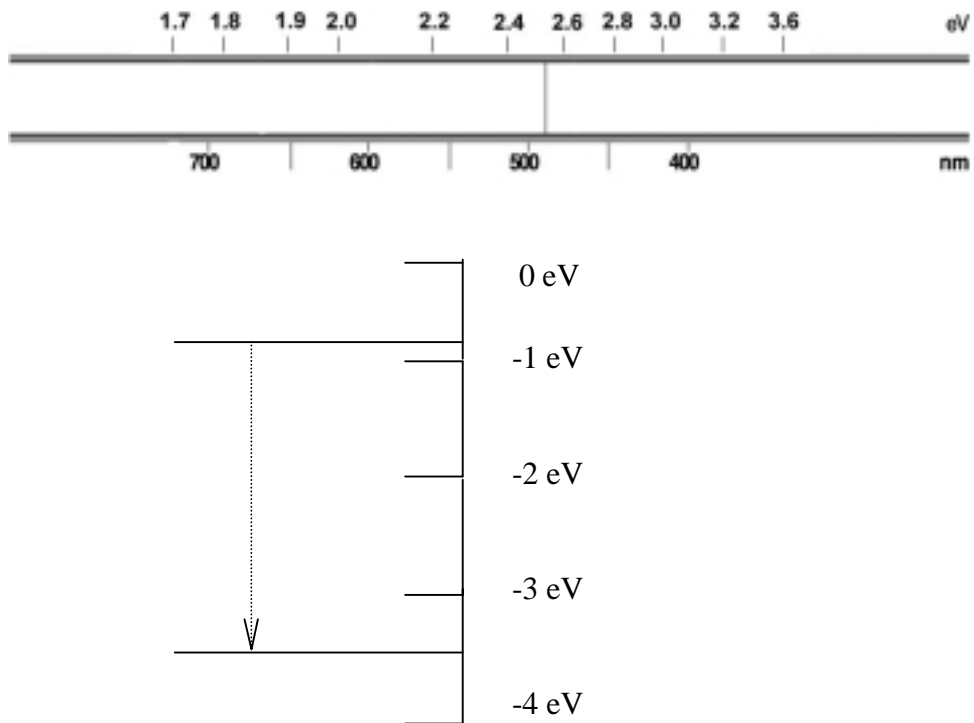
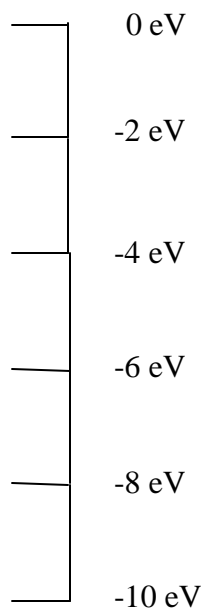


Figure 1-6 The interaction that was shown in the previous figure but combined onto one graph.

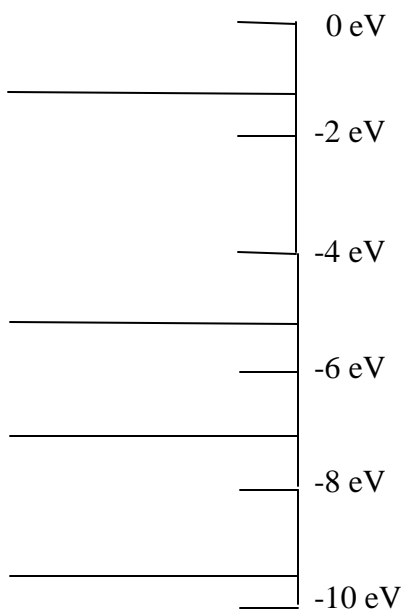
The arrow in Figure 1-6 indicates that electron changed from an energy of -0.85 eV to an energy of -3.40 eV . The sketch above the energy diagram represents what we would see in a spectroscope when the photon is emitted. (One photon is too few to see but it is representative of the energy.)

The process during which an electron changes energy is called a *transition*. Thus, Figure 1-6 represents a transition from -0.85 eV to -3.40 eV .

Draw an energy diagram which represents a transition from -2.3 eV to -4.6 eV .

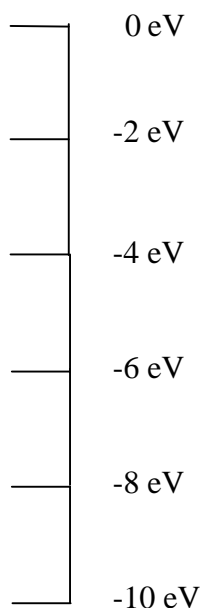


The energy diagram below has four possible energies for the electron. Indicate all transitions that could occur.



Determine the energies of the photons for each transition.

Another type of transition involves the electron gaining energy rather than losing it. Sketch a diagram which indicates that an electron changed from -3.47 eV to -1.1 eV .



Speculate about what type of process could cause such a transition. Explain your answer.

An Energy Model for the Atom

The energy diagram provides us with a way to understand some of the processes in the atom. One of these processes is the emission of light. As we apply energy diagrams to light emission you will learn why the spectra of incandescent lamps, LEDs, and gas lamps are different from each other. You will also learn how gas lamps and LEDs can emit certain colors even though they are not covered with colored glass. The next step is to build an energy model of an individual atom. That model is the subject of Activity 2.

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ACTIVITY 2

Understanding the Spectra Emitted by Gas Lamps

Goal

You will use your observations of gas spectra to build a model of energies in an atom.

In the last activity, we learned that light is emitted when atoms change energies. An electron in an atom loses energy. The energy lost by the electron appears in the form of light. The energy difference determines the energy and, thus, the color of light emitted by the atom.

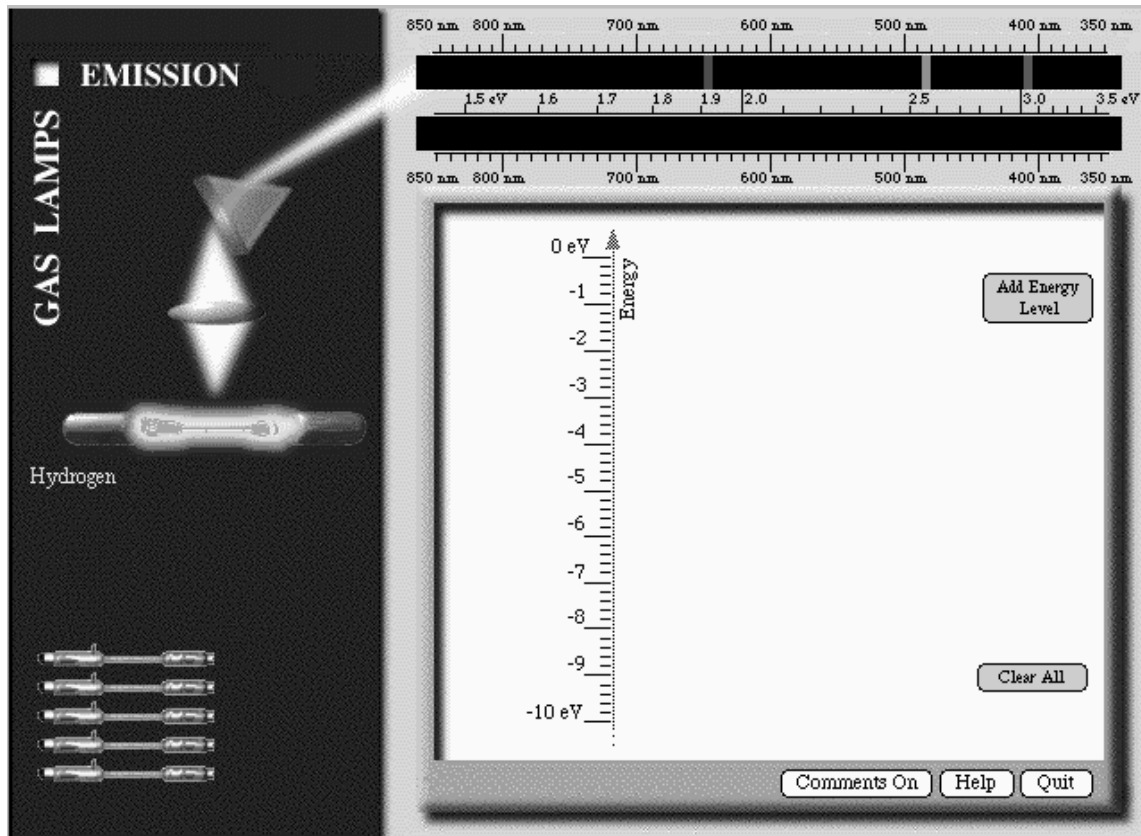
We will now use *Spectroscopy Lab Suite* to see how the spectra of light emitted by gases can help us understand more about the energies in an atom.

In *Spectroscopy Lab Suite*, select *Emission* under *Gas Lamps*. Figure 2-1 shows the screen that appears. In this program, we can

- Select a gas tube and drag it to the socket that is just above the lamps. Some of the light in the spectra for that gas will appear at the top of the screen.
- Add energy levels for an electron in a potential energy diagram by using the Add Energy Level button.
- Move the energy levels by selecting them at the left of the vertical energy scale and dragging them to the desired position.
- Create transitions (represented by vertical arrows) by selecting, the electron's initial energy on the right of the energy scale. (It turns green.) Drag the transition arrow to the electron's final energy. When you reach the final energy, it will turn green.
- This process will enable you to create an energy level model of the light emitting process in an atom. From the results you will be able to learn about energy levels in atoms. A colored spectral line on the screen above the potential energy diagram will indicate the light emitted by the transition. If the light is not in the visible region of the spectrum, it will not appear on the screen.

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- Move any of the energy levels after you have created a transition.

Figure 2-1: *Gas Lamp Spectroscopy Computer Program*

Begin with hydrogen. Follow the procedure on the previous page to place the hydrogen gas tube in the socket. Some of the spectral lines for hydrogen will appear in the top spectrum.

Create energy levels and a transition that will match one of the spectral lines of hydrogen. The spectrum that you create appears on the lower spectrum.

- ? How many energy levels are needed to create one spectral line?
- ? What is the energy of the spectral line as indicated by the eV scale?

? List the energies of the levels that you created.

? What is the difference in energy between the electron's initial total energy and its final total energy?

? How is this energy difference related to the energy of the light emitted by the atom?

Move both of the energy levels but keep the difference in energy between them unchanged.

? Why does the spectral line end up at the same energy?

Now create and move energy levels until the bottom spectrum matches the spectrum of hydrogen as shown in the top spectrum.

Sketch the resulting energy level diagram for hydrogen in the space below.

? How many energy levels are needed to create these three spectral lines?

? How many electron transitions are needed to create these three spectral lines?

- ? What other, if any, possible electron transitions can take place with the energy levels illustrated on your screen?

Compare your energy diagram with the diagrams created by other students.

- ? How are they similar?

- ? How are they different?

At this time none of the energy diagrams is more right or wrong than the others. We do not have enough information to distinguish exactly what transitions or initial and final energies occur in nature. Our model is limited by the knowledge that we have. Thus, all sets of energies and transitions that reproduce the spectrum are equally correct. (Scientists have more information to help distinguish the various possibilities, but that is not needed for our purposes.)

We can create energy diagrams that provide all of the spectral lines rather easily. We need only a few energies to have sufficient transitions for all of the visible light. From this construction we conclude that an electron in an atom can have only a few energies. Otherwise we would see light of many more colors. This conclusion is somewhat surprising. When an electron moves in an atom, *it might seem* that the electron could have any one of many energies. But, nature does not behave that way. Instead electrons in atoms are limited to a very few discrete energies. We call them the allowable energies.

Repeat the steps to determine the energy levels and transitions necessary to produce the spectral lines emitted by another gas that is assigned by your instructor.

Sketch the resulting energy level diagram for the helium gas in the space below.

? How is the energy level diagram for the helium gas similar to the diagram for hydrogen?

? How are they different?

Up to this point, we have learned that light is produced when electrons make transitions in atoms. If they have high energy, they naturally lose it in the form of light as they move to a lower energy level. In a normal situation the electrons will be in a low energy level. They must first be given energy to attain high energies so that it can naturally lose that energy. An external energy source, such as electricity must supply that energy. This process is illustrated in Figure 2-2.

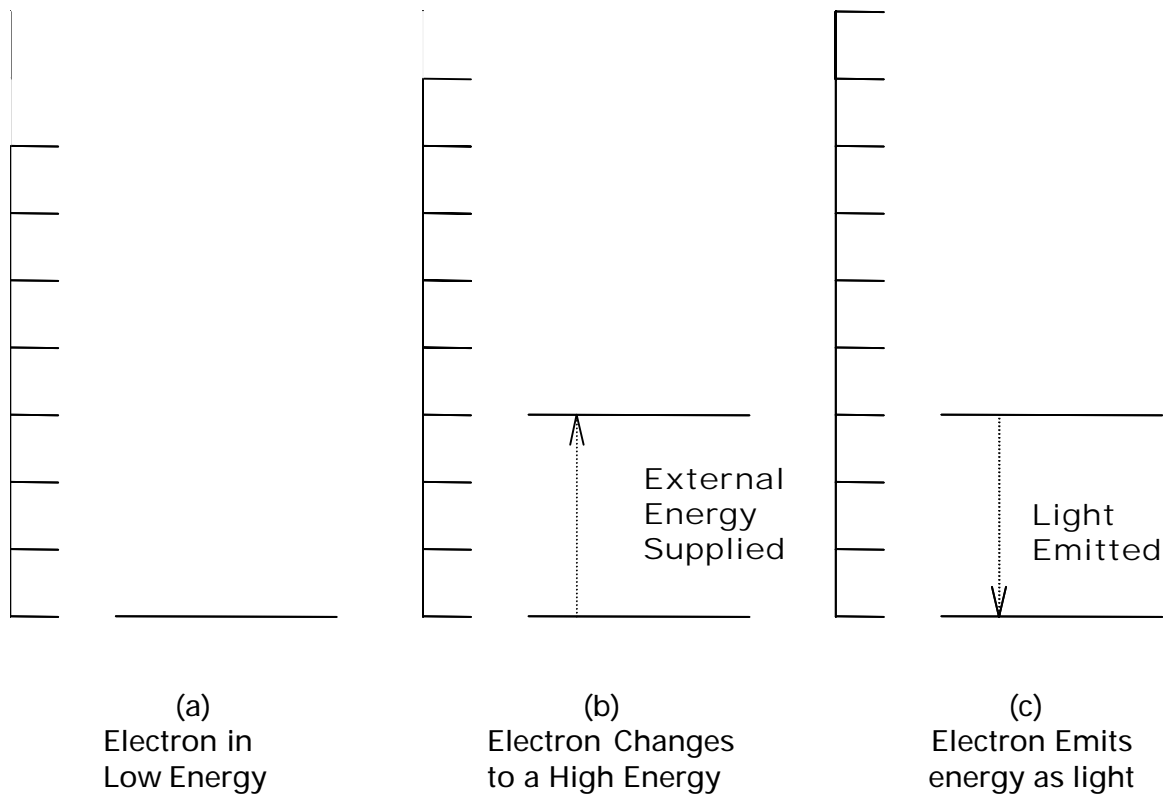


Figure 2-2: Gain and Loss of Energy by Electrons in an Atom

? What external energy source was necessary for the hydrogen and helium gas lamps to emit light?

The larger the external energy provided, the greater the number of electrons that will obtain higher energies. For example, suppose the energy difference between two allowed energies is 2.55 eV. See Figure 2-3.

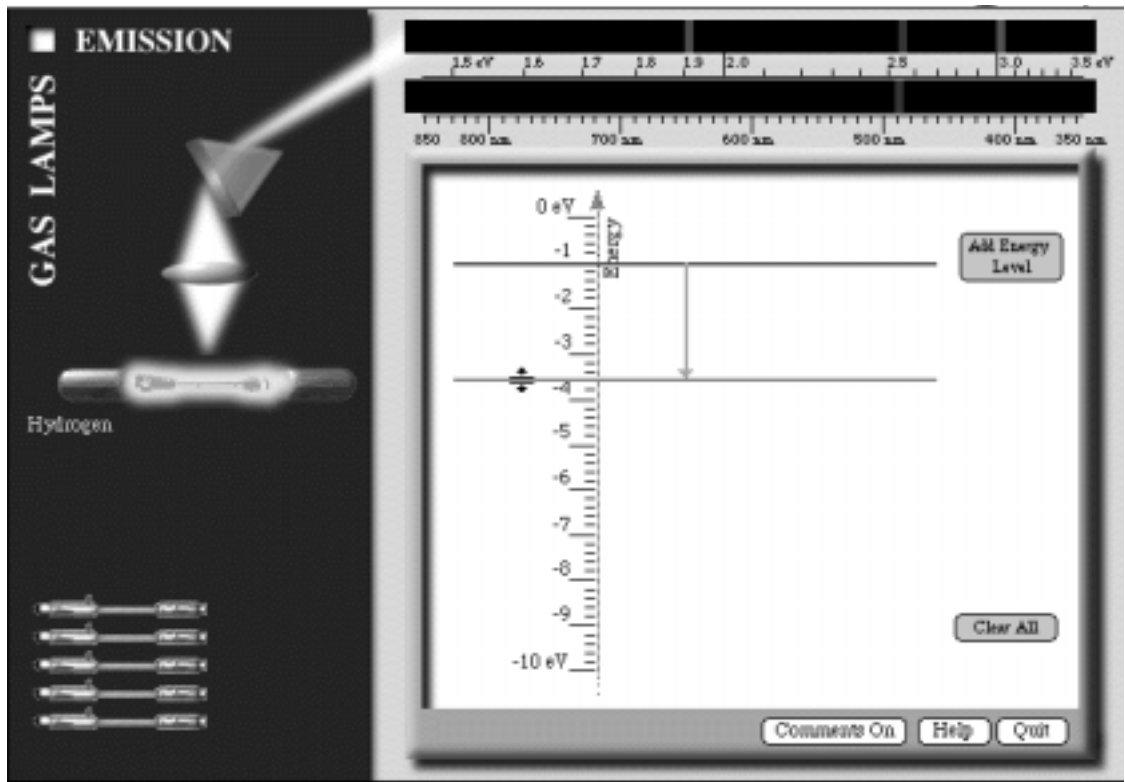


Figure 2-3: Screen capture from the Gas Lamps program.

In other words, 2.55 eV must be supplied by the external source for a *single* electron to change from a low energy to a higher one. If 5.10 eV of energy is supplied by an external source, *two* electrons can have the highest energy. Supplying a larger amount of external energy does not change the allowed energies, but it can change the number of electrons with the higher energy. Recall that the allowed energies for an electron bound to an atom depend on the type of gas atoms found in the lamp. Supplying a larger amount of external energy causes a larger number of electrons to possess the highest allowed energies. Thus, more electrons will make transitions from higher allowed energies to lower allowed energies that result in the greater emission of photons and brighter light.

A different situation occurs if the energy supplied to the atom is not equal to the difference between energy levels. For the example illustrated in Figure 2-3 no transition will occur if we supply less than 2.55 eV. If we provide the electron with 2.40 eV of energy, it cannot use it. No energy level exists for that transition to occur. If we provide more than 2.55 eV but less than 5.10 eV of energy, only one electron can make the transition. When 3.20 eV is available, the electron can use 2.55 eV and the remaining 0.65 eV will end up as some other form of energy.

In hydrogen and helium atoms, unlike the *Gas Lamp Spectroscopy* computer program, the allowed energies for an electron cannot be changed because the type of atom uniquely defines them. The values for the energies are determined by electrical interactions between the nucleus and the electrons.

- ? What differences are there between hydrogen and helium atoms that might account for their electrons having different energies?

The electrical properties of an atom uniquely determine what energies its electrons are allowed to have. So, even though the *Gas Lamp Spectroscopy* computer program allows you to adjust the energies available to the electrons, these energies are fixed at very specific values by the electrical properties of the atom.

Because the atoms of each of the elements have a unique set of energies, the light given off by a material can be used to determine the type of elements present. This property is used to learn about the composition of distant stars as well as substances on earth.

Homework Problem

1. Art historians are frequently faced with determining whether paintings are originals or forgeries. Suppose an art dealer who seems a little suspicious has a painting that she claims was painted in 1704. Because of its age she says that it is very expensive. However, one of the colors looks remarkably like pigments from cobalt blue. As an art expert you know that blue paint that used the element cobalt was not used by artists until 1804. How might you use the ideas presented in this activity to determine whether the painting contains cobalt blue?¹

¹ Adapted from Jacqueline D. Spears and Dean Zollman, *The Fascination of Physics*, (The Benjamin /Cummings Publishing Company, Inc., Menlo Park, CA 1985). Permission granted by the Authors.

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ACTIVITY 3 Light and Waves

Goal

We will look at a property of light and learn how scientists conclude that light behaves as a wave.

The light from atoms indicates that only certain energy transitions occur in each atom. We were able to explain these results using energy diagrams. However, we have not yet explained why only certain energies occur in each atom or why these energies are different for different elements. To get to the reasons will take us a little time. We begin with a short diversion about the nature of light.

Interference of Light: Young's Experiment

Approximately 200 years ago, scientists (then called "natural philosophers") argued about the fundamental nature of light. In his book *Optiks*, Isaac Newton assumed that light consisted of a collection of minute particles. With this model, he explained most of the known optical experiments. Others thought that light must take the form of a wave. In 1801, Thomas Young completed an experiment that seemed to end this controversy.

In this activity, you will begin with an experiment very similar to one done by Young. Place a laser about two (2) meters from a sheet of white paper taped to a wall. With the room as dark as possible, aim the laser so that its beam is perpendicular to the paper. Remember: Never look directly into the beam coming from the laser. Then place a slide with two very small, closely spaced slits in the path of the laser beam, also perpendicular to the beam as shown in Figure 3-1.

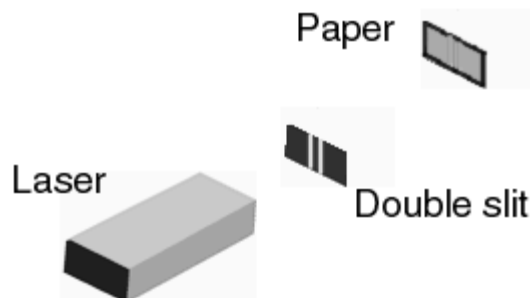


Figure 3-1: Diagram for Young's double-slit experiment.

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You may have to adjust the position and orientation of the slits so that the laser light passes through both slits and forms a clear pattern on the wall.

In the space below, sketch and describe the pattern created by the light where it hits the screen.

We call such a pattern an interference pattern.

From these results Thomas Young concluded that light behaved like a wave and that Newton's particle model of light could not explain these observations. Even though Newton was an intellectual giant essentially everyone agreed with Young. To see why, we need to complete a couple of short experiments.

The first experiment involves a version of the two-slit experiment of Young. We will use something that we know is a wave. It's a wave drawn on a piece of plastic. Along the solid line mark with an X locations where the addition of the two waves is a maximum (constructive interference). Place zeros where the two waves cancel each other (destructive interference). Taping one end of the plastic waves to a wooden block will hold that location fixed and will serve as the two slits. Place the block with waves on the paper so that the two slits are approximately at the two dots below with the waves extending toward the bottom of the page.



Other groups used different amplitudes or wavelengths for this experiment. Compare your results with others in the class and answer the questions below.

How does the distance between zeros depend on

wavelength?

amplitude?

We will use this information soon.

Try a similar experiment with some particles. To simulate objects similar to Newton's particles of light you will use BBs. Place a blank piece of paper in a shallow pan. Over this paper place a sheet of carbon paper with the carbon side down.

To simulate the two slit experiment place the BBs in a small box with two holes in it. Over the carbon paper take your fingers off the holes and let the BBs drop. A carbon mark will appear where the BBs land. [Hint: A much better pattern appears if the carbon paper is placed on a hard surface that is angled so that when the BB's bounce they don't land back on the carbon paper.]

Describe the pattern of particle locations.

? You have now completed three two-slit experiments. One each with light, waves and particles. Use the results of these experiments to describe why Young concluded that light behaved as a wave.

As you see with these experiments, interference — the pattern of light and dark for light — only occurs for waves. Particles can't do it.

Now, you will complete a simulation of an experiment which at first thought seems rather strange. A double slit experiment using electrons. This type of experiment can be done, but the equipment is rather expensive. Open the *Double Slit* program and click on the source tube labeled *Electrons* on the left side of the screen. Click *Start* to see what happens. Describe the pattern below.

- ? Which pattern(s) — light, waves or BBs — does the pattern for electrons resemble?

- ? What can you conclude about the behavior of electrons in a Double Slit experiment?

Similar experiments resulted in a conclusion that electrons behave as waves. This result was surprising but inescapable. As we shall see, it is confirmed in many situations.

First, let us see how this pattern changes with the energy of the electron. Keeping other variables fixed try at least four different energies and record the distance between consecutive dark areas on the screen.

Electron Energy (eV)	Distance between dark areas

? How does the distance between dark areas change as the energy increases?

? For the plastic waves look at how the distance between zeros changed with wavelength. Use that information to describe below how the electron's wavelength changes as the energy increases.

In the next activity we will use all of the results from this one to connect the wave properties of electrons to other properties such as momentum and energy.

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ACTIVITY 4 Matter Waves

Goal

We will use the results of the previous experiments and establish quantitative concepts for electron waves.

In the previous activity, we saw that in certain experiments electrons produce patterns attributed to waves. From this observation, we concluded that electrons have wave-like properties. You also concluded that as the energy of electrons increases their wavelength decreases.

Electrons are a form of matter, so these waves are called matter waves. In this activity, we will relate quantitative features of matter waves with familiar, measurable physical quantities such as energy, mass, and momentum. We will apply these relationships to forms of matter other than electrons, and see how these results can be applied to the electron microscope.

Louis de Broglie was the first person to establish an equation for the relationship between an electron's momentum and its wavelength. He concluded that:

$$\text{Wavelength} = \frac{\text{Planck's Constant}}{\text{Momentum}} \qquad \lambda = \frac{h}{p}$$

where h is a number called Planck's constant (named after Max Planck) and is equal to $6.63 \times 10^{-34} \text{ J} \times \text{s}$ or $4.14 \times 10^{-15} \text{ eV} \times \text{s}$.

When we observe electron diffraction, the electrons' kinetic energy is easier to measure than their momentum, so we write the de Broglie wavelength as

$$\text{Wavelength} = \sqrt{\frac{(\text{Planck's Constant})^2}{2 \times \text{Mass} \times \text{Kinetic Energy}}} \qquad \text{or} \qquad \lambda = \sqrt{\frac{h^2}{2mKE}}$$

This equation is consistent with our results in the previous activity — as the energy increases the wavelength decreases.

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While we have learned about this equation using electrons, it can be used for any type of matter. So, you might wonder why we do not see wave effects for large objects.

Start the *Double Slit* program. To begin to understand and experiment with objects with masses greater than that of an electron, run the simulation of the experiment for electrons. Throughout this experiment keep the same energy.

Next choose *pions* and repeat the process. Repeat for *neutrons*, and finally *protons*. (A pion has a mass 270 times that of the electron while the neutron and proton masses are about 2,000 times that of the electron.) After doing these experiments, feel free to vary any of the parameters except the energy. Then, answer the following questions. Compare the diffraction patterns made using the other types of matter with that of the electron.

- ? How do the distances between dark areas change with the mass?

- ? How does the de Broglie wavelength change?

- ? How do the patterns for protons compare to the patterns for neutrons?

- ? Use de Broglie's hypothesis to explain this similarity or difference.

Now, investigate how the patterns change as the separation between slits changes. Pick one particle and one energy; change the slit separation to answer the following.

- ? How does the pattern change as the slit separation increases?

For patterns to be formed, the separation between the slits must be comparable to the wavelength of the waves passing through them. Thus, as your separation became very large, the pattern was not easy to see.

While wave behavior is exhibited by electrons, pions, neutrons and protons; we do not observe similar behavior for large objects such as gnats or humans. As an example, we will consider why diffraction doesn't cause a gnat to look like several gnats as it flies through window blinds. Suppose the gnat's mass is .001 kg, and its speed is 0.10 m/s.

? What is the gnat's momentum?

? What is the gnat's de Broglie wavelength?

? Approximately what would the spacing between the window blinds have to be for the gnat to create a pattern as it flew through? Why?

? Is it necessary for a gnat to worry about creating a pattern as it flies through the blinds? Why?

? How would a human being's de Broglie wavelength compare to that of a gnat? Why?

Because we know the values of Planck's Constant and the electron's mass, we can use them to simplify the equation to apply only to electrons as

$$\text{Wavelength} = \sqrt{\frac{15 \text{ nm}^2 \cdot \text{eV}}{\text{KineticEnergy}}} \quad \text{or} \quad \lambda = \sqrt{\frac{15 \text{ nm}^2 \cdot \text{eV}}{KE}} \quad [\text{ELECTRONS ONLY}]$$

where the kinetic energy has been measured in electron volts (eV). The result, λ , is the electron's wavelength in units of nanometers. This equation works only for electrons, so use it carefully.

? How does the relationship between energy and wavelength in this formula compare with the relationship that you observed in Activity 3?

A valid question to ask is: "What is waving with these matter waves?" Unfortunately, the answer is not an easy one. We never observe a matter wave directly; we only see results that can be explained by them. The matter wave is an abstraction that allows us to explain observations. In the next activity, we will look at what information is contained in waves of matter. We will examine the features of matter waves that describe simple properties such as the location of an object.

An Application

Because electrons behave as waves, they can be used to "illuminate" objects in a manner similar to light. An electron microscope is an instrument that takes advantage of this situation. Electrons are given energy by accelerating them in a manner similar to the way a TV tube works. Then, using magnetic fields, they are directed at an object of interest. The electrons are focused to illuminate the object, and then to form the image of that object. A schematic diagram is shown in Figure 4-1. This system can be used to look closely at very small objects.

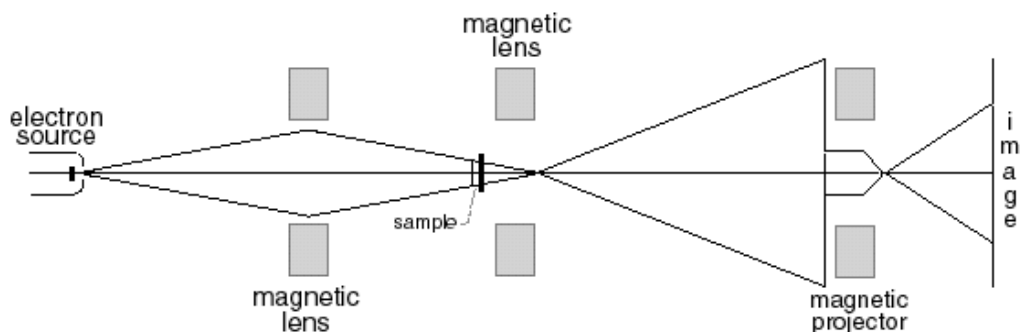


Figure 4-1: Schematic diagram of electron microscope.

The wavelengths of the electrons are related to their kinetic energies. In electron microscopes, wavelengths as much as 100000 times smaller than those of visible light can be achieved. With such small wavelengths, electron microscopes can reveal features that are as small as 0.000000001 meters (1 nm). Below are some electron microscope pictures.

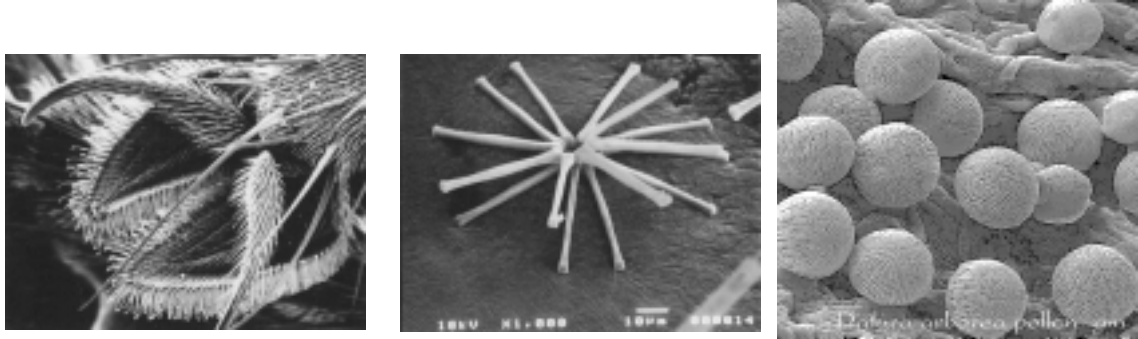


Figure 4-2: Electron microscope images of: (a) the foot of a housefly; (b) a diatom; and (c) pollen.

Homework

A situation where matter waves could become important is the Star Trek transporter. We are not sure how a transporter would really work, but for the purposes of this activity, let us suppose that it decomposes a person into his or her component atoms. Then, it sends the atoms to a new location where the person is reconstructed.

Consider transporting Captain Janeway of the Voyager by such a method. She wishes to reach her new location quickly, so her atoms are sent out of the ship at 10% of the speed of light (3×10^7 m/s). Assume that her atoms have a mass of 10^{-26} kg

? What is the de Broglie wavelength of each matter wave?

Each atom must be transmitted through the titanium hull of the starship. The titanium can be considered as a large number of slits separated by 1 nm.

? Must the designers of the transporter be concerned about diffraction effects as the captain's atoms are beamed through the hull of the Enterprise? Why or why not?

? Would this effect make a good premise for a Star Trek movie? Why or why not?

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ACTIVITY 5
Interpreting Wave Functions

Goal

We will return to the two-slit experiment for electrons. Using this experiment we will see how matter waves are related to where electrons may be located.

In the previous activities you saw that experimental evidence supports the theory that matter, such as electrons and subatomic particles, exhibit wave behavior. Wave behavior is utilized by the electron microscope, which enables us to look at nanometer-sized details in objects. We must consider this wave behavior as we try to understand why atoms have only certain allowed energies. The next steps are to learn how to interpret matter waves, to understand what information they carry, and how we can use that information.

To continue we return to the *Double Slit* program. Choose electrons. Set the slit separation to 15 and the kinetic energy to 20 eV. Click *Start*. You will get a pattern similar to the one shown in Figure 5-1.



Figure 5-1: Simulated electron interference pattern with your prediction for the next electron's position.

? Now, suppose you were to send just one more electron through the slits. On the diagram above, mark one spot on the screen where an electron is likely to strike?

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? How does your prediction compare with those of others in your class?

To test the predictions keep your present pattern by dragging the pattern to the right. Then, start a new experiment. In the new experiment you will experiment with a few electrons at a time. Move the *Particles per Second* slider to the left. Press the *Start* button. When just a few electrons hit the screen, press the *Stop* button.

? How does the location of the dots match your earlier prediction?

In this experiment we restricted your predictions by limiting you to one location. Now, we will broaden your possibilities and have you make another prediction. This time stop the experiment when the # particles = indicates approximately 100 electrons have been added to the experiment. On the pattern below indicate any place where some electrons are very likely to be. Then indicate locations where you are not sure if electrons will appear and places where you feel rather certain that there will be no electrons.



Figure 5-2: Indicate places where electrons are very likely and very unlikely to be.

Then do the experiment with a large number of particles and compare it to your predictions.

As you see, we cannot state with certainty where the electrons will appear. However, we can discuss probabilities. The next electron has a very high probability of appearing in bright regions — locations of constructive interference. Lower probabilities are associated with regions where the interference is between constructive and destructive. The probability of the electron appearing at regions of destructive interference is essentially zero.

Thus, we cannot predict with certainty where each electron will go. It is possible that everybody could give a different prediction for the location where the next electron would appear. Yet, each of the predictions could be equally correct. With electrons, we can really only describe location in terms of probability.

Below is the same electron diffraction pattern you used to predict where an electron would go. On it, label where the probability of detecting an electron is highest, where it is lowest, and then indicate at least two places that have identical probabilities.



Figure 5-3: Simulated electron diffraction pattern with your labels for highest *probability* and lowest *probability*.

So far we have seen two results of the double slit experiment with electrons

- 1) Electrons behave as waves causing a pattern of light and dark regions to develop on the screen.
- 2) It is not possible to predict where any one electron may appear on the screen when it is sent through a double slit. However, we can state the probability of it appearing in various locations.

Now we will learn how results 1) and 2) are related to each other and develop a single concept that is consistent with both results.

Because matter waves are abstract ideas used to describe results, they do not travel through a medium, such as water. In fact, a matter wave is not a physical entity at all. So, scientists generally describe these waves in terms of mathematics or pictures. (We will use the pictures.)

Physicists call the pictures or equations wave functions. A sample of a wave function of an electron is shown in Figure 5-4 below. *The shape of the wave function is NOT the path that the electron traverses.* It is a useful description related to the location of an electron and can be used to predict the probability of finding the electron in any given region of space.

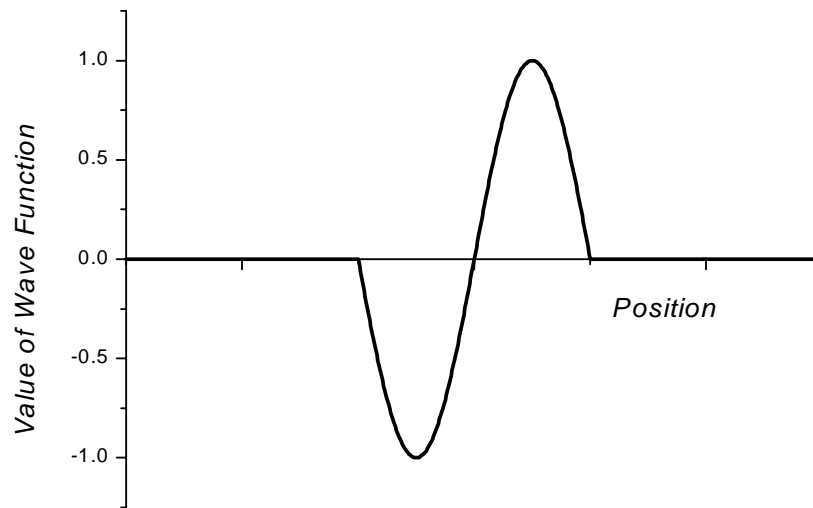


Figure 5-4: Example of a *wave function* for an electron.

To understand how a wave function is related to probabilities we return (one more time) to the electron interference pattern and to the plastic waves. The places where electrons are most likely to be correspond to places where the two waves on plastic added and the amplitude was large. The locations where the probability of finding the electrons is zero correspond to locations where the waves on plastic amplitudes added to zero. In turn the plastic waves each represent part of an electron's wave function. Their addition is the wave function at the screen. A large amplitude is related to a large probability while a small amplitude goes with a small probability.

We have a slight complication. Waves can have amplitudes that are either positive or negative; probabilities can only be positive numbers. The solution is to square the wave function. A square number is always positive. (See Figure 5-5.)

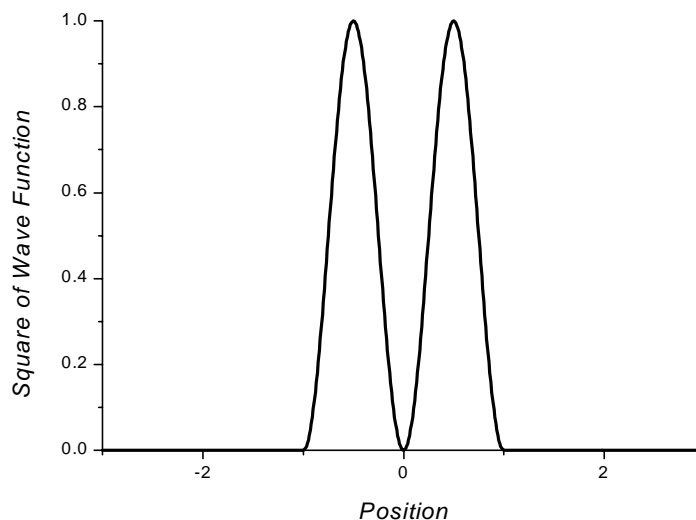


Figure 5-5: The square of the wave function shown in Figure 5-4.

When we square the wave function, we obtain the *probability density*. This graph is related to the probability of finding the electron at each point in space. However, we measure over regions in space. Fortunately, a little math can convert a probability density in a region into a number representing the probability of finding the object in that region. All of this is likely to seem somewhat abstract. We can make it more concrete with the *Probability Illustrator* program.

Click and drag the pencil that appears in the top frame to “sketch” the wave function similar to the one in Figure 5-4. You need not sketch the wave function accurately. Anything that looks similar is acceptable for our purpose. The program sketches a graph of the probability density in the lower frame.

? Does the probability density graph drawn by the program seem to be the square of the wave function that you sketched?

Now do the same for a wave function that is an approximate reflection (about the x-axis) of the previous one. (Something like Figure 5-6.)

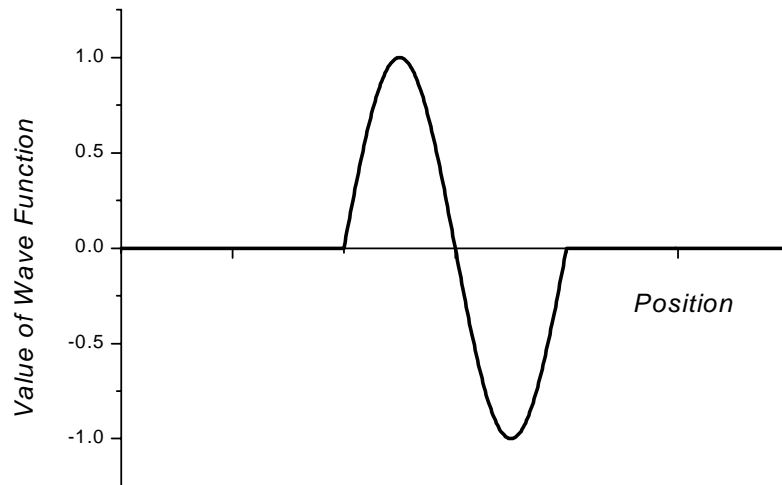


Figure 5-6: Wave function that is the exact inverse of Figure 5-4.

? How do the probability density graphs in the two cases, compare with each other?

? Is this result what you would expect? Why or why not?

In the *Probability Illustrator* the value of the wave function is displayed next to an arrow that is just below the wave function graph. A similar arrow below the probability density graph displays the value of the probability density at that point. You can move these arrows.

The lower graph displays the probability density of finding the electron at each point in space. To determine the probability of finding the electron in a region, move the two arrows at the bottom left of the probability density graph to any two locations. The probability of finding the two electrons between these two locations is displayed at the bottom left corner of the graph.

The probabilities are given as numbers between 0 and 1. A probability equal to 0 means that the electron will never be found in that region. A probability of 1 says that you will definitely always find the electron in that region. To see how these probabilities work record the probabilities of finding the electron in

- a) About one-fourth of the screen on the left side. _____
- b) About one-fourth of the screen on the right side. _____
- c) The left half of the screen. _____
- d) The right half of the screen. _____
- e) The entire screen. _____

? Do the numbers for (a) and (b) or (c) and (d) differ? If so, use the wave function to explore why?

? Is the probability for the entire screen what you expect? Why or why not?

Summary

Wave functions give results that are quite different from measurements we obtain for the location of more familiar, macroscopic objects. We can never be sure about where exactly an electron is at a given instant of time; rather we can only predict the probability of finding the electron in a given region of space at a given instant of time. The wave function of an electron enables us to determine that probability. To obtain the probability density we calculate the square of the wave function.

Probabilities of finding the object within a certain region are determined from the probability densities. An important conclusion is that we cannot state with certainty the location of an electron, only the probability of finding it at each of many locations.

The following essay describes some of the differences between our knowledge of large objects and our knowledge of the very small.

Interlude: From Newtonian to Quantum Views of Nature

Adapted from *The Fascination of Physics* by Jacqueline D. Spears and Dean Zollman © 1986,1996. Reprinted with permission of the authors.

More than 50 years have passed since the wave and particle models merged to become a new model of the physical world. In the early days of this century, physicists voiced strong arguments for and against the wave function and its interpretations. Now, the arguments have become less emotional; the concepts less unsettling. Passing years and new generations of physicists have a way of turning a revolutionary thought into a tradition; the new physics into the old physics. In the midst of this settled acceptance of modern physics, we must realize the enormous impact quantum mechanics and wave functions have upon a physicist's view of "reality." We pause briefly to examine the remarkable transformation from the physics of Newton to that of the modern quantum physicist.

When Isaac Newton introduced his three laws of motion, he provided a structure within which we could understand all motion – from the falling apple to the orbiting planet. Once we knew all the forces acting on an object, we could predict all future motions with complete accuracy. By placing certainty squarely within the grasp of human intelligence, Newton created an enormously comforting view of our universe. This feeling of certainty was stated well by the French mathematician Pierre LaPlace:

An intelligence which at a given instant knew all the forces acting in nature and the position of every object in the universe – if endowed with a brain sufficiently vast to make all necessary calculations – could describe with a single formula the motions of the largest astronomical bodies and those of the smallest atoms. To such an intelligence, nothing would be uncertain; the future, like the past, would be an open book.

Newton's model created an image of a rational world proceeding in a rational way – a world view eagerly embraced by philosophers, theologians, and physicists alike.

Beneath this world view lie two very important assumptions. The first is that all events are ordered, not random. To Newton and his contemporaries, all motion was completely determined by whom-ever or whatever started the universe. These motions obeyed and would continue to obey a series of orderly rules that could be discovered by the careful observer. The second assumption was that the physicist acts as an objective observer of events. Newton and his contemporaries believed that

while the measurer does have some impact on the events he or she measures, this impact is minimal and predictable. Events continue, according to a system of ordered rules, with an existence independent of the observer. All that remained was for science to discover the rules.

During the eighteenth and nineteenth centuries, when Newton's laws were applied to objects as small as molecules, this world view prevailed. In principle, physicists believed, once they knew the momentum and position of each molecule, they could predict all future motions of all molecules. Completing these measurements and calculations for a gram of water, let alone the entirety of the universe, was not humanly possible, so statistical or probabilistic descriptions were adopted. Consistent with Newton's world view, probabilities were needed only to compensate for an information overload, not because of the inherent unknowability of nature.

What does the new world view have to say to us about our knowledge? Implicit in the probabilistic interpretation now given to matter waves is the assumption that, on the microscopic level, events are random. Wave descriptions provide us with information about the probabilities associated with this random behavior; particle measurements convert these probabilities into brief certainties. Further, objective observers have become active participants in the world that they are trying to describe. Physicists now acknowledge that the types of measurements they undertake affect the observations and models they subsequently construct. Words like particle, position, and path have no meaning apart from the way in which the experimenter measures them. These words describe our way of ordering the events we see, not a true underlying structure of nature. Newton's view of an orderly nature that exists independent of how we observe it exists no more.

For many physicists the radical departure from more traditional ideas was difficult to accept. Erwin Schrödinger, whose equations were the Newton's laws of quantum mechanics, remained uncomfortable with the probabilistic interpretation given to matter waves. Albert Einstein, whose quantum explanation of the photoelectric effect won a Nobel Prize, also remained unconvinced. He felt that quantum theory was only a stepping stone to a more complete understanding of matter. In this view, probabilities do not represent nature but rather, people's limited ability to comprehend nature. In a letter to Max Born in 1926, Einstein summarized his and perhaps many others' feelings:

Quantum theory is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us closer to the secrets of the "old" one. I, at any rate, am convinced He is not playing at dice.

Only time will tell whether Einstein's inner voice was the voice of wisdom or the voice of a past, unwilling to give way to the future.

In the next activity, we will learn more about wave functions and how they can be determined from the energies of electrons.

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ACTIVITY 6

Creating Wave Functions

Goal

To make wave functions useful we must be able to create them for physical situations. We will start with electrons moving through space and materials and learn to sketch wave functions.

When physicists wish to describe or predict the motion of large objects, they work with some basic concepts such as Newton's Laws. They consider the forces or energies involved, then write equations, draw graphs and predict changes as the object moves through time and space.

Predicting probabilities for small objects is somewhat similar. The basic concept is Schrödinger's Equation, named for Erwin Schrödinger who first wrote it. This equation describes the changes in wave functions over space and time. It is based on the wave behavior of small objects and conservation of energy.

The mathematical form of Schrödinger's Equation is a little complex, so we will not write it down. Instead we will introduce a series of steps based on the equation. These rules will enable you to sketch wave functions for several situations, then interpret the results in terms of probabilities.

As electrons move, they interact with other objects. For example, an electron in an atom interacts with the nucleus. In an electron microscope, the electrons interact with magnetic fields as they move toward the sample. As we develop wave functions, we must include these interactions as we learn about representing electrons with wave functions.

Because Schrödinger's Equation is based on energy, we will represent interactions with potential energies. Locations where the potential energy changes indicates places where the interaction changes. A potential energy of zero indicates that the electron is not interacting with anything. Positive values of potential energy will be used for interactions in which the electron is repelled by other objects, while negative values will be used for interactions in which the electrons are attracted by other objects. To represent potential energies, we will use graphs of the potential energy versus location — a potential energy diagram. One such diagram is shown in Figure 6-2.

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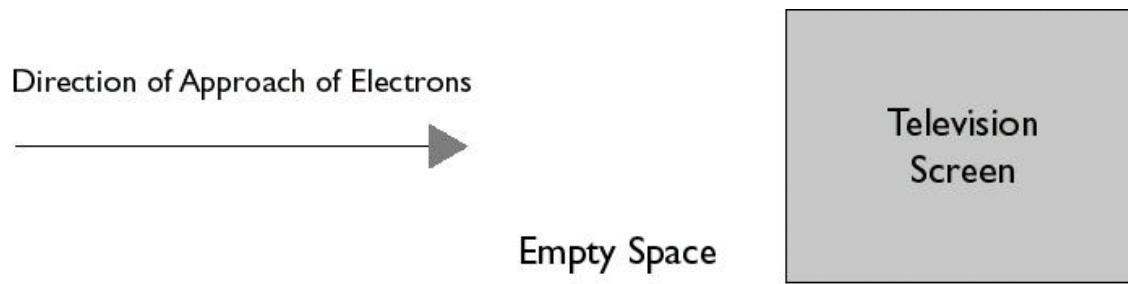


Figure 6-1: A beam of electrons approaches a TV screen

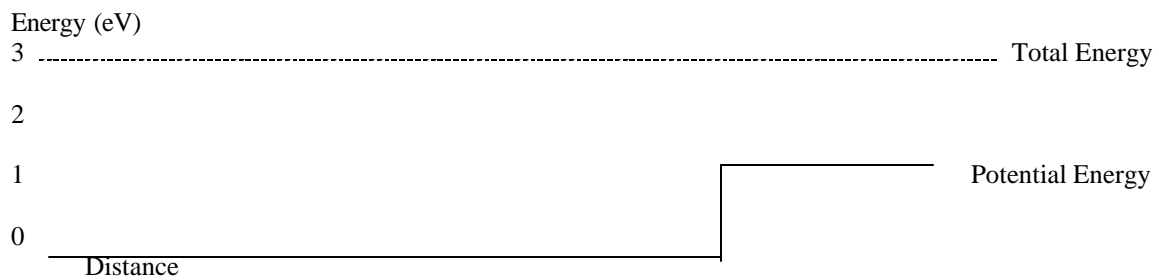


Figure 6-2: Potential & Total Energy of an electron approaching a TV screen.

- ? What is the Kinetic Energy of the electron in the empty space in Figure 6-1?
- ? What is the Kinetic Energy of the electron in the screen?
- ? Calculate its de Broglie wavelength in each region.

As you can see from your calculations above, the de Broglie wavelength changes when the potential energy changes. Thus, we will divide the space into regions based on the changes in potential energy. Any location at which the potential energy changes is a boundary between regions.

To help you draw wave functions, use the *Wave Function Sketcher* program. The program has a window for drawing wave functions and another the energies. Generally, we will establish the energy graph first. This graph is based on the physical situation. Our first situation will be the one described in Figure 6-1.

The vertical dotted line marks the boundary between regions. Move it to the location that you wish to use for the change in potential energy. Now we wish to set the values of the potential energy.

Using the "Potential" tab at the bottom of the screen and adjust the "Height" and "Right Level" both to 1eV. Adjust the "Total Energy" to 3eV. The potential energy diagram should look like Figure 6-3.

You have now completed step 1 in drawing wave functions:

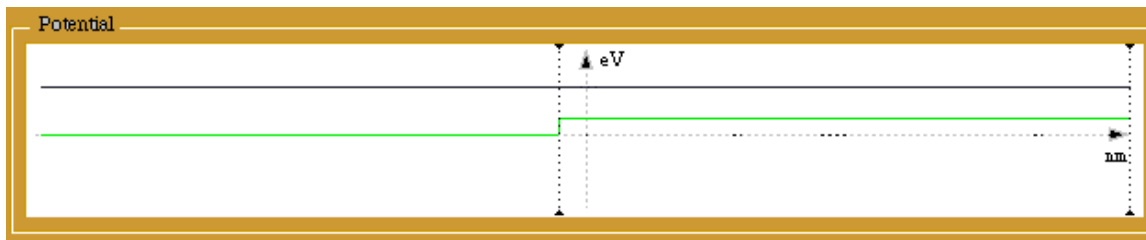
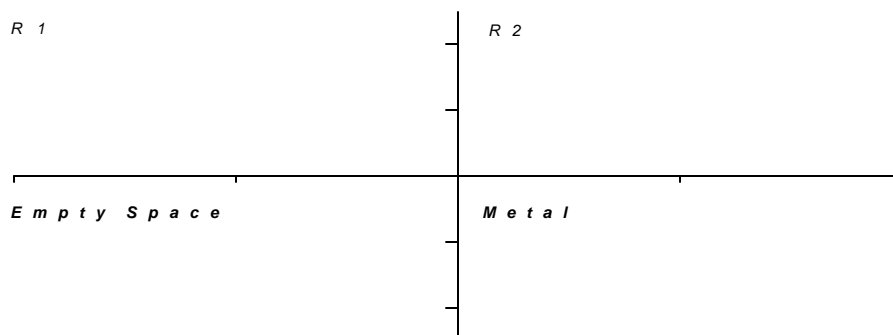


Figure 6-3: The total energy line and potential energy diagram as shown by the *Wave Function Sketcher* computer program.



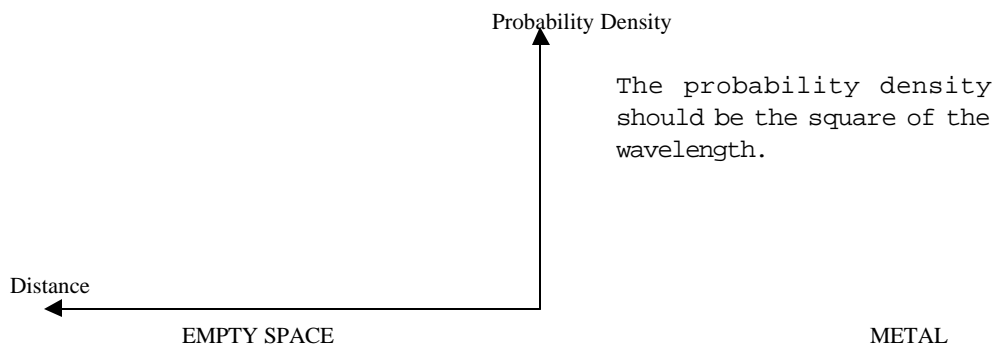
Step 1. Use the physical situation to draw the energy graph and determine the boundaries.

Now, create the wave function for each region.

Click the "Region I" tab, and adjust the "wavelength" of the wave function in Region I (empty space) to the value calculated above. Click the "Check" button to verify if your wavelength is correct.

Sketch the wave function of the electron in empty space, on the graph, as displayed by the Wave Function Sketcher and indicate the de Broglie wavelength on your sketch.

From the wave function graph above, draw the probability density graph of the electron in empty space *only*.



Click the "Region II" tab, and adjust the "wavelength" of the wave function in Region II (i.e. in the metal) to the value calculated above. Click the "Check" button to verify if your wavelength is correct.

Sketch the wave function of the electron in the metal and probability density on the right sides of the graphs above.

This process is step 2.

Step 2. Determine the wavelength (if possible) in each region and sketch the wave function.

(We will need to modify this step later and, then, explain the "if possible.")

At the Boundary

For the next step we need to consider how to connect the wave function in region 1 to the one in region 2. We could imagine several different possibilities. Two are shown in Figure 6-4.

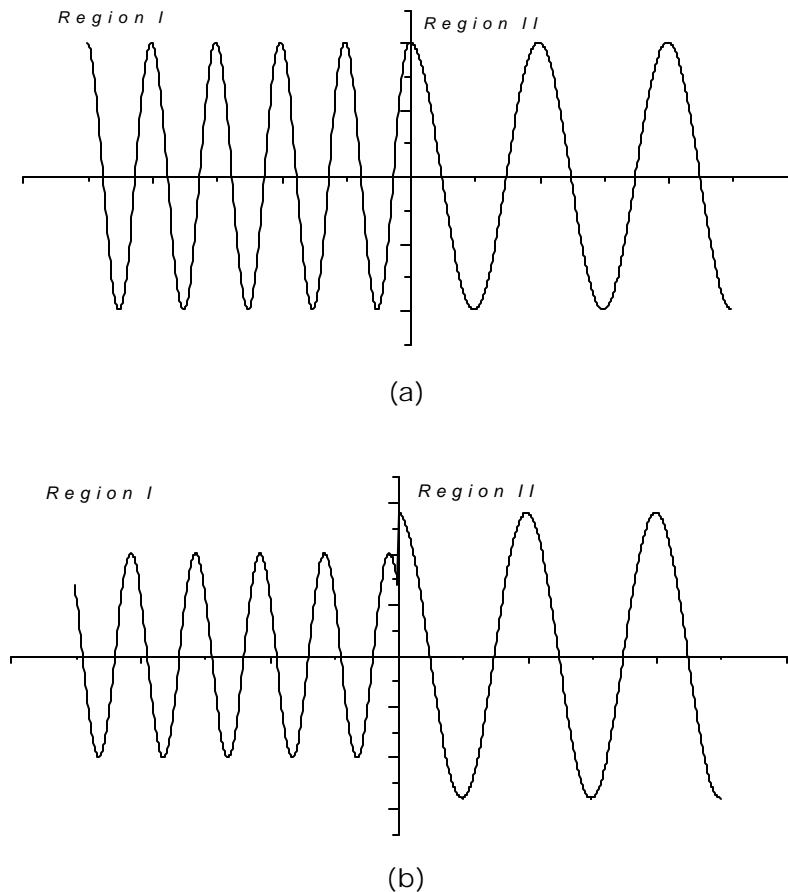


Figure 6-4: You must consider how to join segments to form a wave function.

In Figure 6-4a the wave functions connect smoothly, while in Figure 6-4b they do not connect. Most people immediately think that (a) looks better. We will now discuss why “looks better” is correct in terms of the physics. To understand why, we need to look at the probability interpretation of the wave function and how that interpretation affects the wave function when the potential is changing.

? Can you determine the probability of finding the electron at the boundary in Figure 6-4(b)?

You should find this question difficult. At the boundary the wave function has two different values. Thus, we cannot uniquely state the probability of an electron being detected there. Therefore, we must reject this type of wave function because we cannot use it for determining probabilities. (We recommend that you draw a big X through Figure 6-4(b). Then, when you look back at it later, you will be reminded that it is not acceptable.)

Interpreting the wave function as a measure of the probability of finding an electron at a particular location forces a condition on the wave function. At boundaries where the potential energy changes, the wave function must make a smooth connection between its segments in the two regions. If the segments did not meet at the boundary, then each would give a different probability of detecting the electron there. Since both values of the wave function represent the electron at the same location, two different values for the probability at the same point would be meaningless.

Another possibility which will give only one value for the boundary is shown in Figure 6-5. The wave function is not smooth, but it has only one value at the boundary.

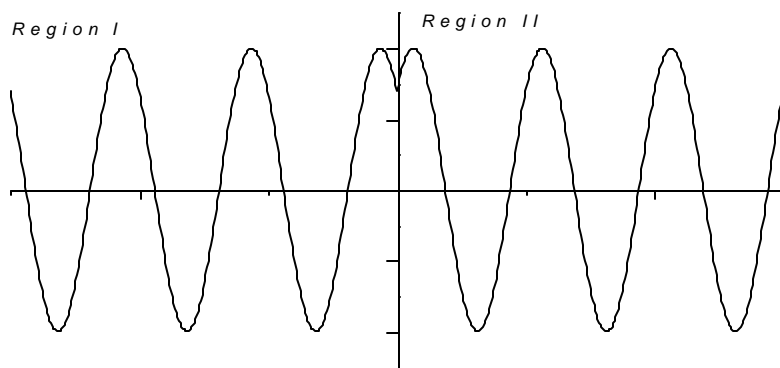
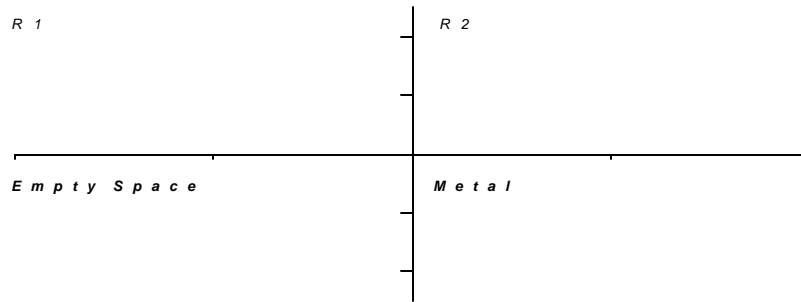


Figure 6-5: Wave functions with kinks are also not acceptable.

This type of wave function also gets a big X. Schrödinger's Equation can be used to calculate the probabilities for the speeds of the electron. A wave function such as the one in Figure 6-5 will give two values for the speed probabilities at the boundary. So we must reject it.

The only acceptable wave functions are smoothly connected at all boundaries. When creating wave functions we must adjust parameters to create a smooth connection at all boundaries. However, we cannot mess with the wavelength; it is determined by the kinetic energies. We can change the amplitude and the phase. Adjust these parameters in your wave function until the shape at the boundary is acceptable. Sketch your result below.



You have now completed the third step in creating a wave function.

Step 3. Adjust the phase and amplitude of each part of the wave function until it is smooth across the boundary.

You have completed the steps in creating the wave function. The next step is to interpret it in terms of probability of the electron at various locations.

Step 4. Interpret the wave function and discuss the probability of finding the electron at various locations.

Complete this step for your wave function. Sketch the probability density below and write a short paragraph about what you can learn from it.

We have now established a way to create a wave function when we know the physical situation. The four steps require that we set up energy diagrams; determine the electron wavelength, adjust the wave function to be smooth across the boundary and interpret the results. From this process we can predict probabilities concerning where electrons will be. We will continue these procedures in the next activity. We will consider a different situation and come closer to understanding why atoms can have only certain energies.

An Application

Now you will apply these steps to sketch wave functions for electrons in an electron microscope. Electron microscopes come in two basic varieties. The electrons in a scanning electron microscope strike a sample and interact with the electrons of the atoms on the surface of the sample. By observing the results of these interactions an image is created. In a transmission electron microscope, the sample is made very thin, so that the electrons pass through it. These electrons are picked up by a detector on the other side, and carry information about the material of the sample. We will consider the latter type here (See Figure 6-6).

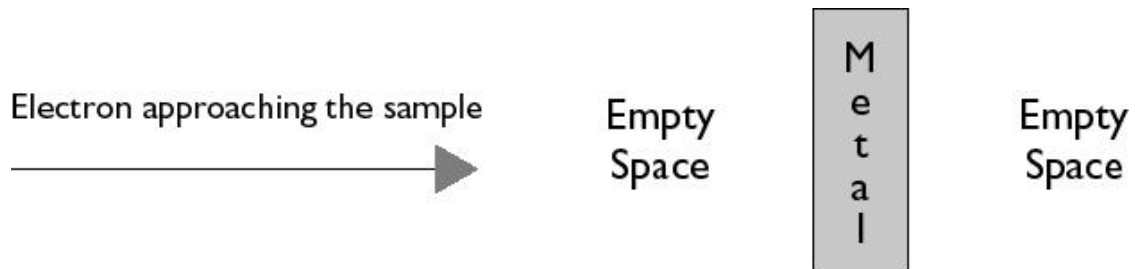


Figure 6-6: Electrons in a transmission electron microscope passing through a thin sample.

In this example the microscope gives the electrons a total energy of 3 eV. They are approaching a sample in which the potential energy due to repulsion from other electrons is 1 eV.

Draw the potential energy diagram of the electrons indicating the potential and total energies inside the sample as well as in the empty space on either side.

Use the *Wave Function Sketcher* and the steps listed above to create the wave functions for the electrons in each region. Sketch your result below. (Be sure to indicate the location of the boundaries.)

Sketch the probability density and write a short interpretation.

- ? If the sample is 10 nm thick, how many wavelengths of the electron wave will have to pass through the sample, before the electron emerges on the other side?

ACTIVITY 7

**More Steps for Sketching
Wave Functions***Goal*

We now consider wave functions for situations where the total energy is less than the potential energy. We will learn that these wave functions have a property that is significantly different from ones we have discussed so far.

In the previous activity we considered electrons which entered a TV screen and had enough energy to go on through. However, the probability interpretation indicates that not all electrons just keep going. If they did, the probability on each side of the boundary would be identical. Some of the electrons bounce back, but most go on through.

In many situations electrons arrive at a metal where they do not have sufficient energy to go through. The potential energy results from a repulsion and is larger than the total energy of the incoming electrons. Such a situation is shown in Figure 7-1.

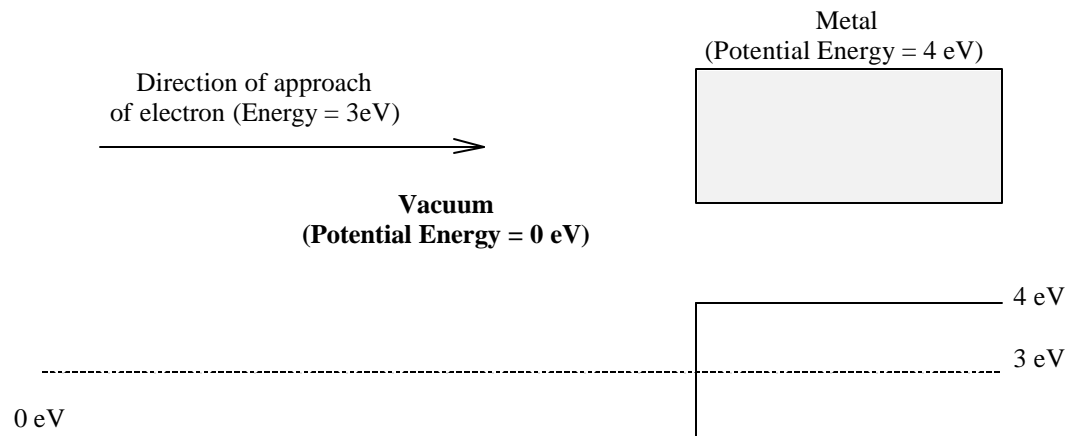


Figure 7-1: Potential & Total Energy of an electron approaching a sample.

For this situation, describe the motion that you would expect for the electrons.

Calculate the kinetic energy and wavelength of the electrons in empty space.

Calculate the kinetic energy of the electron inside the metal.

You have just uncovered a problem. The kinetic energy that you calculated is negative. But, only positive quantities — mass and speed squared — are involved in kinetic energy. It must be a positive number.

Our first conclusion could be that electrons cannot be in the metal. That works if we only consider energy, *but* we must also consider the wave behavior of the electrons. Remember that the probability interpretation requires a smooth connection at the boundary. Wave functions such as the one in Figure 7-2 give zero probability inside the metal but fail the smoothness test. (Another candidate for a big X.)

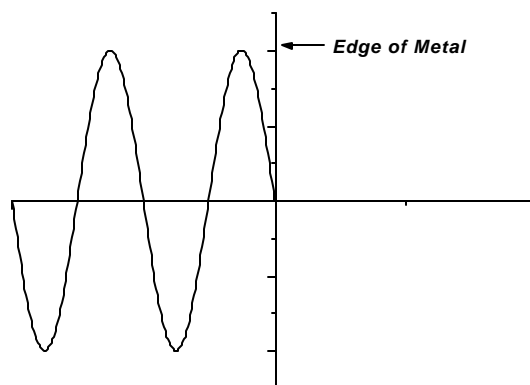


Figure 7-2: This wave function is not smooth at the boundary.

While the wave properties are important, energy conservation must also be taken into account. Suppose electrons interact with a large region where they do not have enough energy to go. If we go far into that region, we do not expect to find electrons (Figure 7-3).

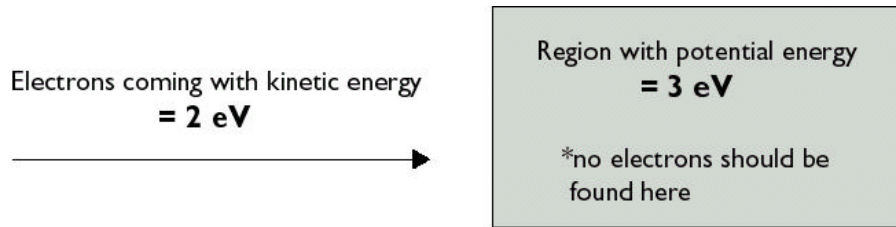


Figure 7-3: Energy considerations tell us that electrons will not be found on the far right for this situation.

So, the wave function in the metal must satisfy two independent criteria:

- (i) It must be consistent with a zero probability of finding the electron far into the metal (energy).
- (ii) It must connect smoothly to the wave function on the left (wave behavior).

A clue to dealing with both of these criteria is given by the behavior of light waves. In special circumstances, light can exist where we think it should not be — but only for a very short distance. When light penetrates into these regions, it is no longer an oscillating wave. Instead, the light's magnitude decreases rapidly. It connects smoothly with the oscillating wave but rapidly decreases its value to essentially zero. Figure 7-4 shows such a situation.

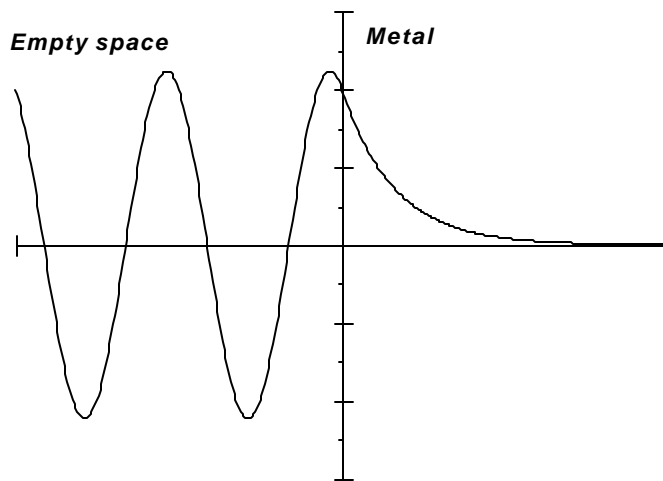


Figure 7-4: The wave function of the electron in empty space and the metal, when the electron's total energy is less than its potential energy in the metal.

The graph in Figure 7-4 also meets our criteria for the electrons. The wave functions in the metal and empty space connect smoothly. It decreases to zero to insure a zero probability of finding the electron in that region.

Generalizing from the wave function in empty space and the metal, as sketched in Figure 7-4 we can arrive at the following recipe for sketching the wave function:

- If the total energy of the electron is greater than its potential energy (i.e. Kinetic Energy is positive), then the wave function is an oscillating wave (as in the empty space in Figure 7-4).
- If the total energy of the electron is less than its potential energy (i.e. Kinetic Energy is negative), then the wave function is decreasing.

A decreasing wave function does not necessarily have to decrease from a positive value to zero, it could also “decrease” from a negative value toward zero. For instance, another valid wave function corresponding to the above physical situation is shown in Figure 7-5.

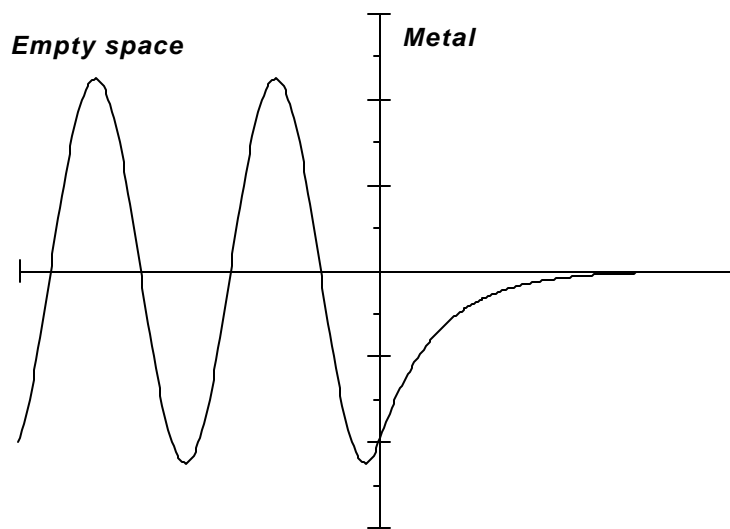


Figure 7-5: Another possible wave function for an electron that enters a metal where its potential energy is greater than its total energy

Sketch the probability densities for the wave functions in Figure 7-4 and 7-5.

How are the probability densities similar?

We have come to the shape of this wave function by logic and analogy. If we were using heavy-duty mathematics, we could get the same result using Schrödinger's Equation. Solving the equation also tells us the dependence of the rate of decrease of the wave function on the *difference* between the total energy and potential energy. But, you can use logic and a little intuition to get the general idea.

Consider the two situations in Figure 7-6:

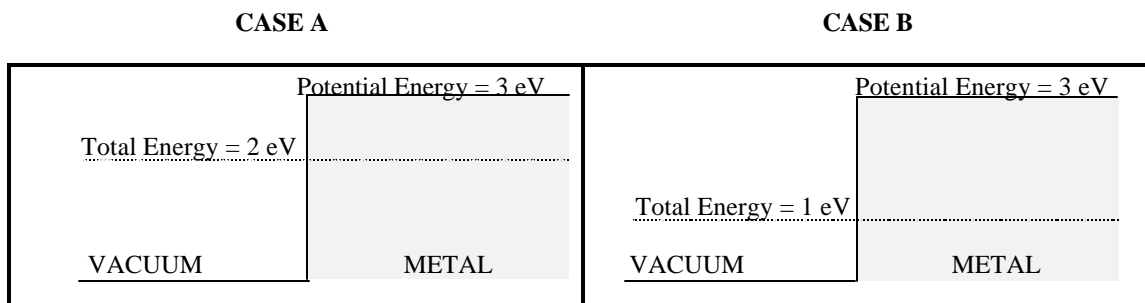


Figure 7-6: In which situation will the rate of decrease on the right be greater?

- ? In which of the two cases in Figure 7-6 will the wave function in the metal decrease more rapidly?

- ? Explain your answer.

Your intuition probably served you well. The larger the difference between the potential and total energies, the more rapidly the wave function decreases.

Using Schrödinger's Equation we can make the conclusion quantitative. We define a decrease length as a distance from the boundary to where the wave function has dropped by a specified fraction. The decrease length is

$$\text{decrease length} = \sqrt{\frac{(\text{Planck's Constant})^2}{8p^2(\text{mass})(V - E)}}$$

In a distance equal to the decrease length the value of the wave function drops to about 0.4 of its value at the boundary (See Figure 7-7). In two times the decrease length the value is $(0.4) \times (0.4) = .16$ of its value at the boundary, and so forth.

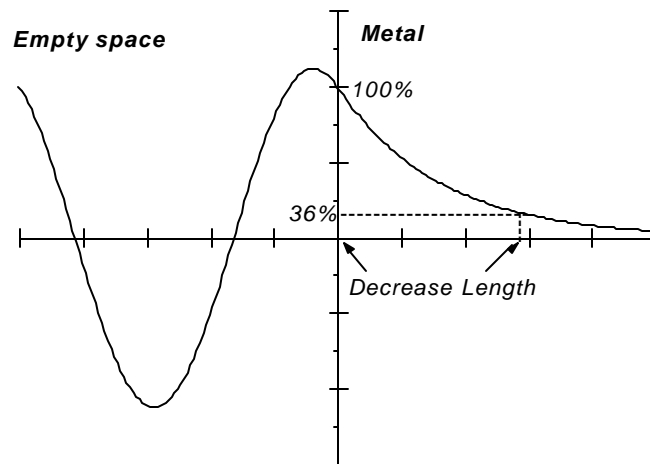


Figure 7-7: The Decrease Length is the distance in which a wave function decreases to about 36% of its initial value.

This equation works for any object. It states that any object has a probability of being in areas where it does not have the energy to be. The physics of large objects tell us that this cannot happen, but wave nature of matter says that it must.

To see how we can reconcile the physics of Newton with that of Schrödinger, consider the gnat discussed in Activity 4. It has a mass of .001 kg and a speed of 0.10 m/s. Suppose that it hits a wall with a potential energy of 1 Joule. (Planck's constant = 6.63×10^{-34} J·s)

- ? What is the decrease length for the gnat? (An approximate answer is good enough.)

- ? Would you expect a gnat to be found on the inside of a wall?

As you can see, the length is so small that Newton would never have noticed. Even with today's technology we could not measure such a small distance.

The equation on the previous page can be used for other objects. As with the de Broglie wavelength the magnitude only becomes useful when the mass is very small. For electrons the decrease length becomes

$$\text{decrease length} = \sqrt{\frac{0.04nm^2 eV}{(Potential\ Energy - Total\ Energy)}}$$

The energies must be in electron volts and the result is in nanometers.

Calculate the decrease length of an electron for Cases A and B in Figure 7-6.

Sketch approximately the wave function and probability density for each case.

The decrease length is an option in the *Wave Function Sketcher*. Set up a situation similar to Figure 7-1 which is repeated as Figure 7-8 below.

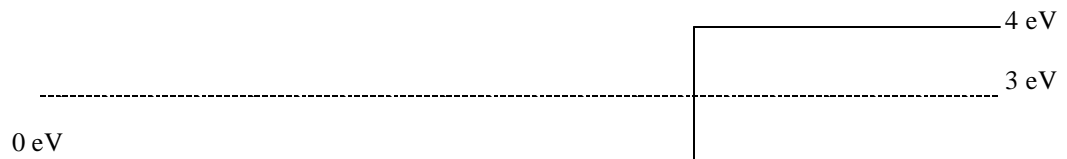
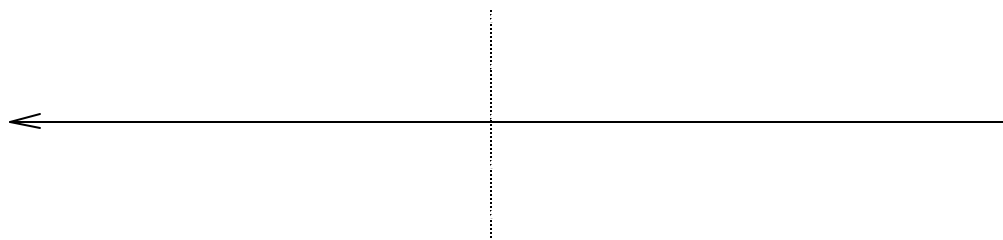


Figure 7-8: A repeat of Figure 7-1.

Using the decreasing options, sketch a wave function that is acceptable for this situation.

Sketch below the wave function as displayed by the program.



Now we must modify the steps for sketching wave function to include both oscillating and decreasing shapes. The shape of a wave function depends upon the Total Energy and Potential Energy. If the potential energy is less than the total energy, the wave function is oscillating. If the potential energy is greater than the total energy, we get a decreasing wave function. Step 2 must be modified to include both possibilities. Below is a list of the steps that will work for all cases.

Creating and Interpreting Wave Functions

1. Use the physical situation to draw the energy diagram and determine the boundaries.
2. Use the total and potential energies to determine if the wave function is oscillating or decreasing, then follow A or B below.
 - A. Oscillating: Determine the wavelength for the oscillations.
 - B. Decreasing: Determine the decrease length.Then, sketch the wave function.
3. Adjust the phases and amplitudes until the wave function is smooth across all boundaries.
4. Interpret the probability density and discuss the probability of finding the object at various locations.

Application

Now you can use the steps to sketch wave functions for an electron in a transmission electron microscope. Consider an electron that has a total energy of 1eV, and it is approaching a metal sample in which its potential energy due to repulsion from other electrons is 3eV. (Figure 7-9)



Figure 7-9: An electron passing through a sample in a transmission electron microscope.

- ? Establish the regions. Then, draw the potential energy diagram of the electron indicating its potential and total energies inside the metal sample as well as in the empty space on either side.

- ? With the steps that you have learned, use the *Wave Function Sketcher* program to sketch the wave functions for the electrons in each region. Draw the wave function below.

- ? Write a brief interpretation of the wave function by concentrating on the probability of an electron being in each region.

- ? Sketch the probability densities.

In this case some of the electrons may be found on the right side of the metal. Conservation of energy would say that none of them should be there. This effect of electrons being where they should not be is called quantum tunneling. It is observed in nature.

Summary

We must include the wave behavior of small objects. We found that matter can penetrate into regions where they are not allowed by conservation of energy. To account for this behavior we modified the steps for sketching wave functions. The result is that small objects are sometimes found where they would not be allowed by conservation of energy. The wave nature of matter forced this conclusion on us. And, experiments have confirmed that it does happen.

ACTIVITY 8

Wave Functions for Electrons in Atoms*Goal*

We apply the steps in creating wave functions to a model of the atom.
From the results we learn why only certain energies are allowed in atoms.

So far we have applied the wave behavior of small objects to beams of electrons. While such beams exist in television and computer monitors, they are not as common as other small objects such as atoms. In this activity we will explore the wave functions associated with electrons in atoms. In the process we will begin to understand why the wave behavior of electrons leads to the spectra of light from atoms.

To apply the steps for creating wave functions we must first establish a potential energy for the electron in an atom. That is: we need to create a model for the atom. We start by noting that the attraction between electrons and the nuclei of atoms is electrical. The potential energy graph for an electron experiencing a one-dimensional electrical force is shown in Figure 8-1a. This graph looks relatively simple and could be the basis for a model of the atom. However, because the energy is constantly changing, we would have trouble applying the steps to wave functions. The graph in Figure 8-1b simplifies the changing potential. It could be used, but the version in Figure 8-1c is even simpler and serves our purposes well. We will use it for our potential energy of an electron in an atom.

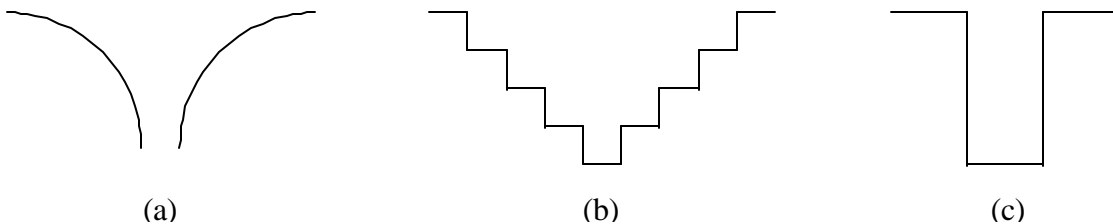


Figure 8-1: Three possibilities for a potential energy diagram of a one-dimensional atom.

The electron stays in the atom. Thus, its total energy will be negative. (See Activity 2.) An energy diagram, such as Figure 8-2, represents both the electron's total and potential energies.

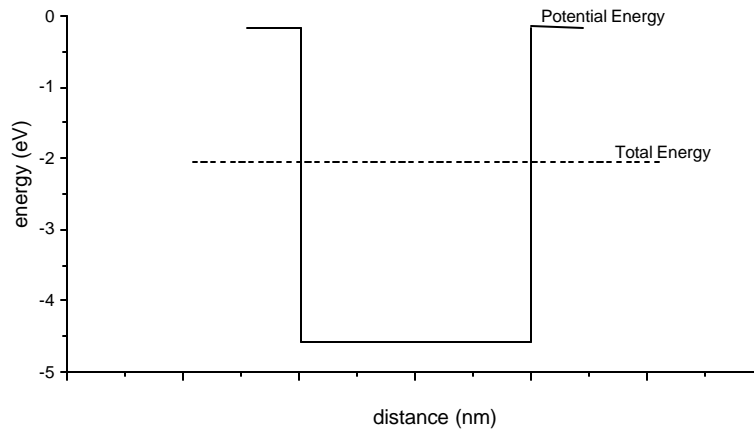


Figure 8-2: The potential and total energies for an electron in a model of an atom.

We will use this diagram to explore wave functions of electrons in atoms. Start the *Wave Function Sketcher*. Create a potential energy diagram similar to Figure 8-2. Follow the steps to a wave function and attempt to create a wave function for all three regions. Be certain to calculate the wavelength using the de Broglie relationship and keep it fixed.

If you have difficulties sketching the wave function, describe the problems below. Sketch your best wave function below even if it is not quite smooth at both boundaries.

? Compare your results with others in the class. How are they similar and different?

? You probably found making smooth connections at both boundaries difficult. Get one right and the other one is messed up. A few groups *may* have been able to draw it. If any group has been successful, compare their results to the unsuccessful ones. What differences can you detect?

? What would you need to change to create an acceptable wave function?

To see that we can draw wave functions for this situation, set up the following parameters.

Potential Minimum = -4.32 eV Potential Width = 0.71 nm Total Energy = -1.32 eV

Sketch your wave function below.

You probably found this wave function much easier to draw. That's because the parameters are "just right." You can see by looking at the wave function how it fits nicely into the available space. Other similar examples are shown in Figure 8-3.

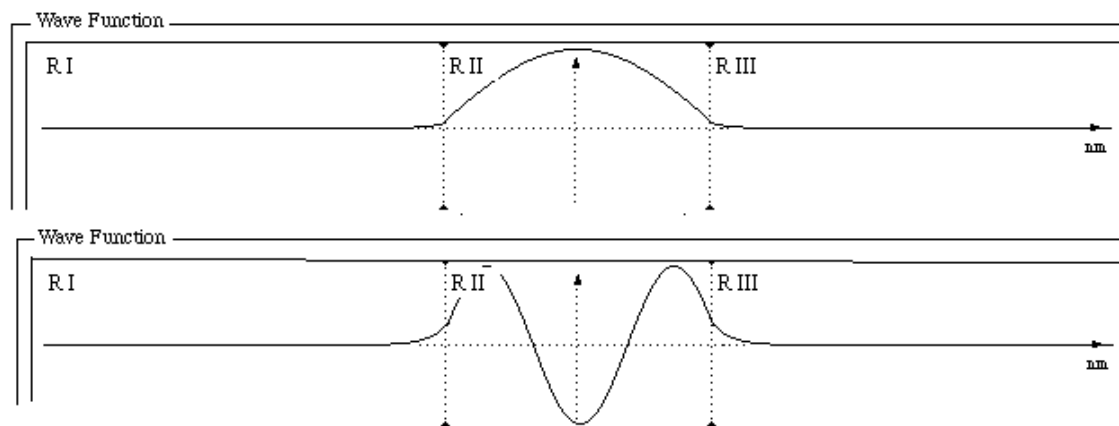


Figure 8-3: Some wave functions which just fit in the potential energy with minimum = -4.32 eV and width = 0.71 nm.

Our conclusion is: Only certain sets of parameters work to give an acceptable wave function. The potential energy minimum, its width and the electron's total energy cannot be any value. They must have the right numbers.

As we discussed in previous activities the potential energy parameters are determined by the physical situation. For the atom the force of attraction between electron and nucleus determines both the minimum value and the width. For example, suppose we are considering hydrogen atoms. Then the width and minimum will be set by the electrical charge on an electron and a proton. Thus, we have only one variable — the total energy — which we can change. We select the total energy so that the wavelengths just fit. Then we can create an acceptable wave function.

The wave function of an electron in an atom is restricted. Its wavelength must be just right to fit in that atom. Wavelength is related to energy. Thus, the energy can have only certain values. An electron in an atom has very limited possibilities for energy. This conclusion comes directly from the wave behavior of electrons.

In the next activity we will pursue this limitation on energies. Then, we will have everything we need to explain our observation in Activity 1: Atoms emit only certain energies of light.

Name:

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WAVES
of matter

Visual Quantum Mechanics

ACTIVITY 9

Energies in Atoms

Goal

We will investigate wave functions of electrons in atoms. By looking at several possibilities we will see why different atoms have different spectra.

In the previous activity, you tried to create the wave functions for an electron in an atom. It was possible to get an acceptable wave function for only a limited set of parameters. This result led us to conclude that only “special” energies were possible for the electron in an atom. In this activity, we study more about the energies of electrons in atoms and learn what makes certain energies special.

The process used in the previous activity is rather tedious. So we will switch to a computer program. The program is programmed to follow a procedure similar to Activity 8. It is faster than we are and does not get tired of doing the same thing many times.

Start the *Bound States* program (Figure 9-1). It starts with a potential energy similar to our model. Select a total energy for the electron by clicking anywhere on the potential energy diagram on the left side of the screen. A red line appears on the potential energy diagram that indicates the value of the total energy. For any energy, the program can determine two possible wave functions. One of these wave functions has a zero in the middle of the negative potential energy region, while the other has a maximum there. Both wave functions are solutions for the Schrödinger equation. Physicists label these wave functions as symmetric and antisymmetric. Those terms have little meaning for our purpose, but we have to give them a name. So we use symmetric (maximum in the middle) and antisymmetric (zero in the middle).

Kansas State University

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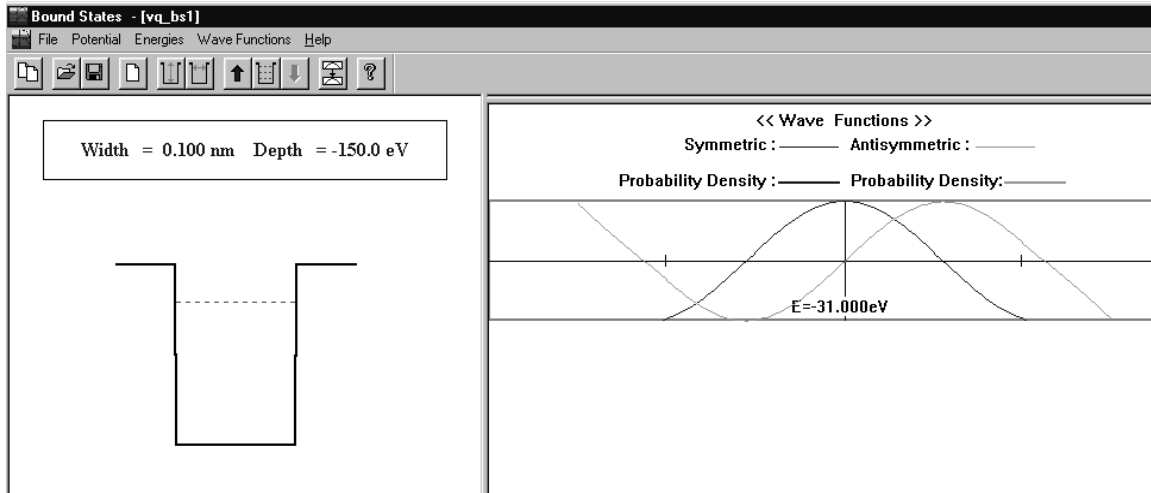


Figure 9-1: Bound States program screen

In this mode *Bound States* is both smart and dumb. It is smart because it can solve Schrödinger's Equation. For any energy it creates a wave function with smooth connections at each boundary. It is dumb because it pays no attention to physics. The wave function may not make sense.

To think about the physics consider the location of an electron in an atom. Is that electron likely to be near the nucleus or very far away?

Now, look at the wave function in Figure 9-2.

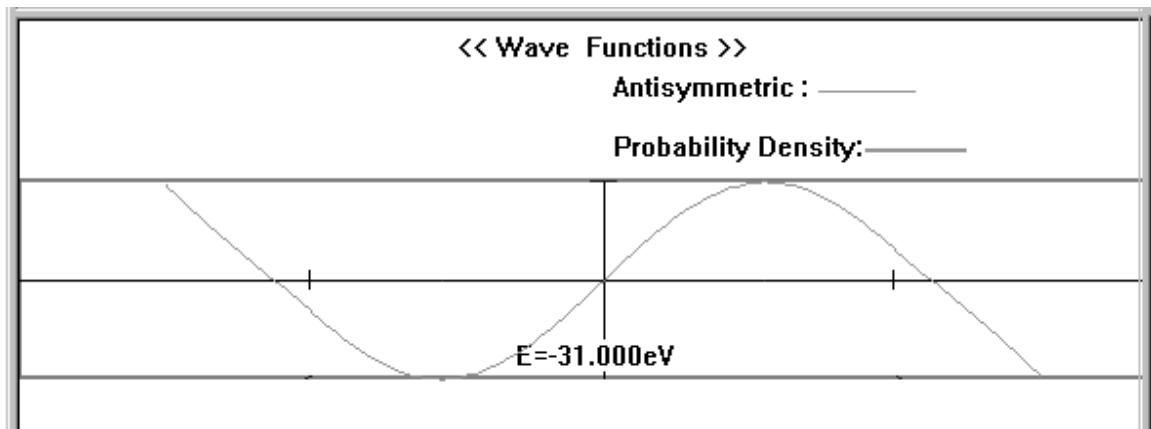


Figure 9-2: An unacceptable wave function. The vertical marks on the axis are the edge of the atom.

We must reject this wave function. It does not fit our steps. Its value does not decrease to zero when the potential energy is greater than the total energy. Both ways say the same thing. Common sense tells us that an electron should probably be near the nucleus. Wave functions that say otherwise are no good. In other words, we reject any wave functions that predicts a high probability of the electron being very far from the nucleus.

So, the program can be dumb. But, it can help us find acceptable wave functions. Use the up (\uparrow) and down (\downarrow) arrow keys on your keyboard to slowly change the energy level. The energy level will change by the **Energy Step**, (= 0.1 eV) each time you press an arrow key. To increase or decrease the **Energy Step**, use the left (\leftarrow) or right (\rightarrow) arrow key on your keyboard and change the Energy Step to 1.00eV or 0.01eV. Change the energy level until you notice that at least one of the wave functions is consistent with the requirement that the wave function goes to zero outside the atom.

Sketch this wave function.

- ? Compare your wave functions with those of others in the class. What are the similarities?

- ? What are the differences?

- ? Press the *Keep* button on the right of the screen to store the wave function in one of the windows. Continue using the arrow keys and find another acceptable wave function. Sketch it below.

? How does it differ from the previous acceptable wave function?

We could continue this process and find all acceptable wave functions and energies. However, the computer can be made smarter. It can complete the process automatically. It can quickly try hundreds of wave functions, reject all that have high probability densities outside the atom and keep the rest.

Select **Energies/Search Allowed Energies** from the pull-down menu. Horizontal lines corresponding to each of the allowed energies appear.

? Record the value for all allowed energies and the potential energy depth and width.

The energies which are shown on the screen are obtained by solving Schrödinger's Equation for the potential energy on the screen. In obtaining these energies we have used

- the wave behavior of electrons,
- the probability interpretation of wave functions, and
- a simplified potential energy for the atom.

With these assumptions we see that only certain well-specified energies are allowed for the electrons.

From Activity 2 we know that changes from one energy to another causes light to be emitted. From the results here we see *why* only certain energies are allowed. The wave nature of the electron requires it.

Our present potential energy model is somewhat simplified. Thus, we do not get the correct values for the energies. Really doing it right would require three dimensions and a more accurate potential energy. However, the basic conclusions would not change; only the numbers would change.

You can view the wave function corresponding to any allowed energy. Click on the total energy outside the vertical potential energy lines. The wave functions corresponding to the energy level appears to the top right. The program shows both wave functions. Only one is acceptable. You must decide which one.

Sketch the acceptable wave function for the lowest energy level.

“Keep” the wave function. Now view the wave functions for the *highest* energy.

Sketch the acceptable wave function for the highest energy level.

? How are the wave functions of these two allowed energies that you have sketched above similar to each other?

? How are they different?

We must address one final issue — the reason that different elements emit different energies of light. The electrical charge of the nucleus is different for each element. In our model that means that the potential energy is different for each element. Let us see how the allowed energies and wave functions depend upon the potential energy.

Use the **Potential/Change Depth** and **Potential/Change Width** menus to create a different potential energy. Use **Energies/Search Allowed Energies** to determine the allowed energies for this new atom.

? Record the values of all allowed energies and the potential energy depth and width.

? How do these allowed energies differ from the previous ones?

Other groups in the class will have changed their potential energy in ways different from yours. Collect information from other groups so that you can answer the following questions.

? How does the number of allowed energies change as the potential energy depth is increased?

? How does the energy difference between the allowed energy levels change when the depth of the potential energy is increased?

? Can you use this information to describe why different elements have different spectra? If you can, do so. If not, discuss it with your instructor.

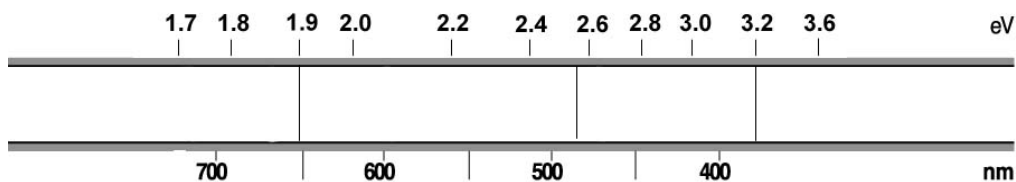
Summary

We began this set of activities with the observation of the light emitted by atoms. By trying to understand light we have learned about the wave nature of matter and the probabilistic nature of knowledge. Along the way we have seen that small objects can sometimes be where they would not be allowed by using only energy considerations, that electrons can be used to see very small objects and that the spectra of elements is explained by this wave nature.

The theory that incorporates the wave nature of matter is now called *quantum mechanics*. The explanation of spectra was an early triumph for quantum mechanics. However, the story does not end there. Quantum mechanics was also needed to explain the way atoms join to form molecules and the organization of the periodic table. All types of small solid devices, such as computer processors and memories are available today only because the designers understood quantum mechanics. Thus, a large fraction of our daily lives are in some way dependent on quantum mechanics. If scientists and engineers did not understand its principles, our lives would be quite different. The concepts that you learned here are likely to increase in importance during the 21st Century.

Homework Application

Suppose you look at a gas lamp through a spectroscope and see the following spectrum.



Assume that the spectrum is created by transitions *only* between various upper energy levels and the lowest energy level in an atom and that the lowest energy is -6.0 eV.

? Sketch the energy levels of the atom.

ACTIVITY 10

Wave Functions for Lonely Electrons*Goal*

We continue to refine the wave function we use to describe an electron. We will discover that the electron's wave properties actually prevent us from making exact measurements of its position and momentum simultaneously. This result leads to philosophical issues related to the nature of knowledge.

In our descriptions of electrons so far, we have been assuming that electrons move in the beam and all of them have identical energies. This situation has been useful, but it is somewhat artificial. For a real world we need to describe individual electrons as well as beams.

First, let's see why our present version has problems when dealing with an individual electron. Suppose we wish to use a wave function that travels straight across this line of the page. It moves from left to right without changing its speeds. It interacts with nothing. So, the potential energy is zero. One electron, no interactions, and no energy changes. Sounds like a simple situation.

The *Wave Function Sketcher* would give us the result in Figure 10-1.

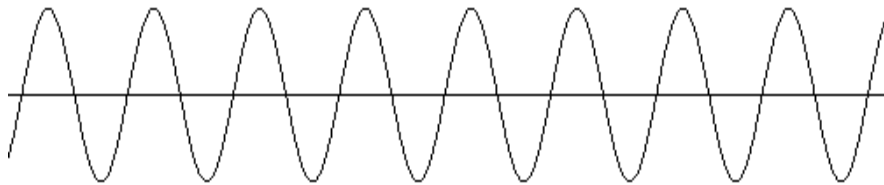
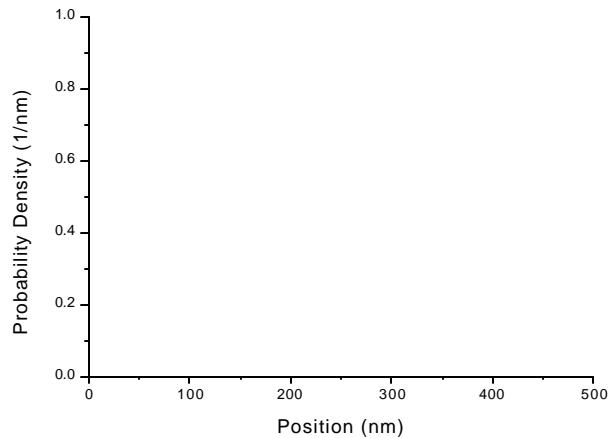


Figure 10-1: The *Wave Function Sketcher* result for the situation described above.

? Does this wave function match the description above? Why or why not? (If you have difficulty with this question, think about the probability of finding the electron at various locations.)

The difficulty is that the probability density is rather uniform across the page. Also, it does not change with time. Let's create a better wave function for a single electron. In the diagram below, sketch an approximate graph of probability density versus position for an electron in the middle of the page. The approximate shape of the wave function, not the numerical value at any point, is the important feature.



- ? Explain why you drew the wave function as you did both *near* the location of the electron, and far *from* it.

- ? How would you expect this wave function to change as the electron moved across the page?

A possible representation for the electron moving across the screen is shown in Figure 10-2. (*Quantum Motion* can show animations of this.)

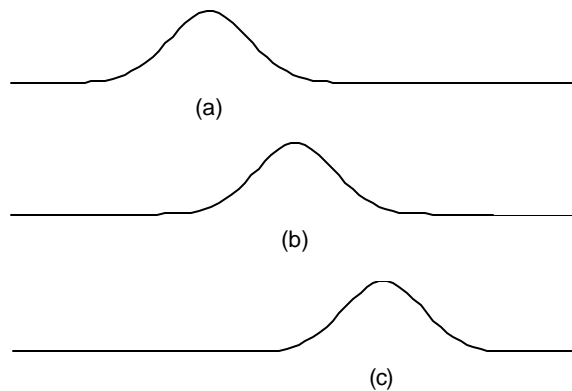


Figure 10-2: Three snapshots of a wave function representing an electron traveling across the page.

This wave function has a probability different from zero only in a small region. It moves as the electron moves. So, it fits the need for our purposes.

All measurements indicate that the electron is a very small object. It is essentially a point in space. So, the “bumps” on the wave functions in Figure 10-2 indicate that the probability of finding the electron is different from zero at several different locations.

Unfortunately, such a wave function does not just “pop out” as a solution to Schrödinger’s Equation. So we must learn to build it from other solutions.

The ability of waves to interfere both constructively and destructively enables us to combine several different waves to create *waveforms* with useful shapes. Because waves interfere with one another both constructively and destructively at the same time but in different locations, we can never create a wave function that is not zero at one location, but zero everywhere else. So, to represent a single electron, we must construct a wave function that, when squared, gives a probability density with one maximum, decreases sharply away from that maximum, and then is zero at relatively large distances for where we expect the electron.

Even simple waveforms like the disturbance on the rope can be constructed by adding simple waves together. Our goal will be to combine simple waves to produce a wave function that, when squared, gives a smooth probability density graph similar to Figure 10-3:

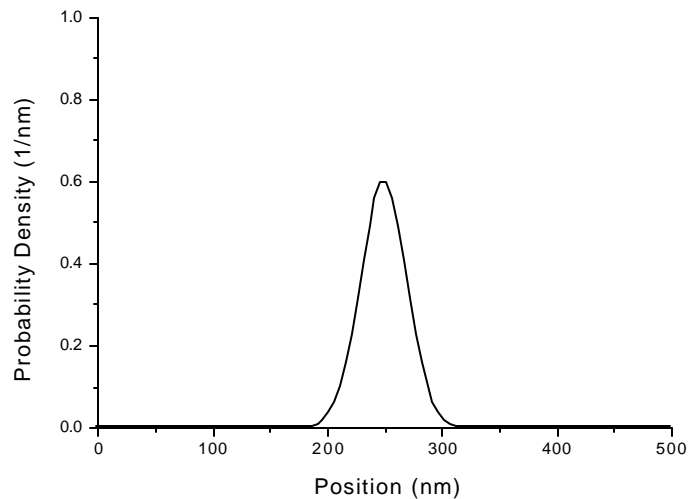


Figure 10-3: Typical shape for the probability density of a *localized* electron.

To see how we might create such a wave look at the addition of waves shown in Figure 10-4.

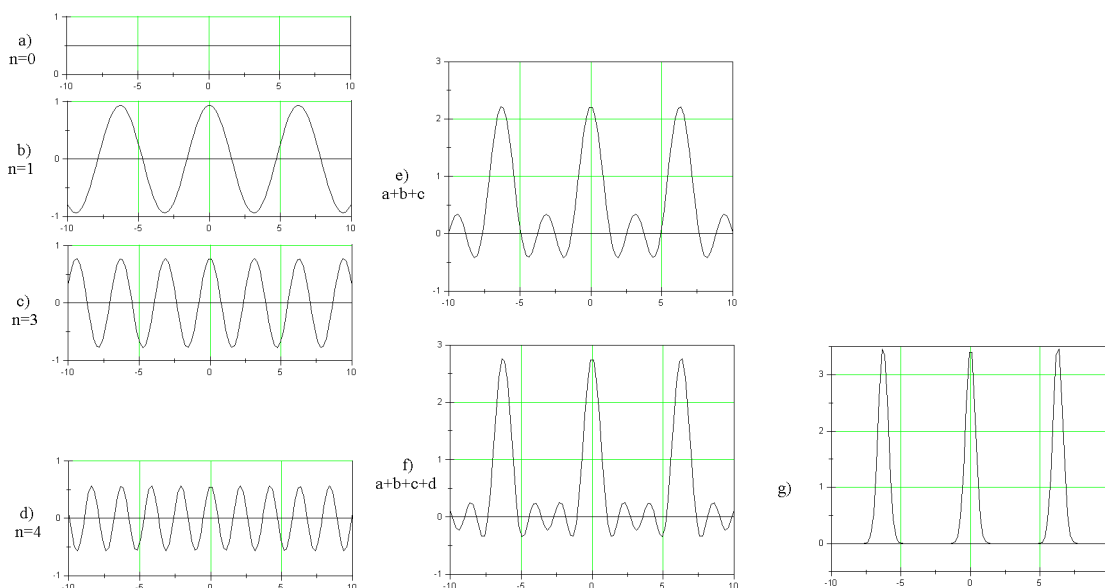


Figure 10-4: the three waves (a-c) on the left, gives the one on the right (e). Adding even more waves can result in a wave form similar to Figure 10-3.

By adding hundreds of waves with carefully selected wavelengths and amplitudes we can come very close to the form in Figure 10-3.

From the de Broglie relation we know that wavelength is related to momentum. So, when we add wave functions of different wavelengths, we are adding wave functions, which represent objects with different momenta.

To create wave functions like this use the *Wave Packet Explorer*. This program allows us to add wave functions of different momenta (wavelengths) rather easily.

Click in the upper left window (*Amplitude vs. Momentum* graph).

A vertical line appears representing a wave function whose amplitude is proportional to the length of the line, and whose momentum is the value at the line on the horizontal axis. The wavelength is calculated from using the de Broglie relation. The wave function appears directly below.

Repeat this procedure by first clicking on the graph in the upper left window, at any value of position and amplitude and creating a new wave function. Each time, a simple wave function is added to all the wave functions that were already present. The individual waves are shown on the graph on the bottom left while the sum of these waves is just above these.

? How does the resulting added wave function change as you increase the number of simple wave functions?

Now, instead of adding just a few wave functions, add a large number of them. Click many times in a definite pattern to create a large number of wave functions. Sketch the resulting wave functions below.

Compare your resulting wave functions with others in the class. Find a couple that are quite different from yours. Describe how their momentum-amplitude graph is different from yours.

Some of these patterns are probably similar to Figure 10-3. Try an amplitude-momentum distribution similar to the one in Figure 10-5.

To come even closer you can add a very large number of momenta. Hold down the shift key and the left mouse button and drag the cursor across the amplitude-momentum graph. This process adds together all of the wave functions with momenta and amplitudes in the shaded region.

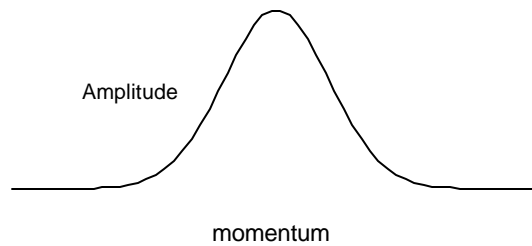


Figure 10-5: An amplitude-momentum distribution that gives a wave function similar to the one in Figure 10-3.

(You can look at any one wave function in all of the collection by moving the mouse over the line in the momentum graph. Both the momentum line and the corresponding wave function in the bottom graph will turn green.)

Sketch the position wave function below.

? What is the probability interpretation of the wave function?

? Does this wave function have a large or small range of momenta associated with it?

Now try a wave function which has a very narrow range of momenta. Create it by dragging over a small range.

Sketch the position wave function below.

? How is the probability interpretation different from the previous wave function?

Now, try to create

- a wave function that represents a high probability of the electron being in a very small region of space, and
 - a wave function that represents an electron that has equal probabilities of being in many different regions.
- ? How do these wave functions (and the momenta represented by them) differ?

We can create a wave function that represents an electron in a small region of space. To do so we must add together many different simple waves. Each simple wave has a different wavelength. Because wavelength is related to momentum, each simple wave has a different momentum.

Thus, we need many different momenta to create a wave function for one electron. We interpret this result to mean that the electron may have any one of the momenta. So, instead of having just one momentum, these localized electrons have a probability of having many different momenta. The probability of each momentum is related to the amplitude of the wave function with that momentum (Figure 10-6).

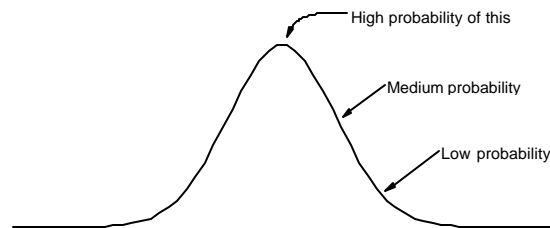


Figure 10-6: The probability of each momentum depends on the amplitude on the amplitude-momentum graph.

We must make a similar statement about position. Even when we add lots of waves with lots of different momenta, we still get a wave function similar to Figure 10-7.

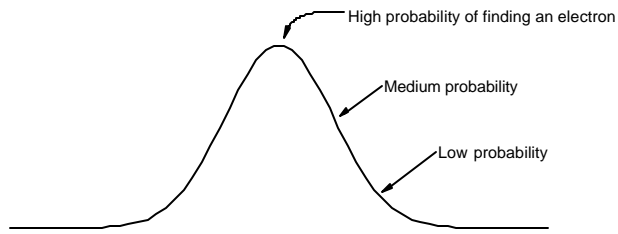


Figure 10-7: The relative probability of finding the electron at various locations for a localized wave function.

Summary

We can create a wave function that localizes the electron to a small region of space. However, not just to one point in space. In doing this we increase the possible range of momenta that the electron can have. We must talk about probabilities for both the electron's location and its momentum.

Name:

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ACTIVITY 11

Moving Lonely Electrons

Goal

We look at wave functions for moving electrons and describe the probabilities for its location.

We have now seen how to create a wave function for a single electron. This wave function provides information about both the location and momentum of the object. However, it cannot give us exact values for either quantity. We say that the wave function leaves us with an *uncertainty in position* and an *uncertainty in momentum* for the electron.

For the moment we will concentrate on the uncertainty in position. Use the *Quantum Motion* program to look at the wave function of an electron moving across the computer screen. We will use the same situation as in Activity 10:

- one electron,
- no interactions,
- no energy change

So, set the potential energy to zero everywhere.

The *Quantum Motion* program solves Schrödinger's Equation for the situation. It puts solutions that change with time on the screen. Thus, the wave functions represent a moving electron.

Press Play and watch what happens. Describe how the wave function changes.

Return to the starting position. Sketch the wave function below and indicate locations of high, medium and low probability of finding the electron.

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Now, play the solution, but *pause* when the wave function reaches the middle of the screen. Again, sketch it and indicate locations of high, medium, and low probability.

Continue until the wave function is near to the right side. Once again sketch it and indicate locations of high, medium, and low probability.

? Compare the three wave functions that you sketched. What changed as the electron moved?

? Other groups probably paused their wave functions at slightly different locations. Compare your conclusions to theirs. Are they similar?

You probably noticed that the locations of highest probability became small. The range of locations with medium and low probability became greater. (See Figure 11-1).

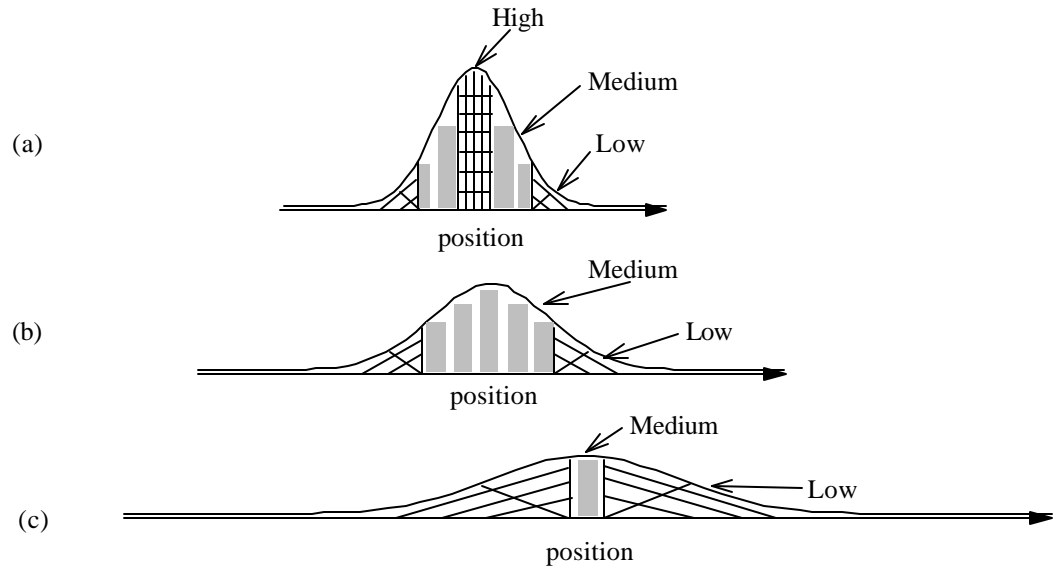


Figure 11-1: The wave function spreads out of time. Thus, the range of probability locations increases.

We can understand this result by returning to the momenta involved. We start with a wave function as in Figure 11-1(a). The electron can have many different momenta. (Remember we needed all those momenta to construct its wave function.) Of course, it also has a range of possible locations. So, we don't quite know where it is, and we don't quite know how fast it is moving — an uncertainty in position; an uncertainty in momentum. (See Figure 11-2)

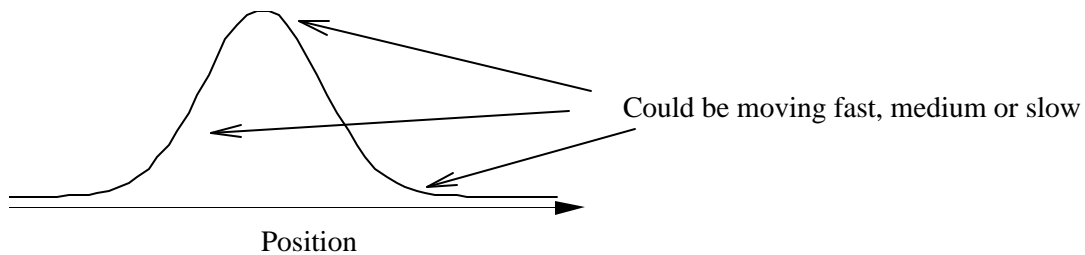


Figure 11-2: When we start, we have an uncertainty in both position and momentum.

A short time later we look to see where the electron is. We expect to still have an uncertainty in position because there was such an uncertainty when we started. In addition, the electron had an uncertainty in its momentum at the start. This uncertainty means that we do not know exactly how fast the electron is moving. The location of an object at a later time depends on both its starting point and its speed. In the case of the electron both the start and the momentum have uncertainties. As time passes the initial uncertainty in position is compounded by the uncertainty in momentum. So, as an electron moves the uncertainty in position increases.

This change in uncertainty is represented in *Quantum Motion* by a change in the wave function. As time passes, the wave function spreads out. This spreading indicates a wider range of positions is probable as the electron moves.

The rate of the uncertainty in positions depends on the uncertainty at the start. Two possible starting wave functions are shown in Figure 11-3. Wave function (a) is restricted to a very small region of space. Wave function (b) is not as restricted.

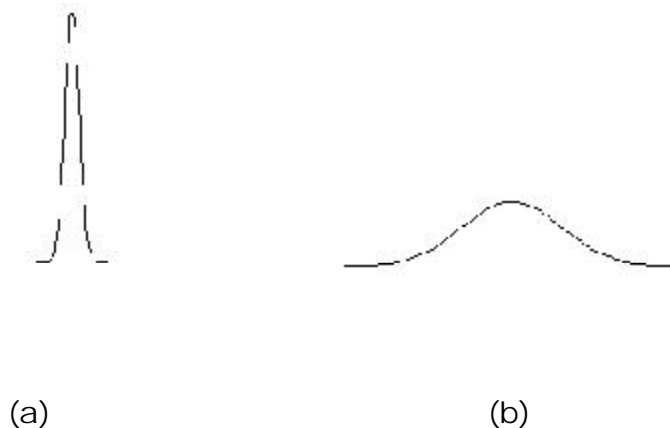


Figure 11-3: Wave functions that (a) represent an object restricted to a very small region of space and (b) not very restricted.

- ? Which one needs a larger number of momenta to create it? Refer to your experiences in Activity 10 to explain why.

- ? Use your answer above to determine which one will *change* the range of possible locations more quickly.

A very small uncertainty at the beginning results in a very rapid change in the wave function. A greater uncertainty at the start means the change in uncertainty is not so great. How well we know the position at one location determines how well we can know it later.

Summary

The uncertainty in position and momentum of a wave function results from restricting the object to a small region of space. The wave function changes as the object moves. Because of the initial uncertainties, our knowledge of position decreases with time.

Name:

Class:

WAVES of matter

Visual Quantum Mechanics

ACTIVITY 12

It Was Probably Heisenberg

Goal

We look more carefully at the uncertainties in position and momentum. We will learn how the uncertainty in one variable is related to the other. Finally, we will speculate about the value of the Heisenberg Compensator in the StarTrek Transporter.

In Activity 11 we saw that the uncertainty in momentum at one time affects our uncertainty in position at a later time. Now, we wish to investigate the relation between the two variables at any one time.

To begin let's use the *Wave Packet Explorer* to create wave functions with different uncertainties in momentum. Your instructor will select a different wave function from Figure 12-1 for each group to create.

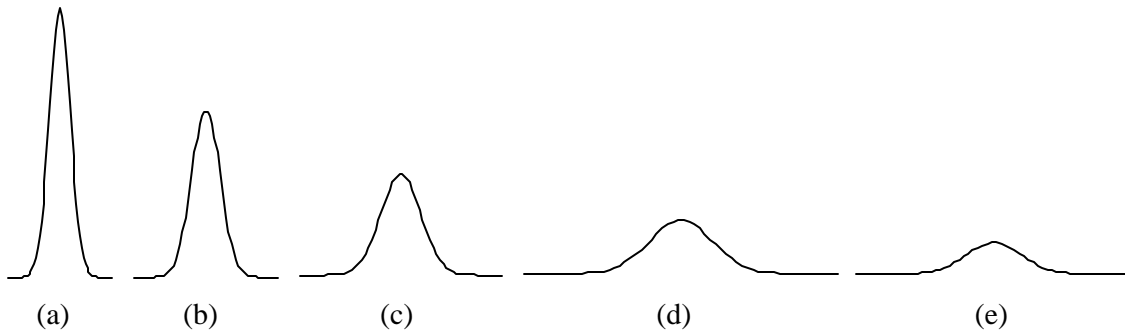


Figure 12-1: Wave functions with different uncertainties in position.

- ? Compare the result of all groups. Which one needed to include the largest number of momenta?

- ? the smallest number of momenta?

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? How is the uncertainty in position related to the uncertainty in momentum?

This exercise indicates that as one of the uncertainties increases the other decreases. This conclusion is built into the wave nature of matter. It does not depend on our measurement instruments. (We have not discussed measurement here — only creating wave functions.)

Heisenberg's Uncertainty Principle

The mutual dependence of the uncertainty in position and the uncertainty in momentum was first stated by Werner Heisenberg. His statement is known as *Heisenberg's Uncertainty Principle*. It says that the uncertainty in position and the uncertainty in momentum are closely related. If one decreases, the other increases by the same factor. Mathematically, the Uncertainty Principle says:

$$(\text{Uncertainty in position}) \times (\text{Uncertainty in momentum}) = \text{constant}$$

Or, if you like symbols for uncertainty in position (Δx) and uncertainty in momentum (Δp).

$$\Delta x \cdot \Delta p = \text{constant}$$

The constant is Planck's constant (h) divided by 2π :

This principle states that we can never know both the exact position and the exact momentum of an electron at the same time. The best we could do is that the uncertainties are related by this equation. We would get these results only with perfect measuring instruments. We can always do worse. For this reason, Heisenberg's Uncertainty Principle is usually stated as an inequality:

(Uncertainty in position) x (Uncertainty in momentum) is at least $\frac{h}{2\pi}$.

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi}$$

? For example, suppose we establish the position of an electron precisely to within a tenth of a nanometer (0.000000001 m). Using $\Delta x = 10^{-10}$ m, what would be the minimum uncertainty in the electron's momentum (Δp)? (The value of Planck's constant is $h = 6.63 \times 10^{-34}$ J·s)

? With this uncertainty in momentum, what would be the corresponding uncertainty in the *speed* of the electron? (The electron's *mass* is 9.11×10^{-31} kg.) Recall that momentum = mass x velocity.

? A reasonable speed for an electron might be around 10^6 m/s. Is the uncertainty in speed that you calculated significant when compared to this speed? Explain.

The Heisenberg Uncertainty Principle is applied to all types of matter. One of the “controversies” in the early Star Trek series involved these uncertainties. Some Trekkies claimed that the Uncertainty Principle means that transporters would not be feasible. The uncertainties meant that the transporter would never be able to measure the location of the matter in people well enough. So they could not be beamed elsewhere.

Maybe these uncertainties are so small that they are unimportant in the transporter. Suppose a gnat is flying around in the transporter room. When it flies over the transporter pad, we measure its position and momentum. Then try to transport it to a location near somebody's compost pile. We measure the gnat's position only fairly accurately. The uncertainty of its position is within one millimeter (10^{-3} m). According to Heisenberg's Uncertainty Principle, what would be the *minimum* uncertainty in the gnat's momentum (Δp)?

So, the gnat's uncertainty position and its momentum are rather small. We could conclude that the Uncertainty Principle is not a problem. But, the transporter does not “work” by beaming whole objects. Instead it measures the locations of all atoms (maybe even all protons, neutrons, and electrons). Then, it disassembles the object and reassembles it elsewhere. Look at the uncertainty calculation for an electron. How likely is it that the gnat can even be reassembled intact? Explain.

? How would the Uncertainty Principle affect our ability to transport objects even larger than gnats? (They will have many more atoms.)

The Star Trek writers worked around this problem with a component they called a "Heisenberg compensator." This device somehow allows precise measurement of both position and momentum. They admit that they don't know how such a device would work. However, they recognize that it is needed to overcome these limitations of quantum physics. Its failure also makes for interesting stories. It should be noted that the Star Trek writers have also been insightful and creative enough to invent "subspace." This spatial continuum is hidden within our own, familiar three-dimensional space. It allows the transporter to "beam" people and objects instantaneously from one site to another since, in subspace, travel is not restricted to speeds below the speed of light. Fortunately, fiction is not restricted by the concepts of physics.

Today we lack the technology to construct "Heisenberg compensators." But, we cannot rule out the possibility that something similar may one day be developed. Talk of transporters may sound fantastic, but some current research indicates that "quantum teleportation" (as it is known to scientists) may actually be possible with individual atoms. Researchers at IBM have shown that teleportation of one atom is theoretically possible, but only if the original is destroyed. One atom is far from a person (or even a gnat) and many problems are involved in scaling up. But, it is fun to think about it.

Perhaps the most important aspect of the Uncertainty Principle is its philosophical implications. It states that humans cannot know everything about an object. Even if we try to imagine the "perfect" measuring instruments, we cannot determine the exact position and exact momentum simultaneously. Such a device can never be built. Thus, the Uncertainty Principle places limitations on humankind's knowledge of everything.

This limitation on our knowledge is inherent in nature. We observe the spectra of atoms. To explain them we conclude that matter behaves as waves. If this description is effective, it must describe individual electrons. This description leads to the Heisenberg Uncertainty Principle. Then, we find we cannot know everything precisely, even if we had perfect measuring instruments.

This lack of ability to know is a major departure from the physics of Newton. In 1787 Pierre Simon LaPlace, a mathematician, considered Newton's Laws carefully. If he knew the initial position and velocity, and the forces on an object, he could measure these variables for every object in the universe. Then suppose he could calculate really fast. He could know all of the future because he knows Newton's Laws. The only thing stopping him is good measurements and slow computers.

Quantum mechanics (particularly the Heisenberg Uncertainty Principle) says, “not true.” Even with perfect measuring instruments we cannot know enough. The nature of matter, not of measurement limits our knowledge.

Homework

12-1. Suppose we determine the position of an electron to within 10 nanometers (10×10^{-9} m). What will be the best possible measurement of its momentum?

12-2a. Suppose you are able to determine the speed of a 100 kg pole vaulter to within .04 m/s. Within what accuracy would you be able to determine her position?

12-2b. Based on your answer above, do you think that we need to worry about the Uncertainty Principle in measurements that we make in our everyday lives? Explain your answer.

Nanotechnology is the science of building tiny machines by constructing them atom-by-atom. This construction concept is rapidly generating interest among scientists and engineers. Claims and promises are being made that this technology will revolutionize our lives. However, we need to think about what happens when the “machines” become extremely small. Perhaps the Uncertainty Principle will make nanotechnology difficult or even impossible to use. Explain why nanotechnologists may need to worry about the limits imposed by the Uncertainty Principle.